

Galaxy Dynamics as testbeds for Dark Matter and Galaxy Evolution

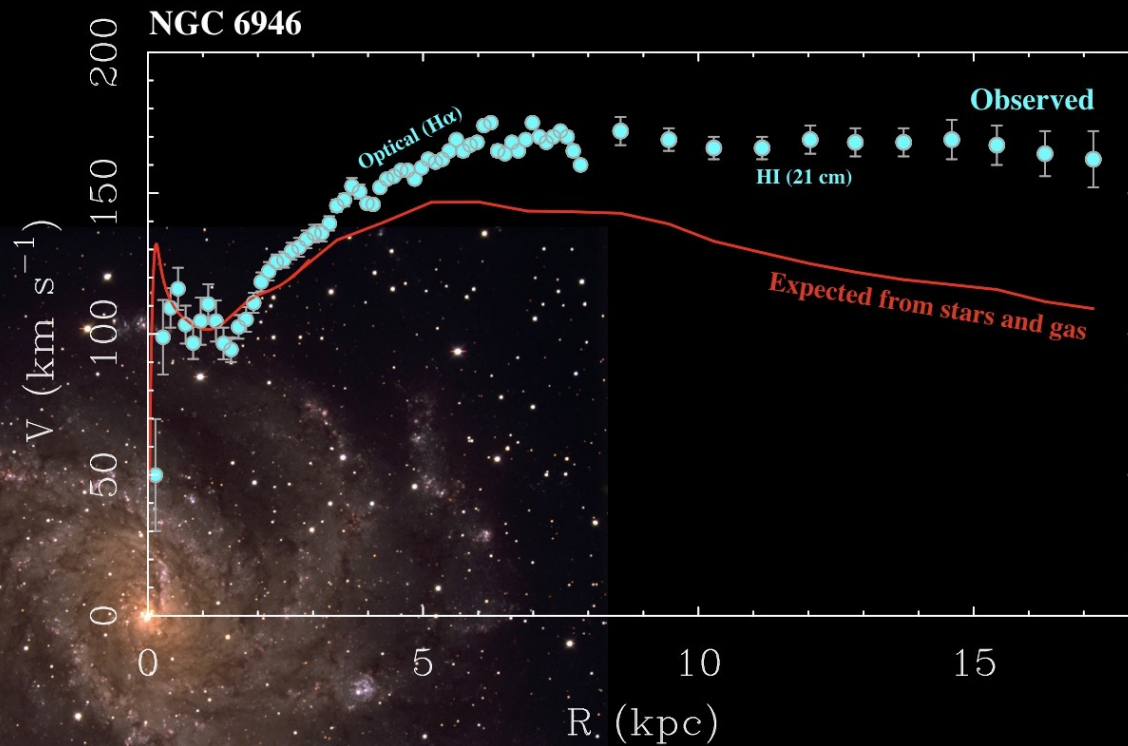
Federico Lelli

INAF – Arcetri Astrophysical Observatory



In collaboration with the SPARC team: Stacy McGaugh,
James Schombert, Harry Desmond, Pengfei Li, Marcel Pawlowski.

Why Studying Galaxy Dynamics?

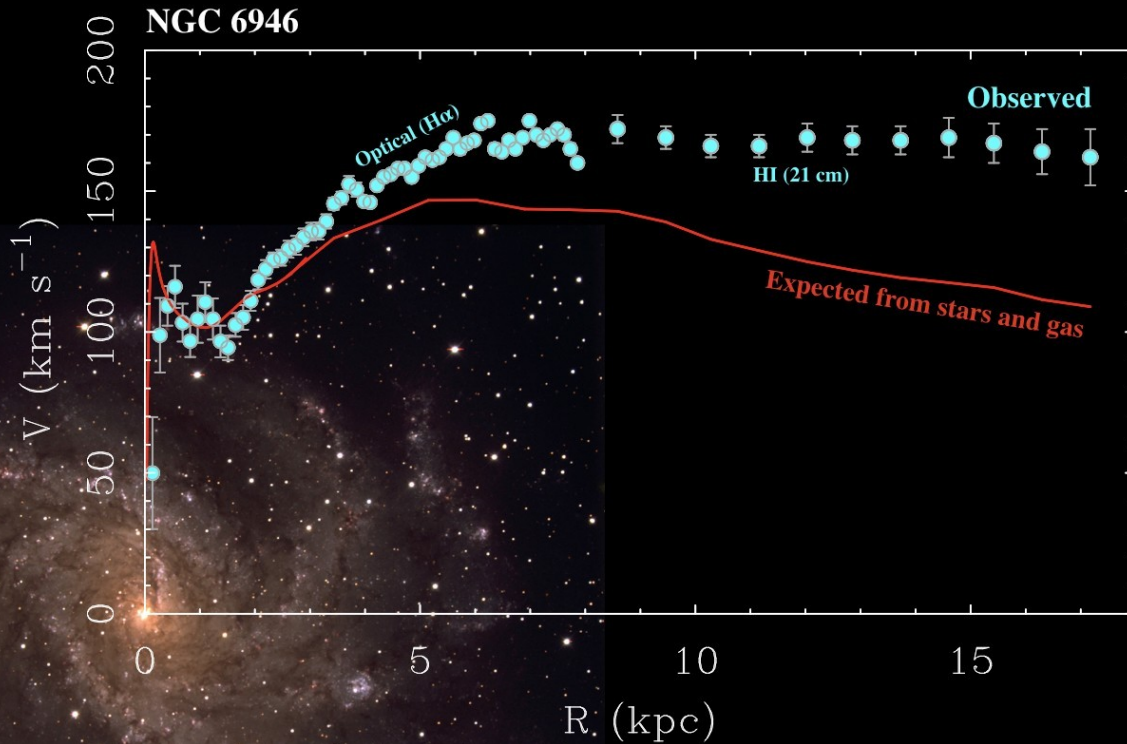


Data from SPARC
(Lelli et al. 2016)

Evidence of “missing mass”

- Dark matter halos → Cosmology
- Alternatives to particle dark matter (e.g. MOND, modified gravity, etc.)

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Galaxy Formation & Evolution

- Dynamical scaling laws (e.g. Tully-Fisher)
- Angular momentum \leftrightarrow Galaxy Morphology
- Disk Stability \leftrightarrow Star Formation
- Gas Turbulence \leftrightarrow Stellar Feedback
- Non-circular motions (bars, inflows, outflows)

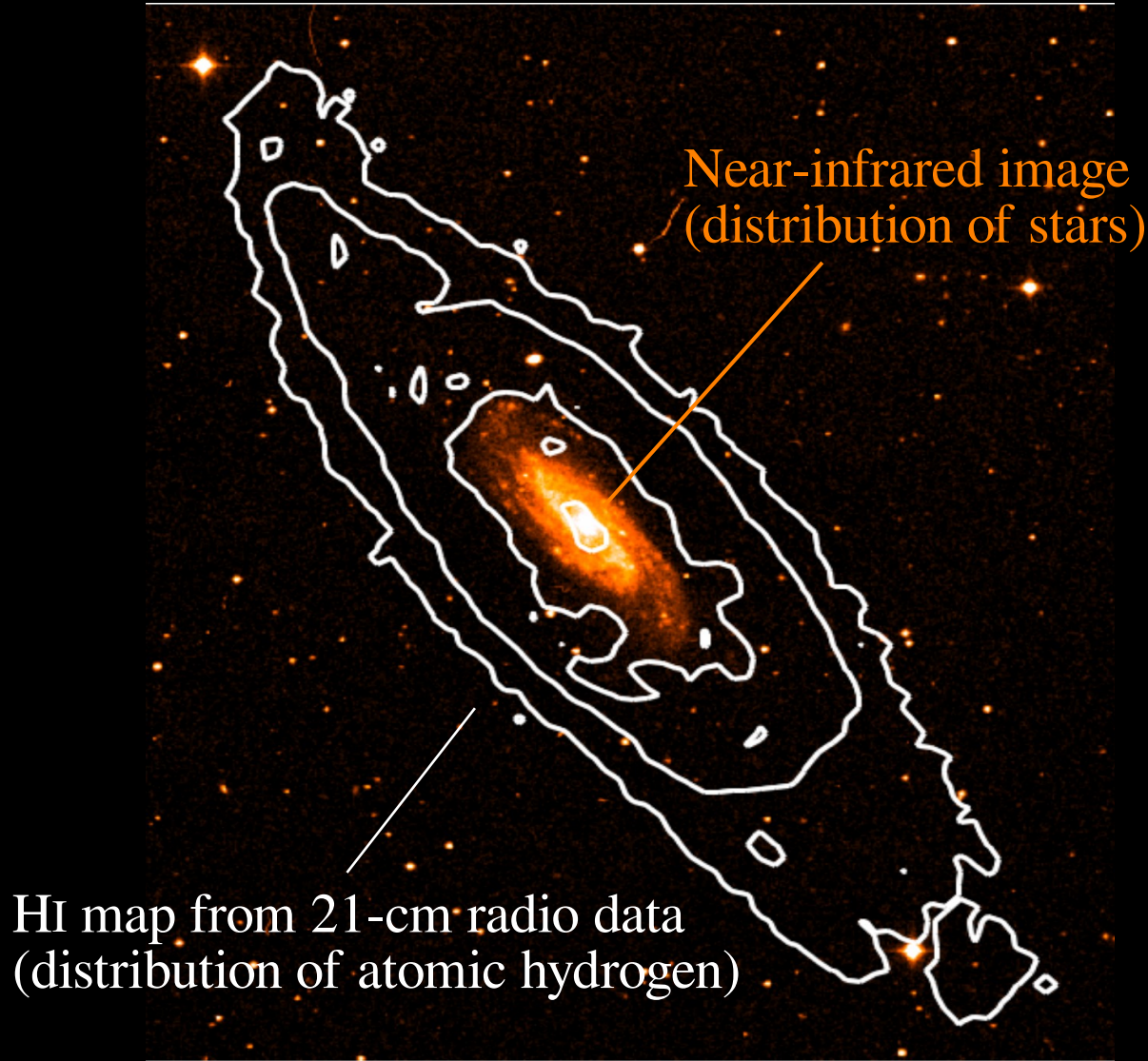
Outline:

1. Intro: Galaxy Rotation Curves
2. The SPARC project
3. Empirical Laws of Galactic Rotation

1. Intro: Rotation Curves

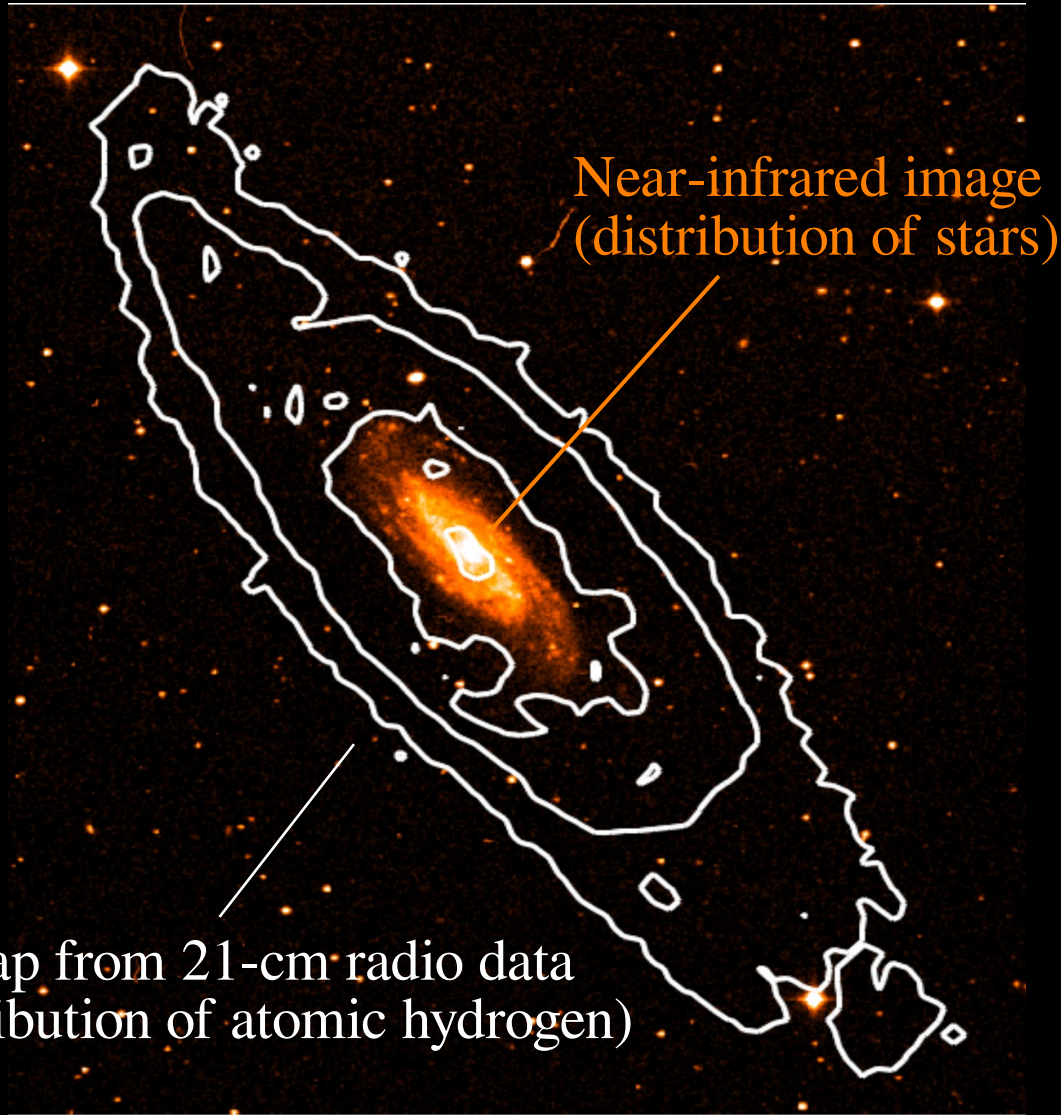
How do we measure rotation curves?

Distribution of baryons (gas & stars)

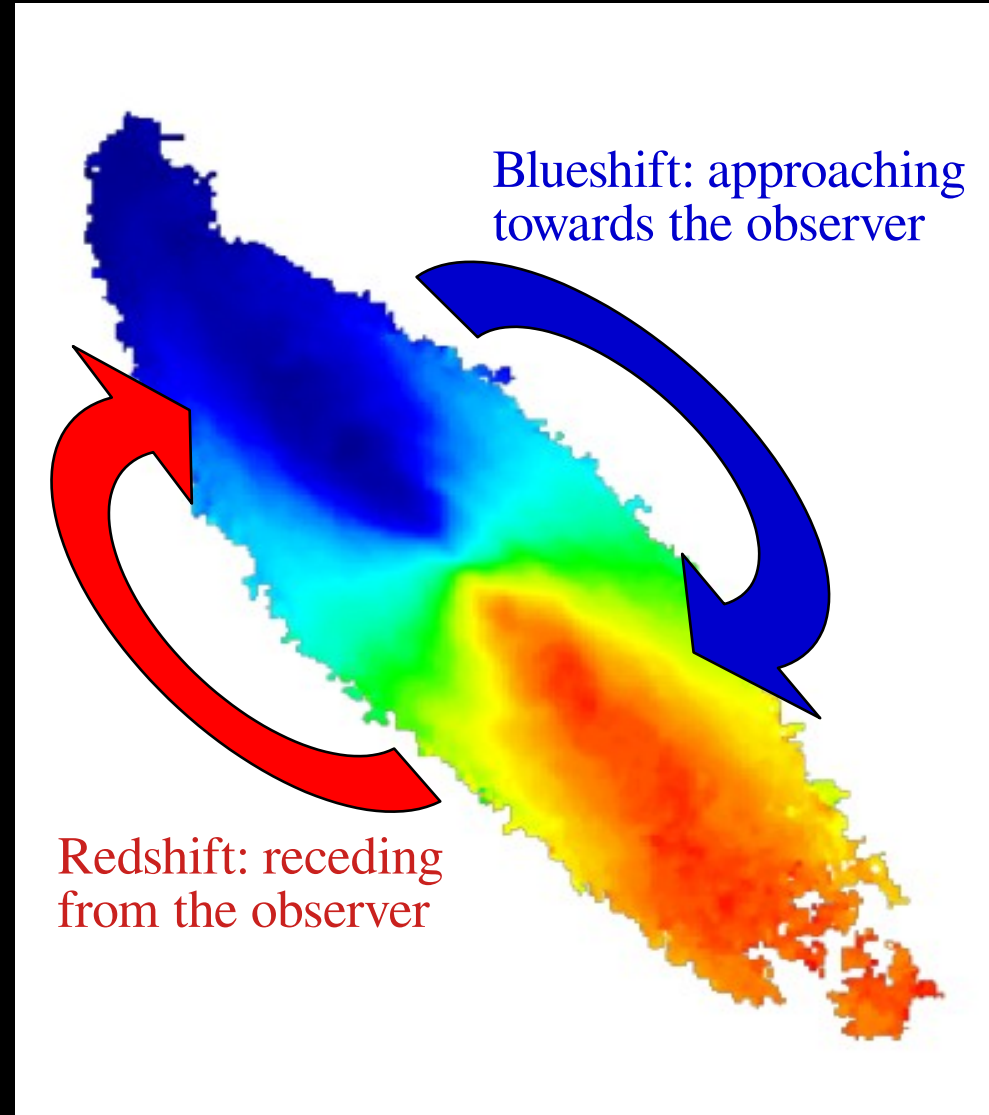


How do we measure rotation curves?

Distribution of baryons (gas & stars)



Gas Velocity along the Line of Sight

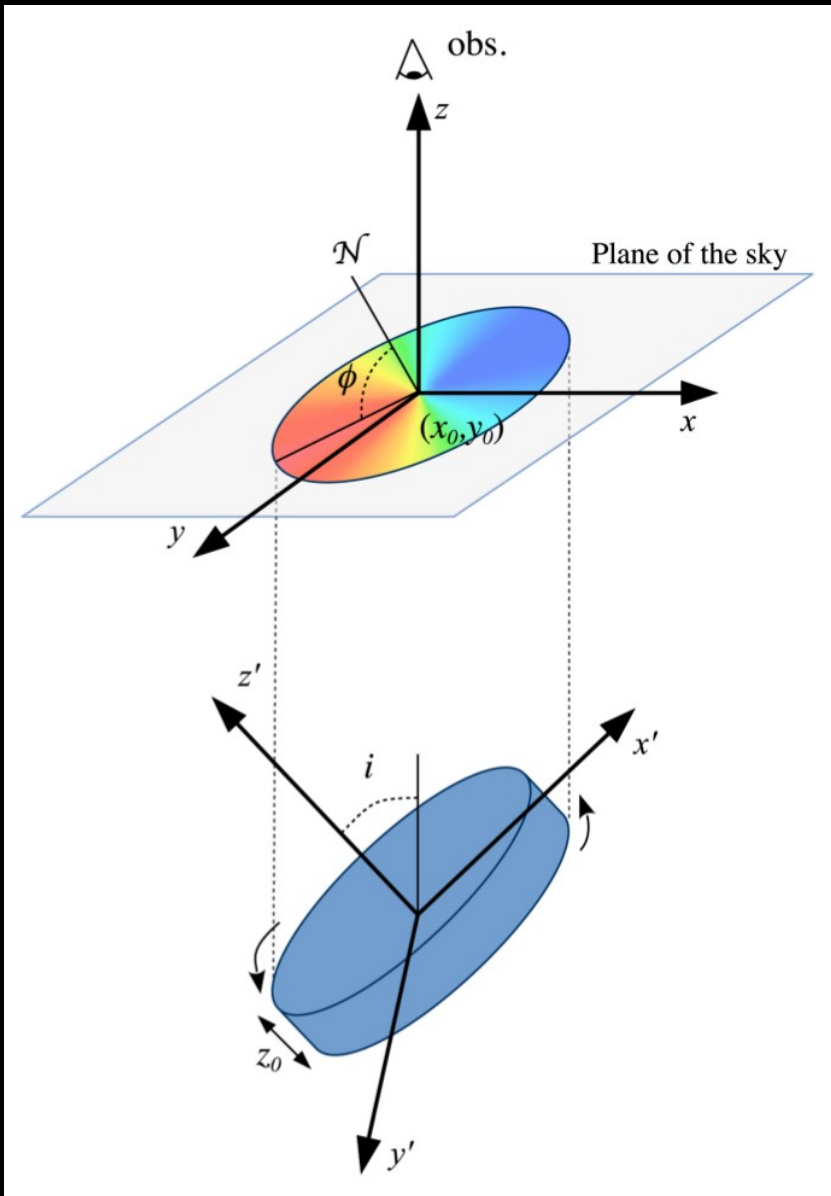


Deprojection from sky-plane to galaxy-plane

For a thin disk with circular orbits:

$$V_{LoS}(x, y) = V_{sys} + V_{rot}(R) \sin(i) \cos(\phi)$$

$$\cos(\phi) = f(x_0, y_0, PA) \rightarrow \text{Disk Geometric Parameters}$$

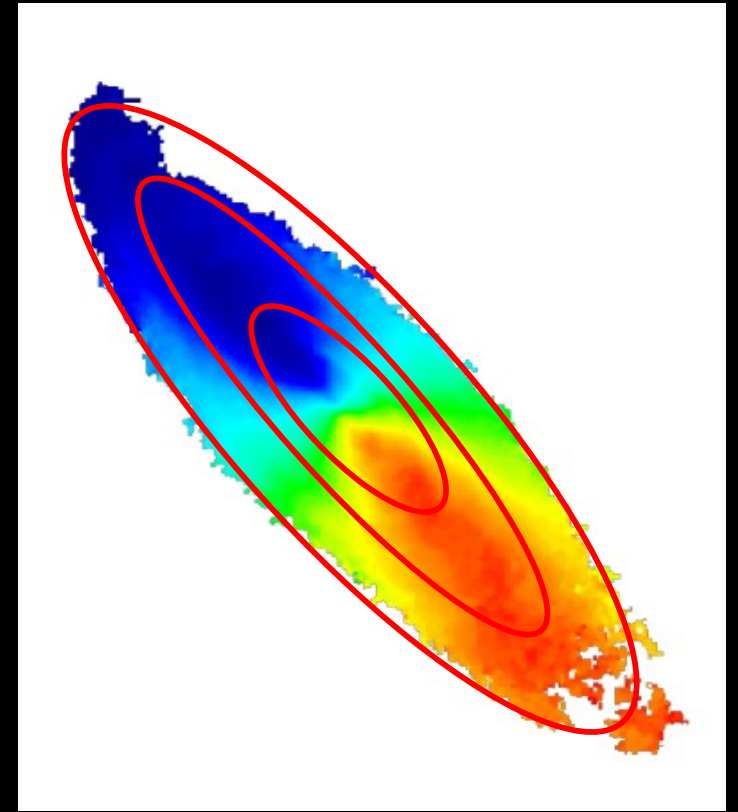
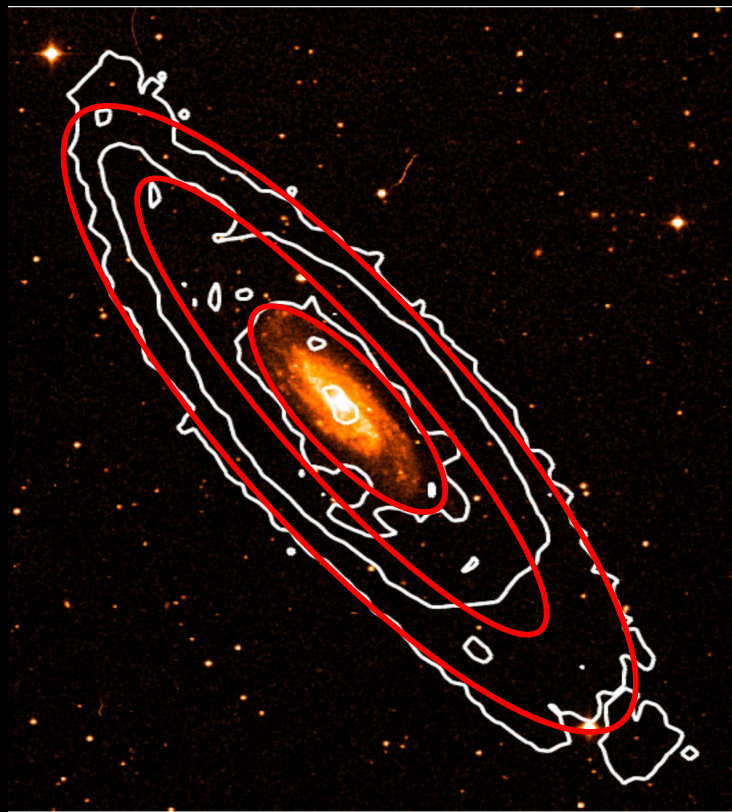
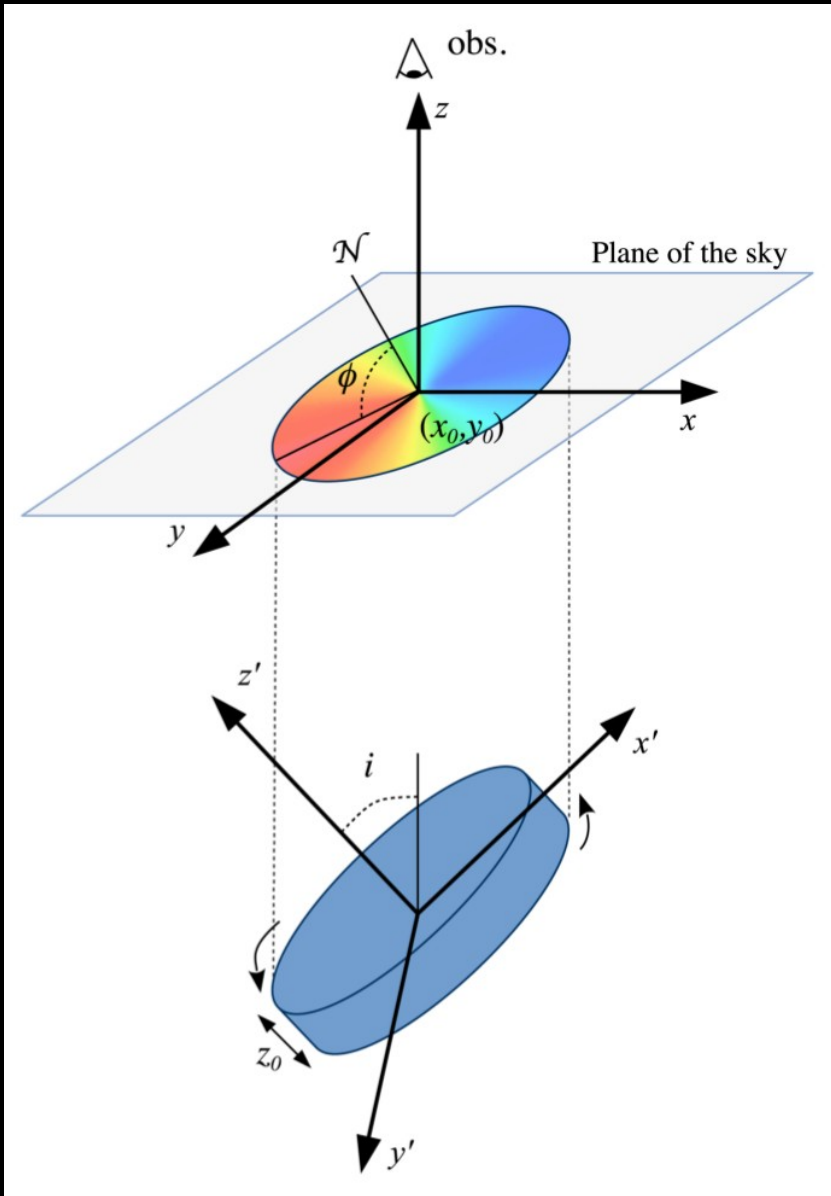


Deprojection from sky-plane to galaxy-plane

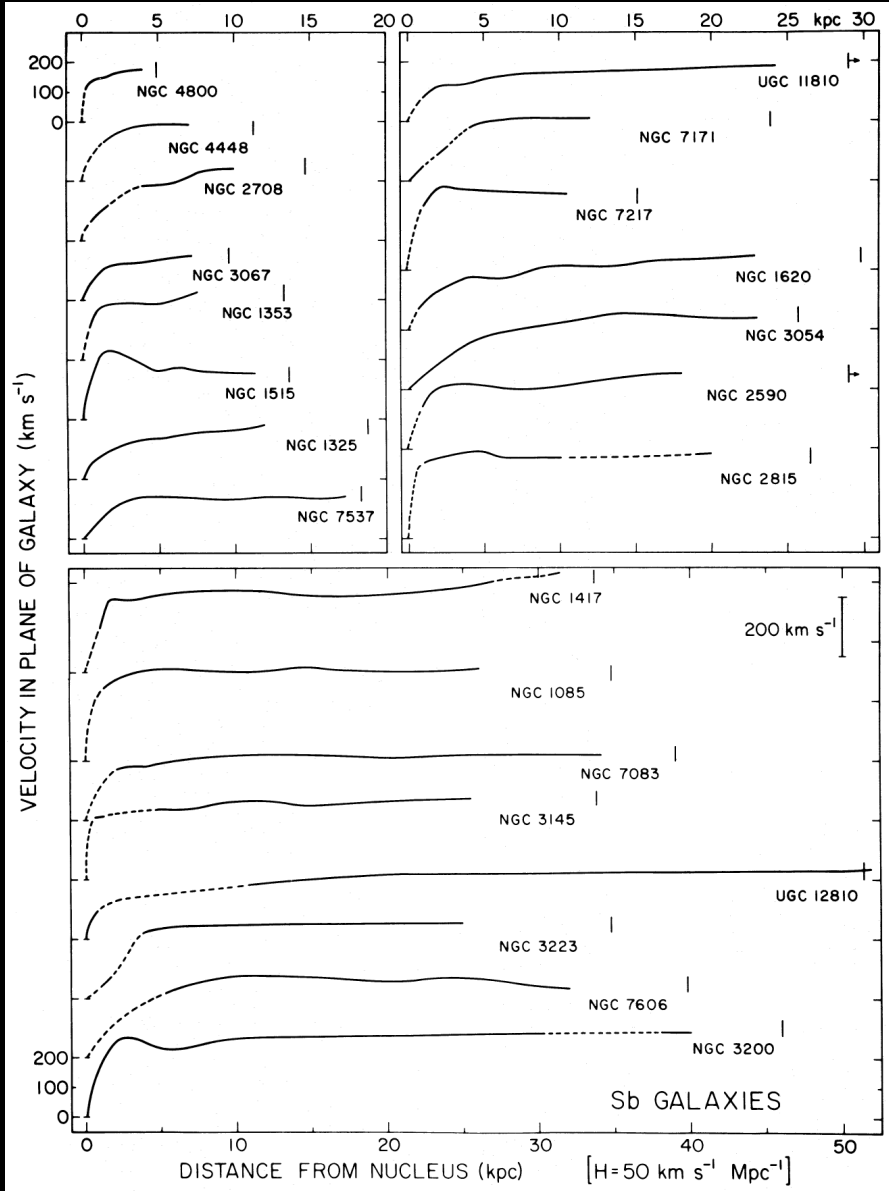
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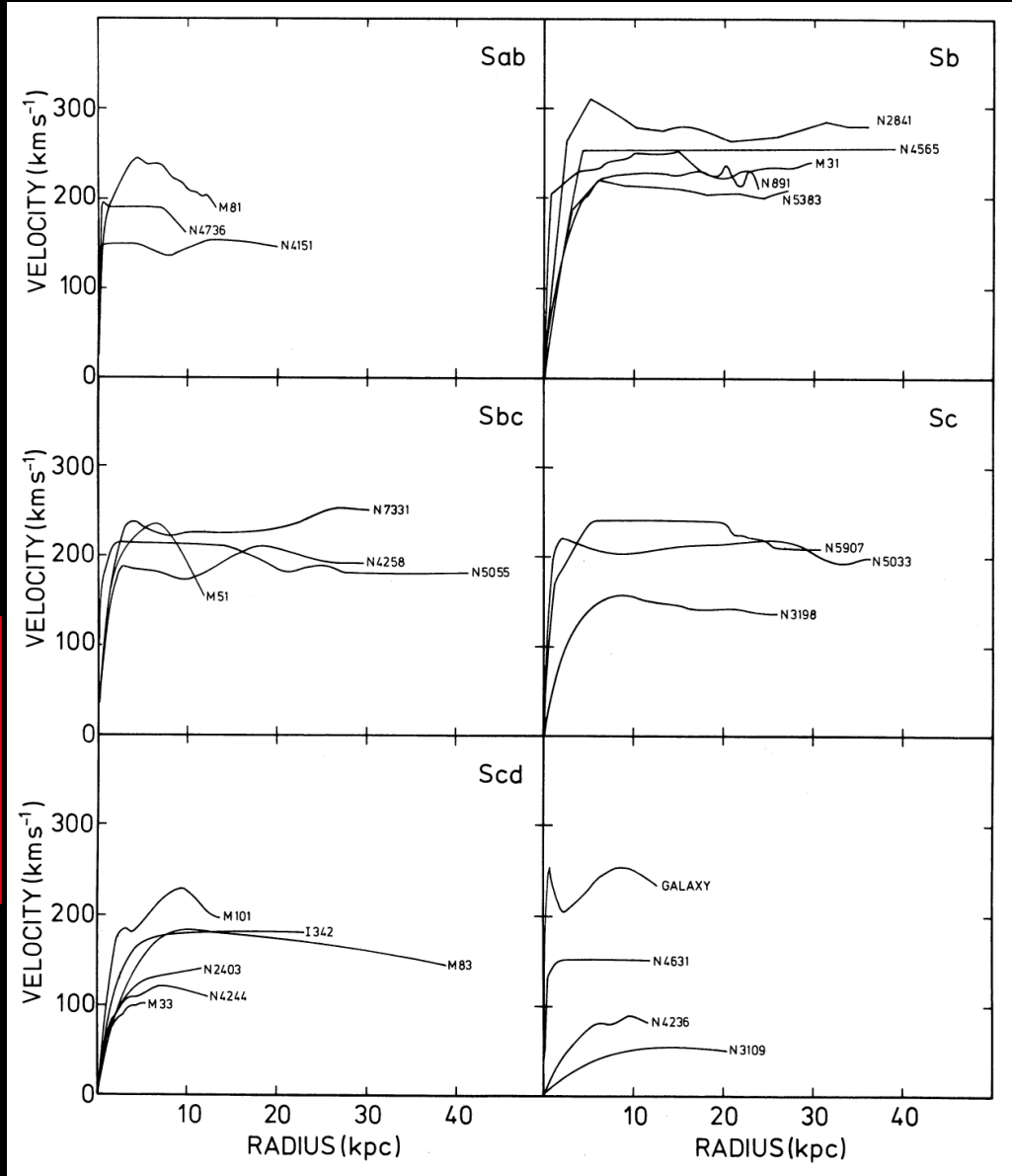
Rotation Curves become Flat at Large Radii



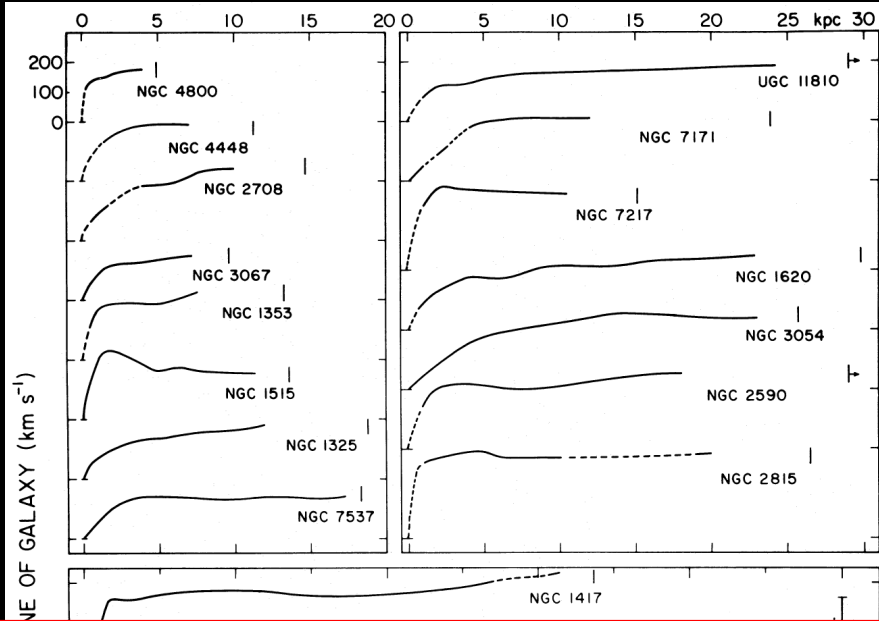
Vera Rubin
(1978, 1982)
Optical Spectroscopy
of Ionized Gas ($H\alpha$)



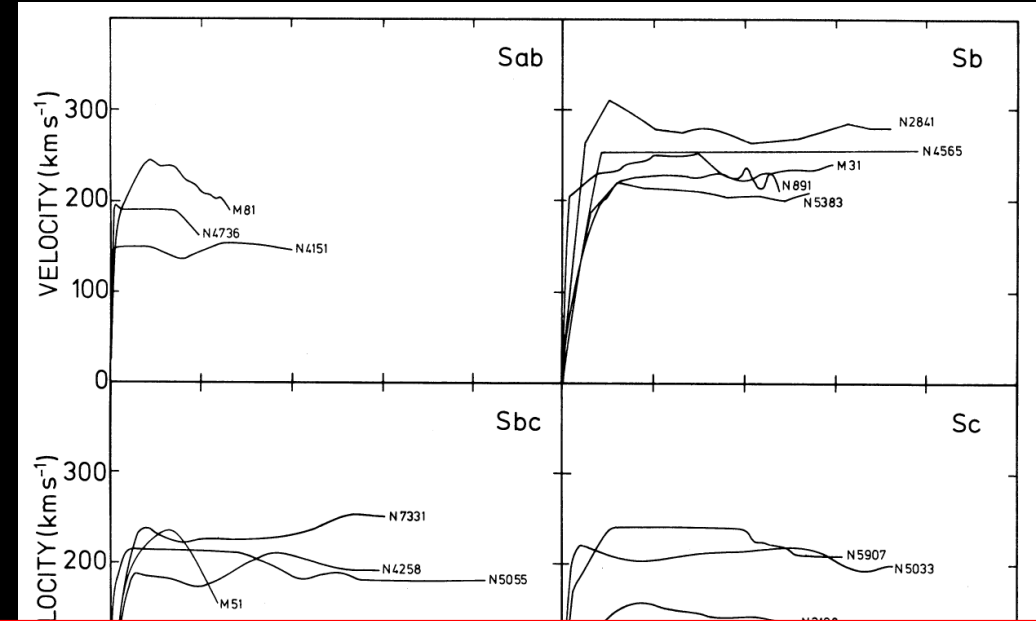
Albert Bosma
(1978, 1982)
Radio Interferometry
of Atomic Gas (HI)



Rotation Curves become Flat at Large Radii



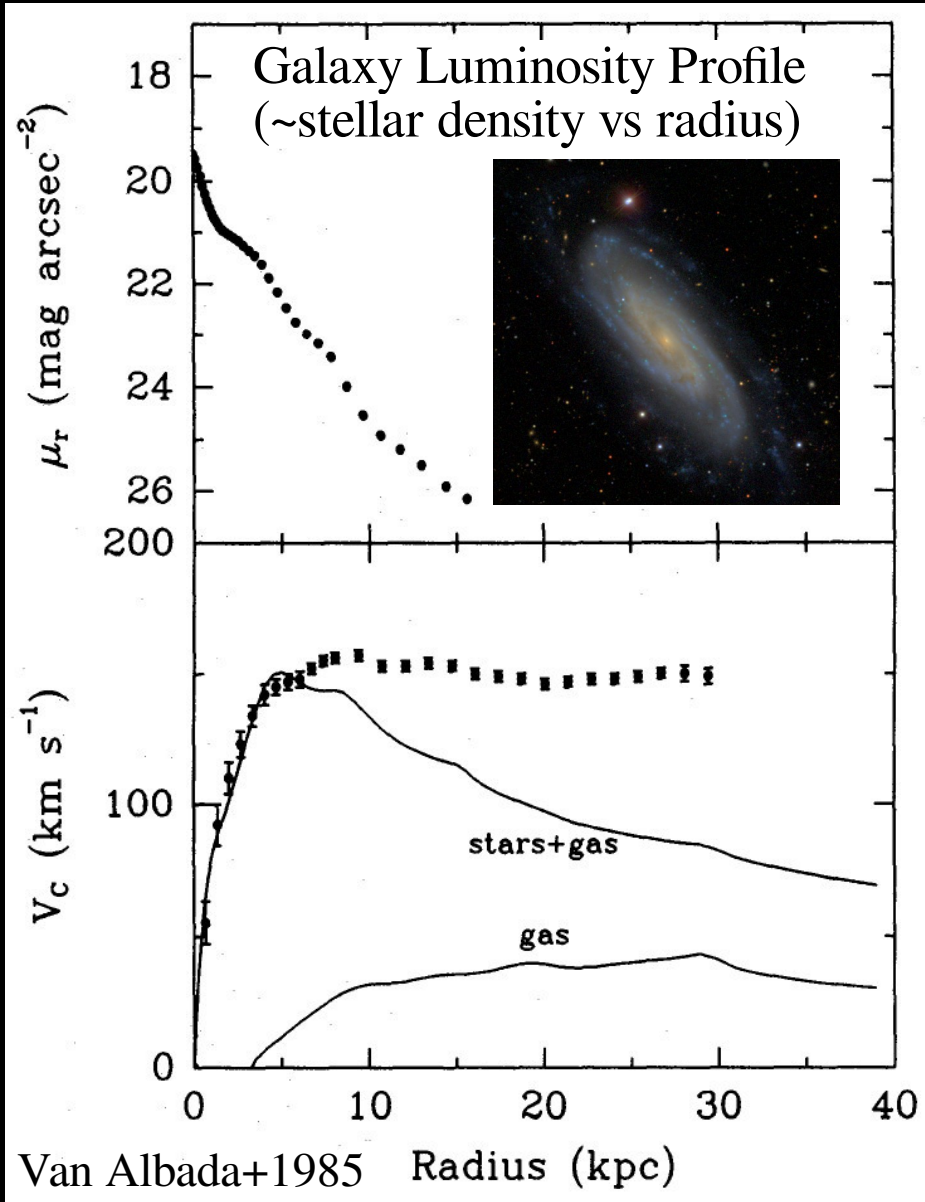
Vera Rubin
(1978, 1982)
Optical Spectroscopy
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Flat rotation curves are only the beginning of the story...

There is much more to learn from the relation between the shapes of rotation curves and the baryon distribution!

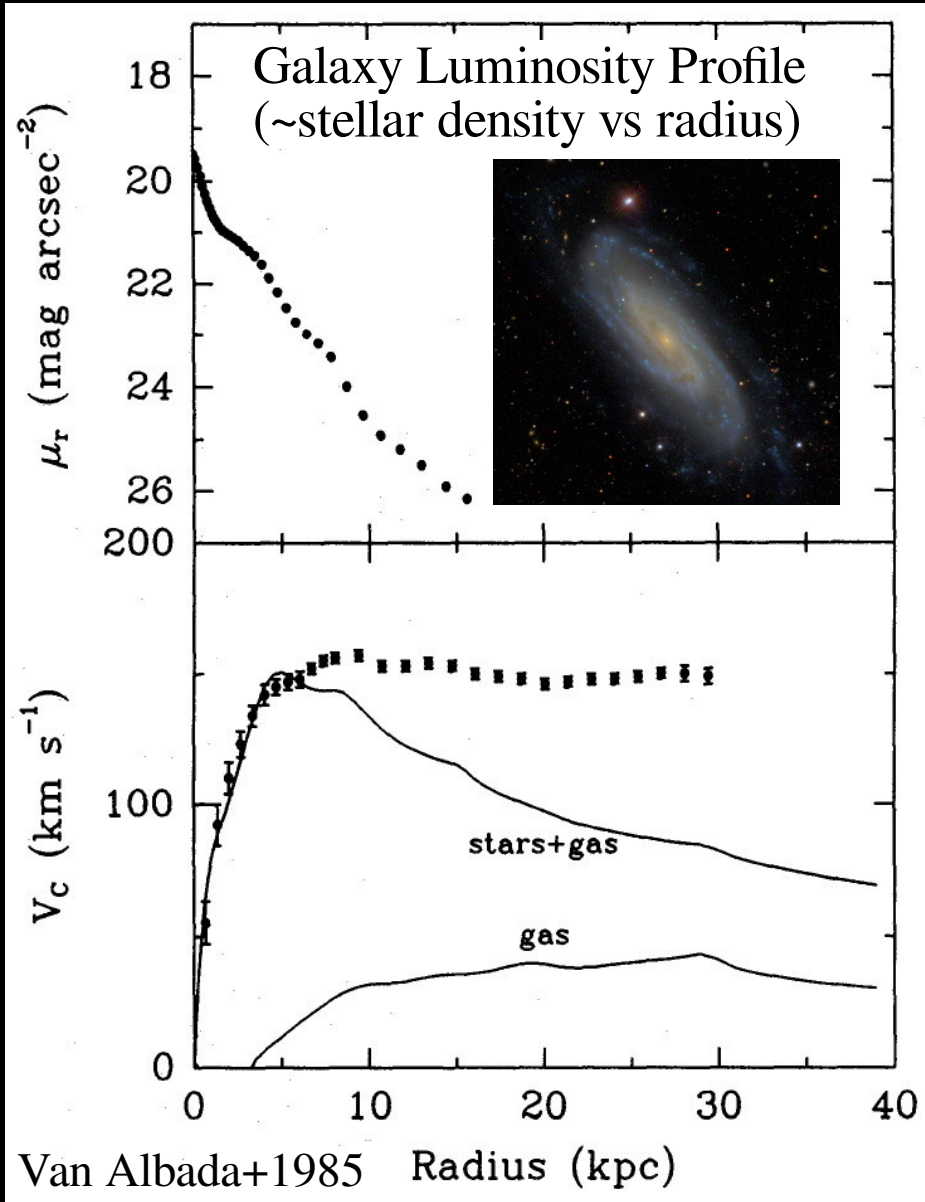
Building a Newtonian Mass Model



- Solve (numerically) Poisson's equation in cylindrical coordinates for each component ($i = \text{stars, gas}$):

$$\nabla^2 \Phi_i(R, z) = 4\pi G \rho_i(R, z)$$

Building a Newtonian Mass Model



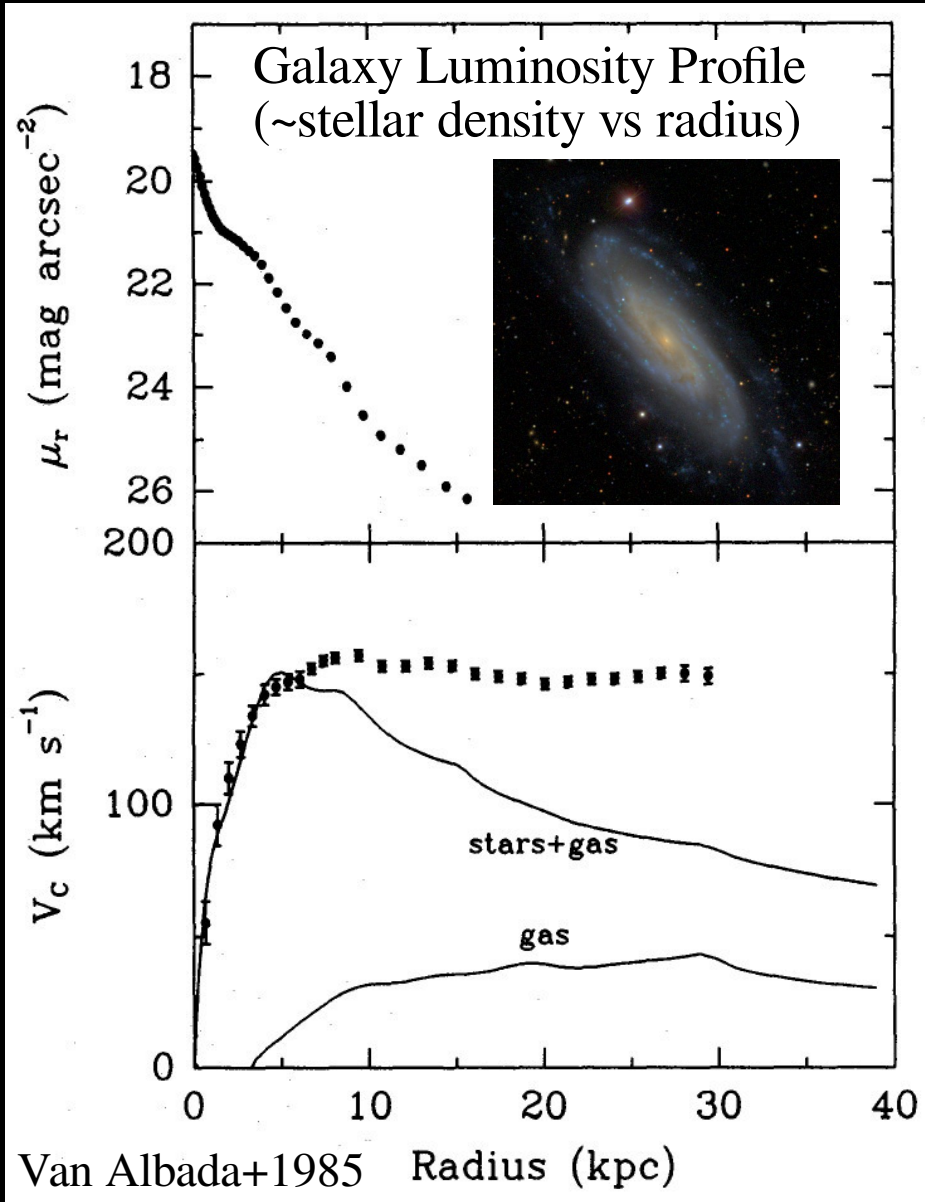
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- Find expected circular velocity in disk mid-plane:

$$\frac{V_i^2(R, z=0)}{R} = -\frac{\partial \Phi_i(R, z=0)}{\partial R}$$

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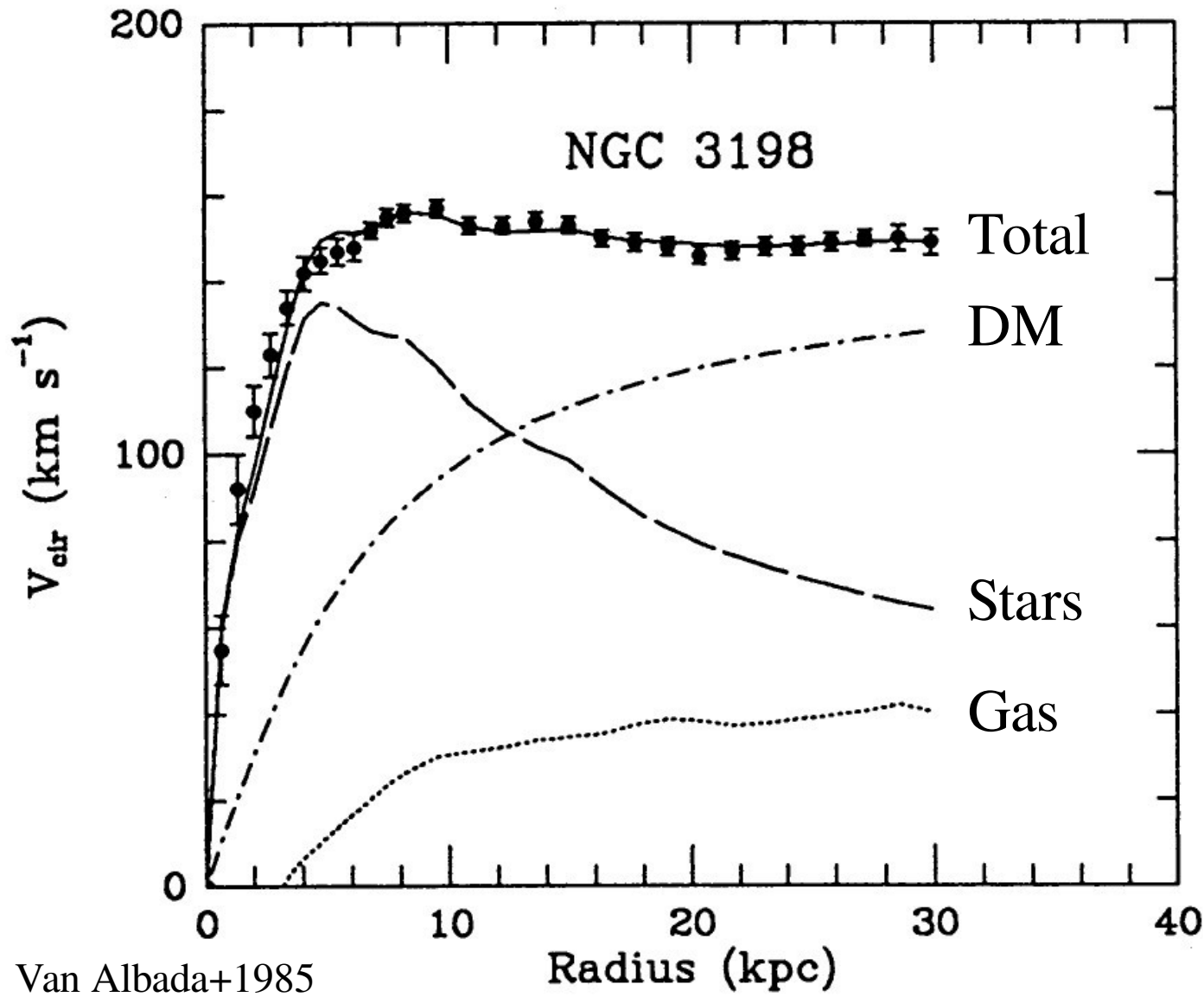
- Sum the gravitational fields ($g_i = V_i^2/R$):

$$V_b^2(R) = Y_s V_s^2(R) + Y_g V_g^2(R)$$

$Y_s = M_s/L$ estimated from stellar population models

$Y_g =$ known for HI from atomic physics (spin-flip) + small corrections for H₂, He, heavier elements

Adding a Dark Matter Halo



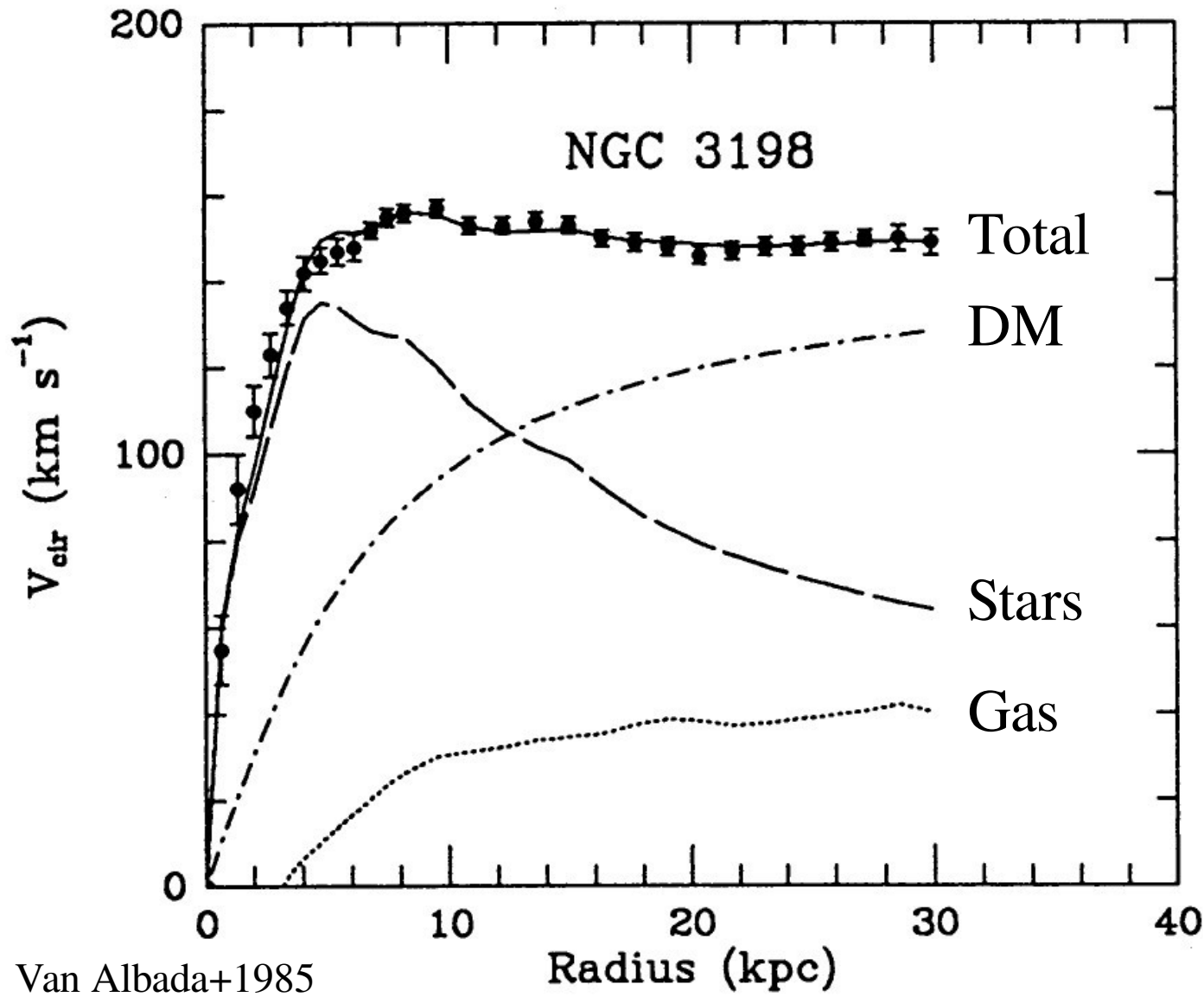
- Assume spherical DM halo profile:

$$\rho_{DM} = \rho(r; \rho_0, r_s)$$

- Add it together with the baryons:

$$V_b^2 = Y_s V_s^2 + Y_g V_g^2 + V_{DM}^2(\rho_0, r_s)$$

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For spiral galaxies like the Milky Way, baryons dominate in the inner parts while DM is needed in the outer regions → **the sum of the two gives the flat part!**

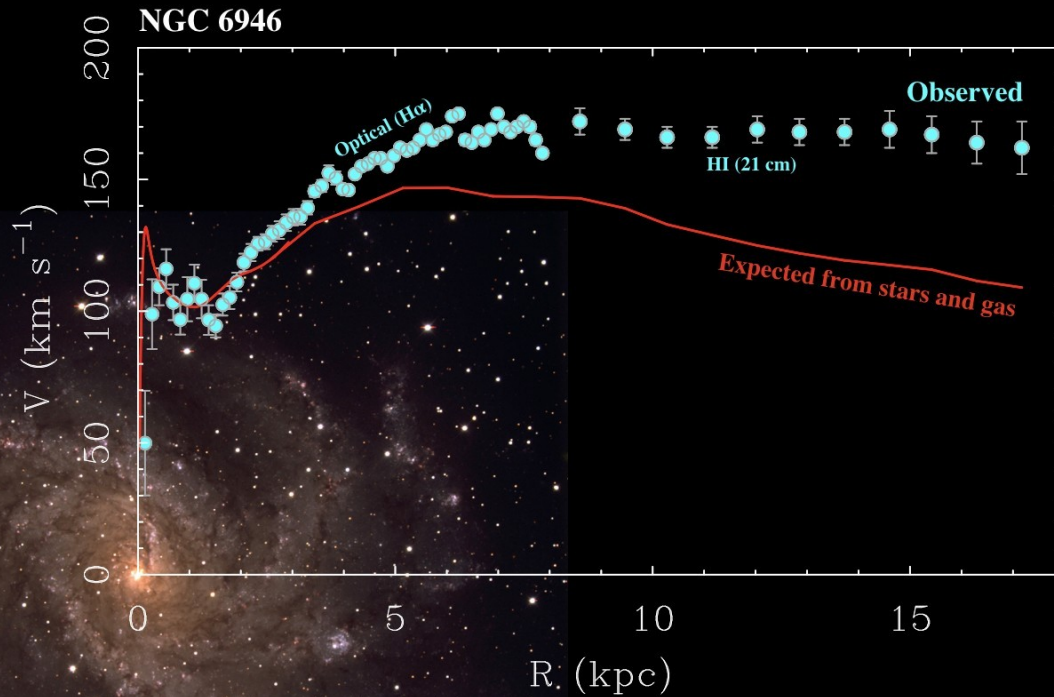
Why are rotation curves flat? Unclear!

This is called “disk-halo conspiracy”

(van Albada & Sancisi 1986)

2. The SPARC project

Database for 175 Disk Galaxies (spirals & dIrr)



- **HI rotation curves from the literature** (> 40 papers or PhD thesis over 40 years)
- **$H\alpha$ rotation curves for 30% of sample** (long-slit, IFU, and Fabry-Perot data)
- **Spitzer Photometry at $3.6 \mu\text{m}$**
Best tracer of the stellar mass distribution

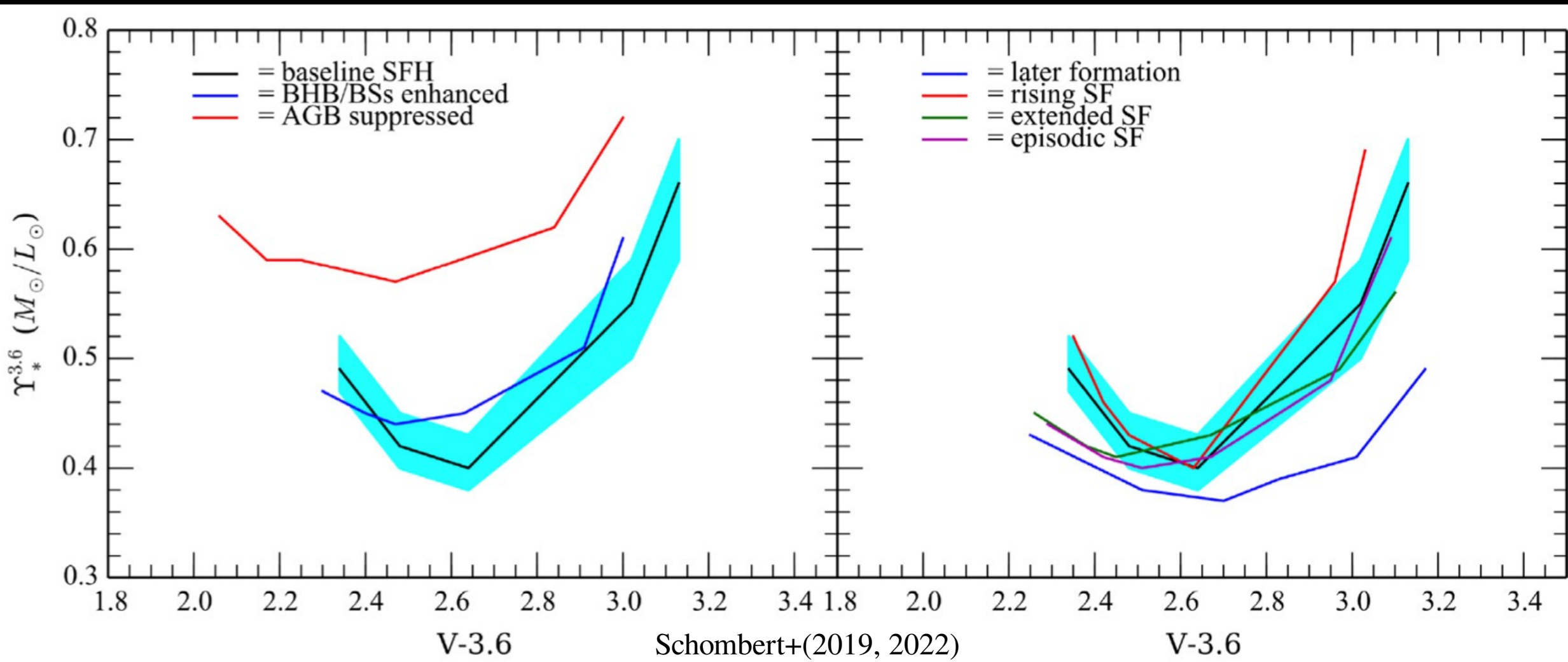
Public data: astroweb.cwru.edu/SPARC

Lelli, McGaugh, Schombert (2016)

Complex Stellar Pop. Models at 3.6 μm

Changing the stellar evolution model:

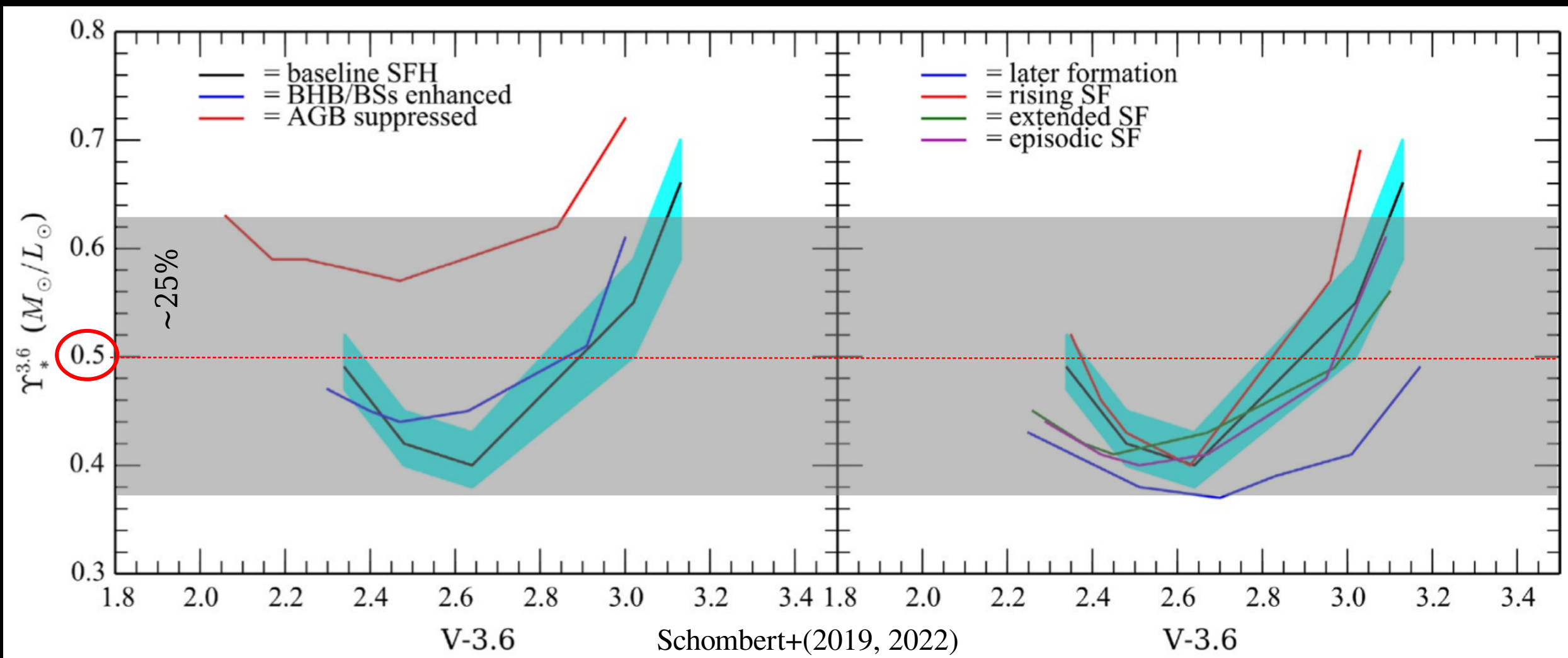
Changing the star-formation history:



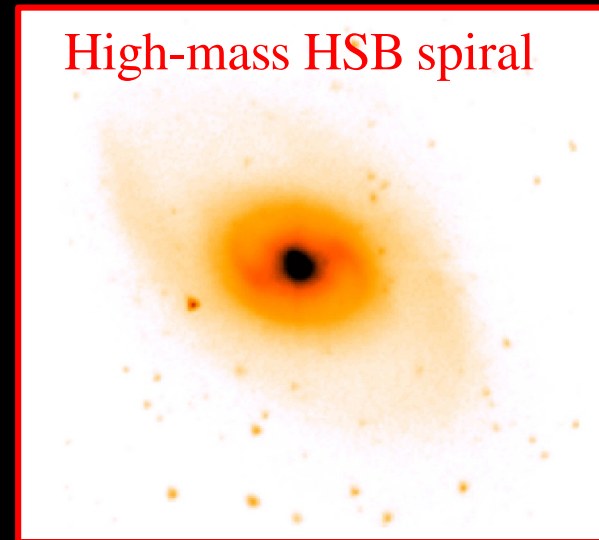
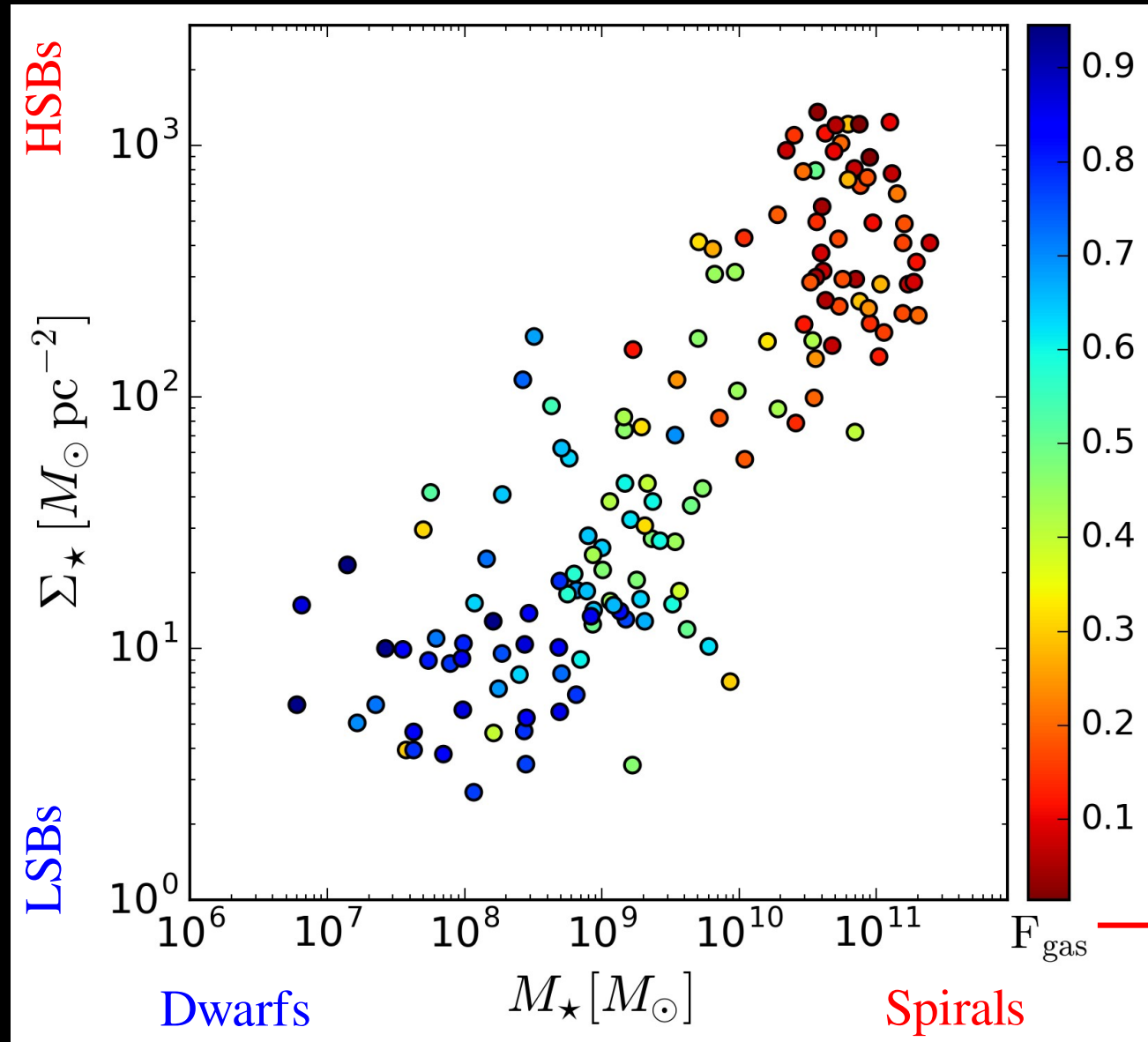
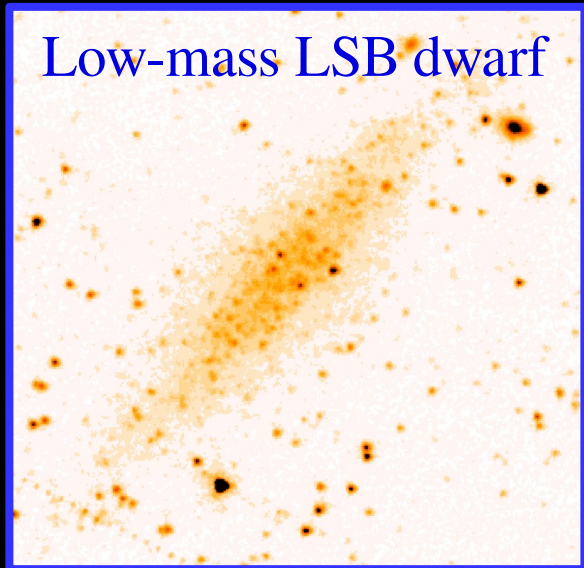
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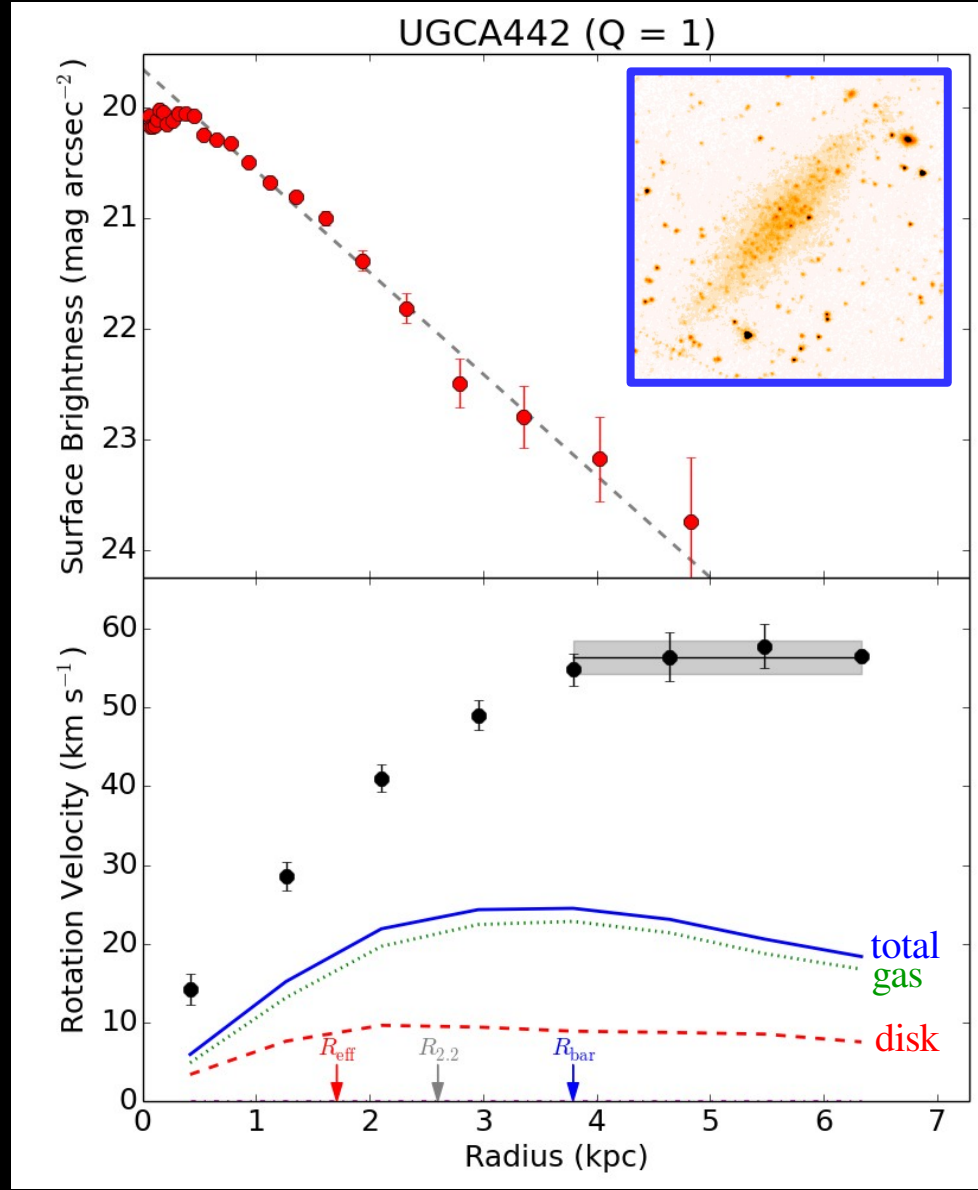
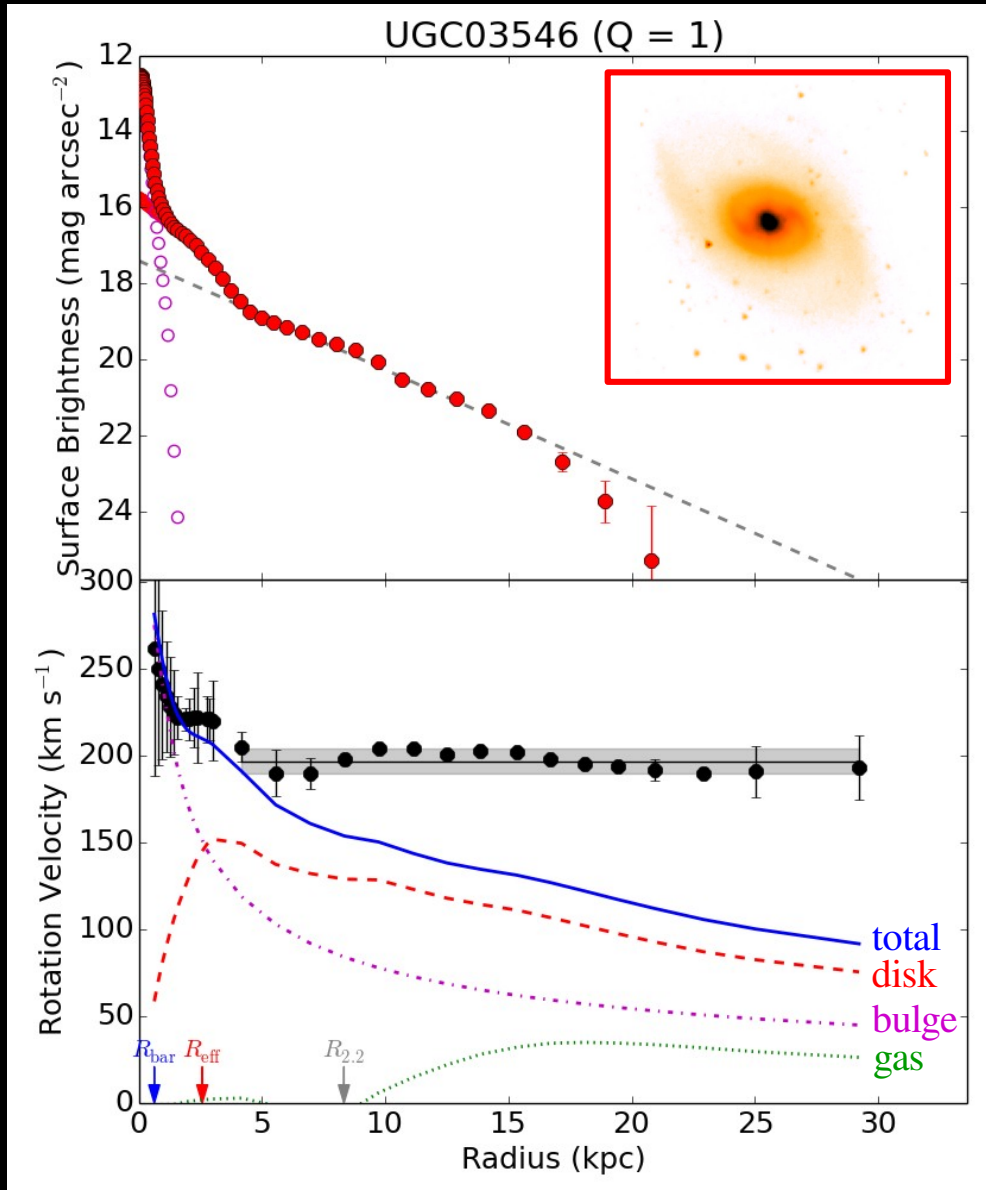
Broad Range of Galaxy Properties



$M_{\text{gas}} / M_{\text{bar}}$

High-Mass HSB Galaxy

Low-Mass LSB Galaxy

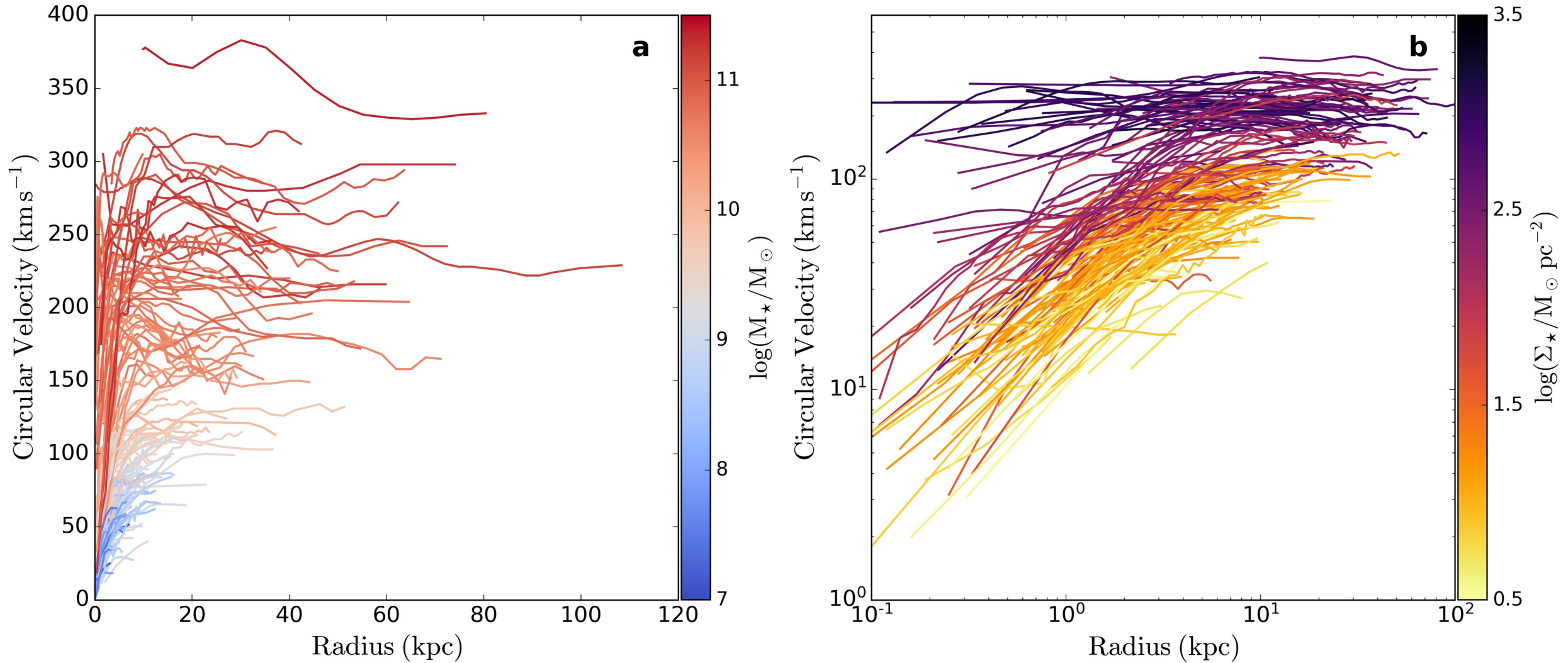


For disks:
 $\Upsilon_* = 0.5 M_{\odot}/L_{\odot}$

For bulges:
 $\Upsilon_* = 0.7 M_{\odot}/L_{\odot}$

Lelli+(2016)

Rotation Curves Overview



Lelli+(2022, Nature Astronomy)

SPARC

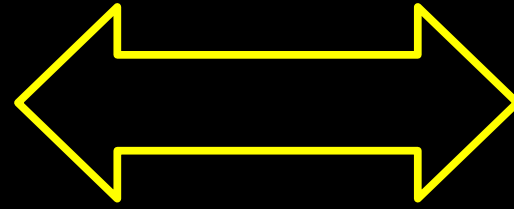
Spitzer Photometry & Accurate Rotation Curves

1. Basic Data & Structural Relations: Lelli+2016a, AJ
2. Baryonic Tully-Fisher Relation (I): Lelli+2016b, ApJL
3. Central Surface Density Relation: Lelli+2016c, ApJL
4. Radial Acceleration Relation (I): McGaugh+2016, PRL
5. Radial Acceleration Relation (II): Lelli+2017a, ApJ
6. The Cusp-vs-Core Problem: Katz+2017, MNRAS
7. Testing Emergent Gravity: Lelli+2017b, MNRAS
8. Radial Acceleration Relation (III): Li+2018, A&A
9. Maximum-Disk Models: Starkman+2018, MNRAS
10. Missing Baryons: Katz+2018, MNRAS
11. Scaling Relations for DM Halos: Li+2019, MNRAS
12. Halo Mass - Velocity Relations: Katz+2019, MNRAS
13. Stellar M/L ratios (I): Schombert+2019, MNRAS
14. Residuals in BTFR: Desmond+2019, MNRAS
15. Tully-Fisher Relation (II): Lelli+2019, MNRAS
16. The Halo Mass Function: Li+2019, ApJL
17. Catalog of DM Halo Fits: Li+2020, ApJS
18. H_0 from Tully-Fisher Relation: Schombert+2020, AJ
19. Testing the SEP in MOND (I): Chae+2020
20. Testing the SEP in MOND (II): Chae+2021
21. Cautionary Tale in Bayesian Fits: Li+2021, A&A
22. Tully-Fisher Relation in the LG: McGaugh+2021, AJ
23. Adiabatic Halo Compression: Li+2022, ApJ
24. Stellar M/L ratios (II): Schombert+2022, AJ

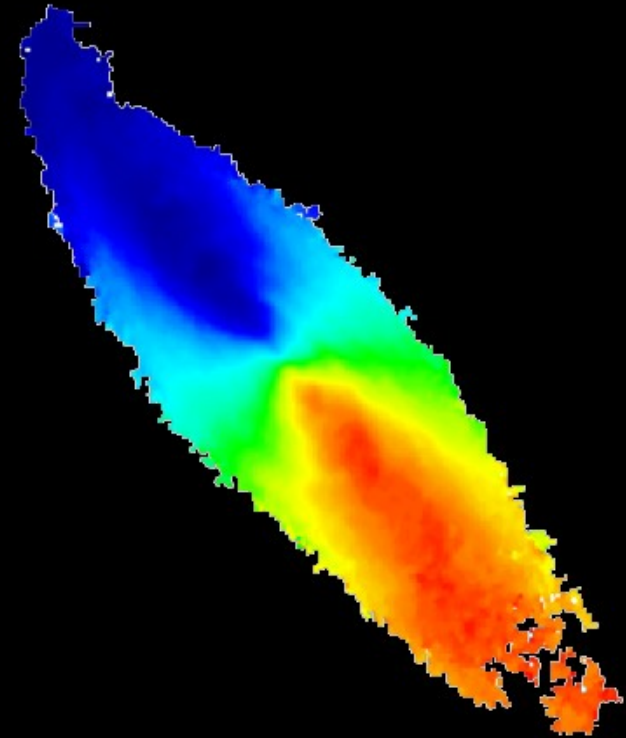
3. Empirical Laws of Galactic Rotation

Dynamical Law: Correlation with small scatter

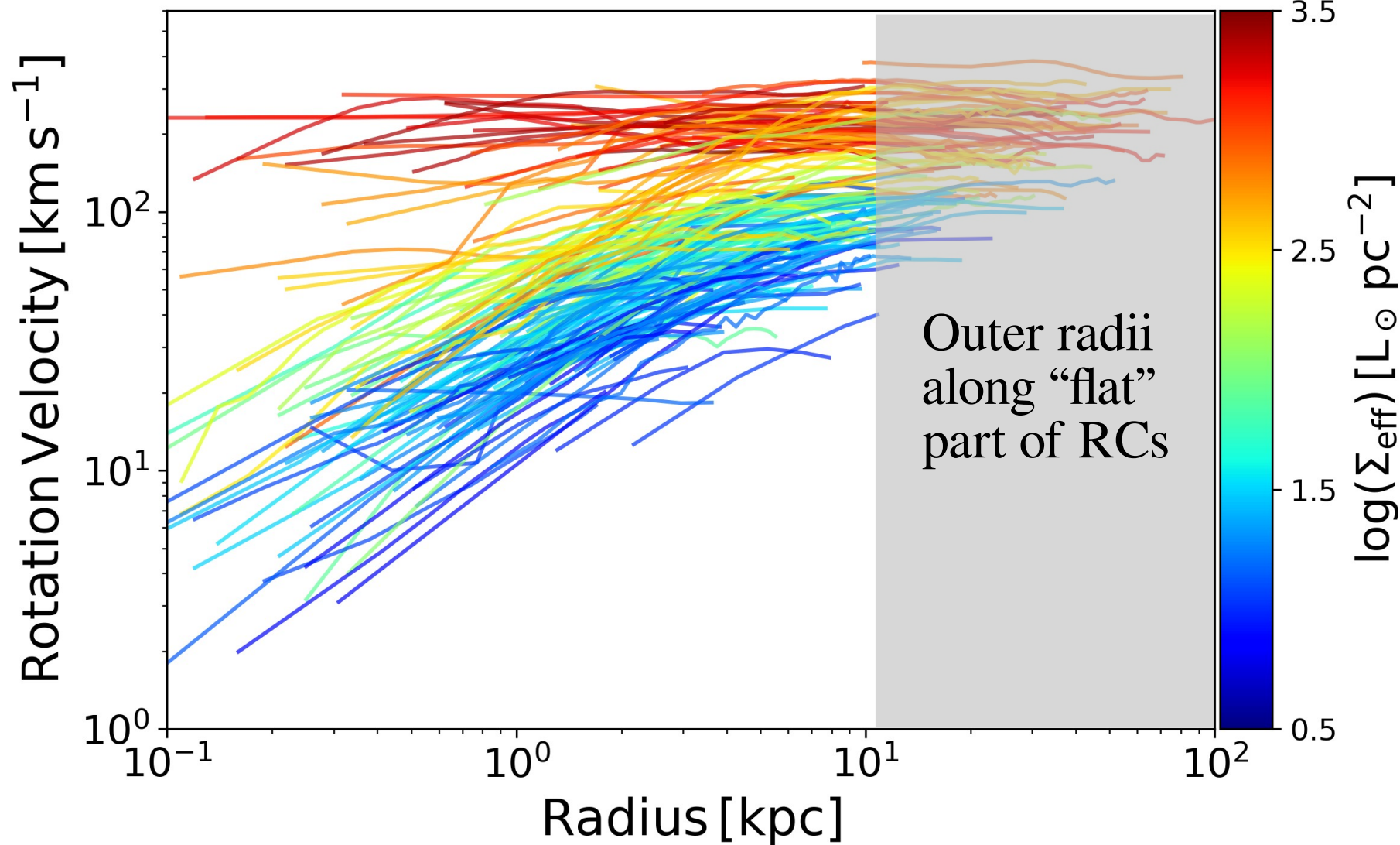
Baryonic quantity
(gas and stars)



Dynamical quantity
(from gas kinematics)



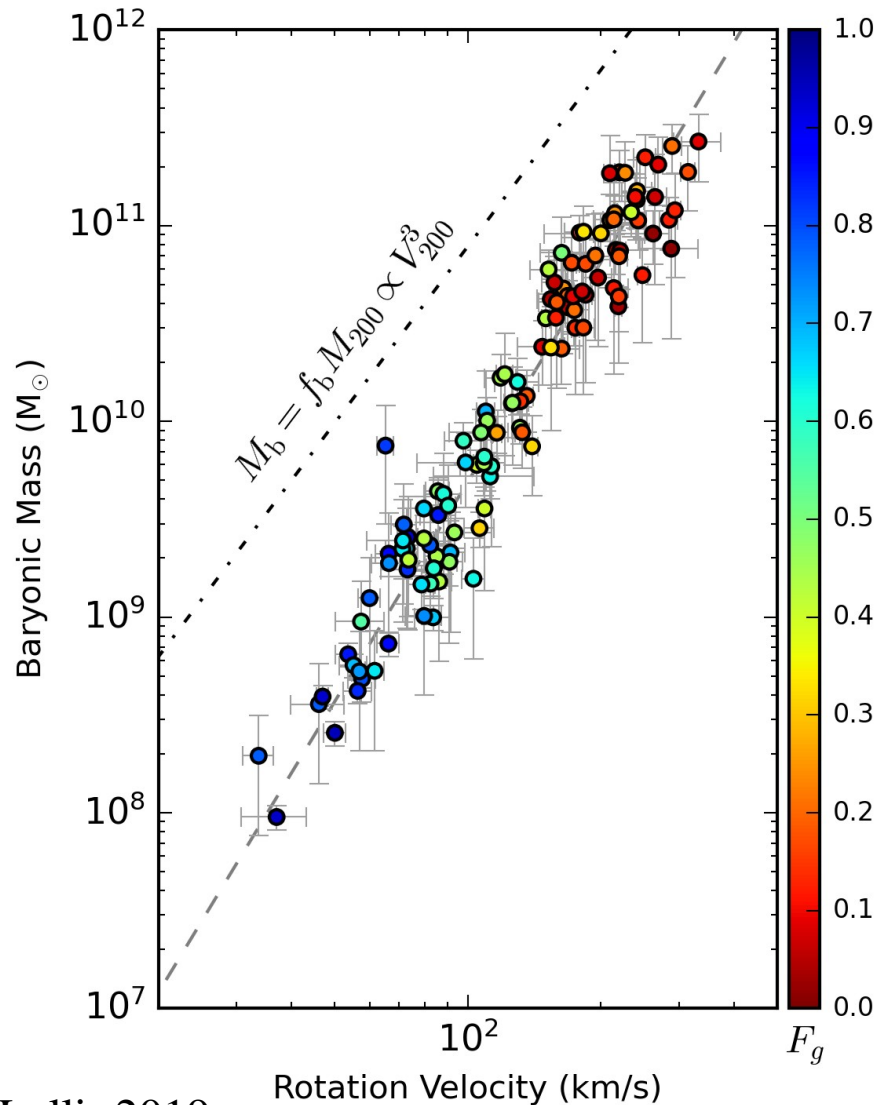
Three Dynamical Laws



1st Law: $R \rightarrow \infty$

$$V_{\text{flat}} \leftrightarrow M_{\text{bar}}$$

1st Law - Baryonic Tully-Fisher Relation (BTFR)

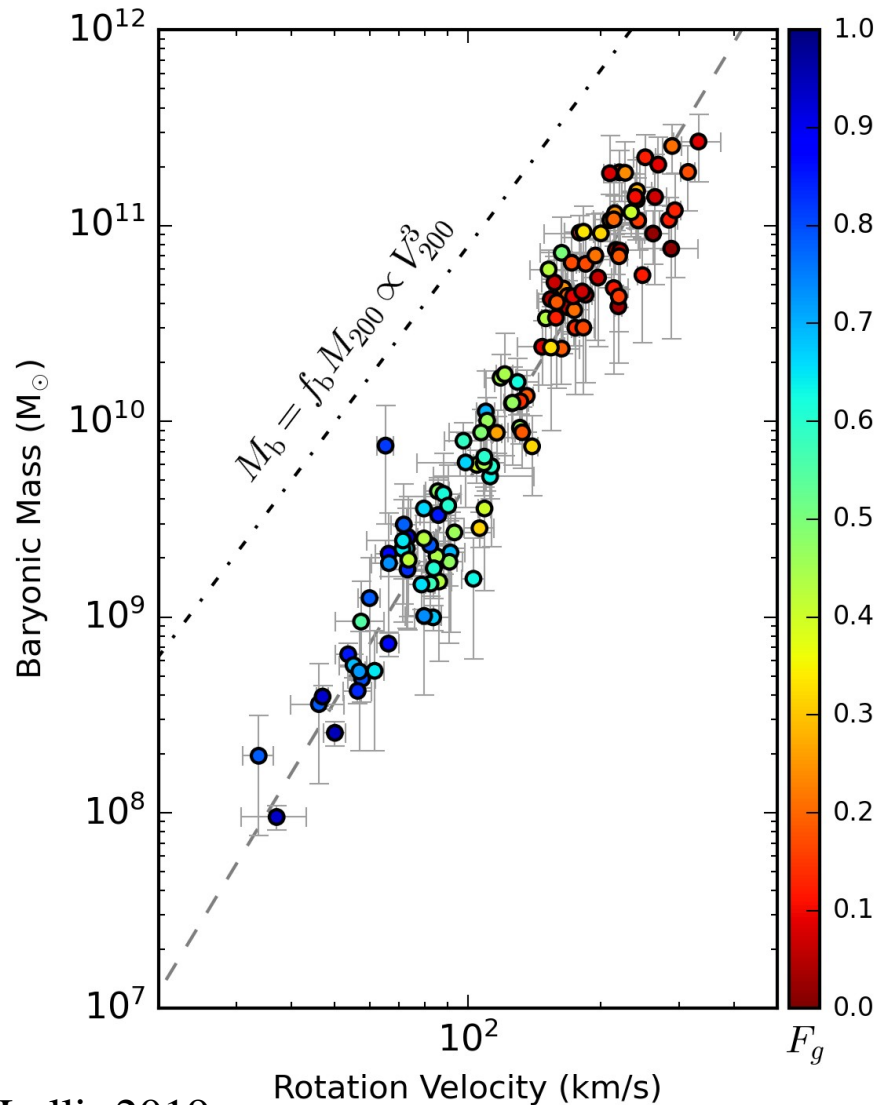


Lelli+2019

Observables:

- V_f = average velocity along the *flat part* of the rotation curve (set by baryons + DM halo)
- M_b = total baryonic mass (gas plus stars)

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Lelli+2019

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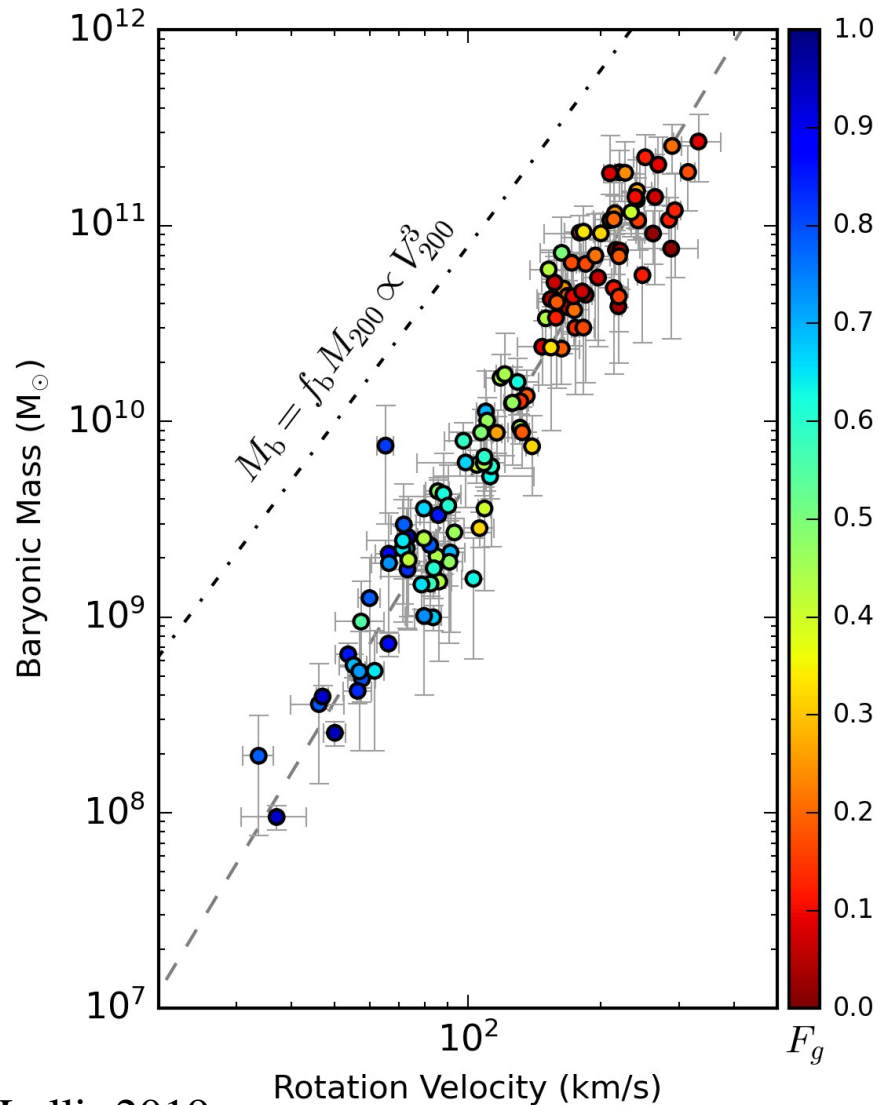
Key Properties:

- Best-fit slope is ~ 4 within the errors

$$M_b = N V_f^4 \Rightarrow N = \frac{1}{G_N a_{BTFR}} \quad a_{BTFR} \sim 10^{-10} \text{ m/s}^2$$

- Scatter is very small (consistent with obs. errors)
- No residual correlation (size, surface density, etc)

1st Law - Baryonic Tully-Fisher Relation (BTFR)



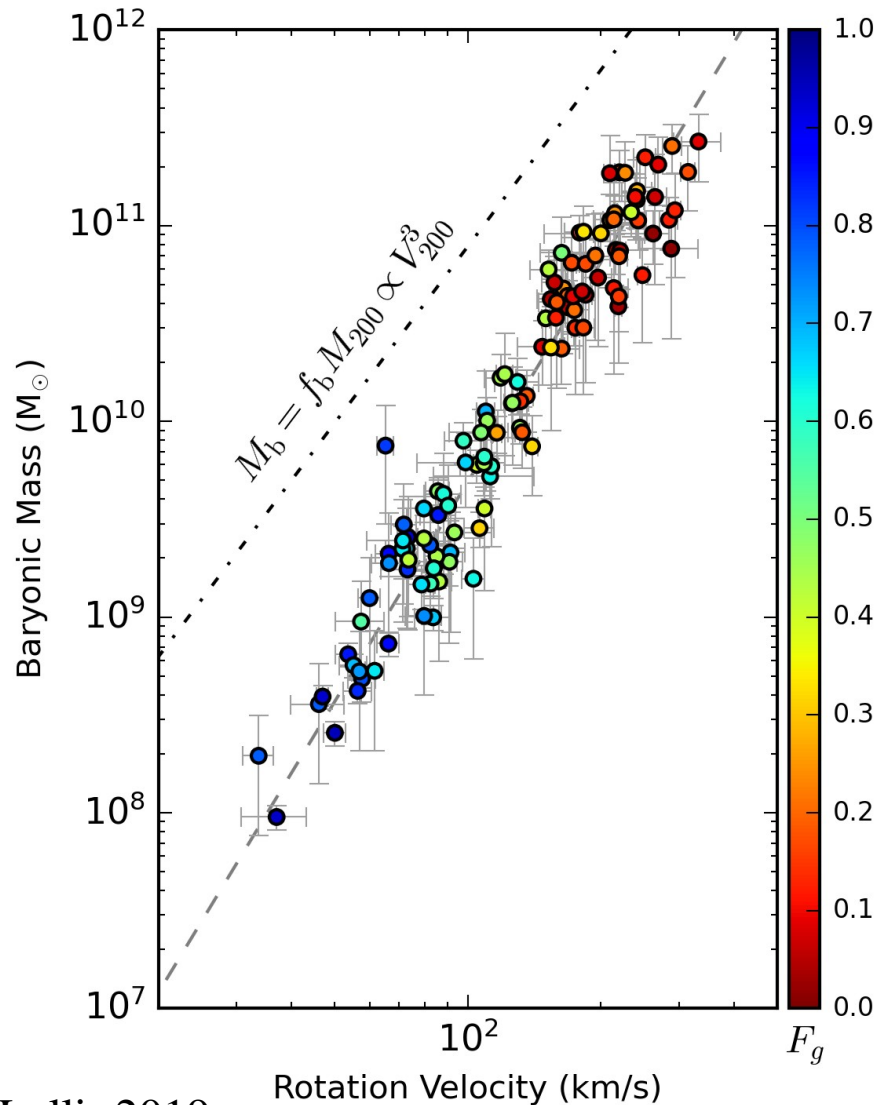
Lelli+2019

In a Λ CDM cosmology:

$M_{200} \stackrel{\text{def}}{=} \text{mass within which } \rho_{\text{halo}} = 200\rho_{\text{crit}}$

$$M_{200} = \sqrt{\frac{1}{100} \frac{1}{G_N H_0}} V_{200}^3$$

1st Law - Baryonic Tully-Fisher Relation (BTFR)



Lelli+2019

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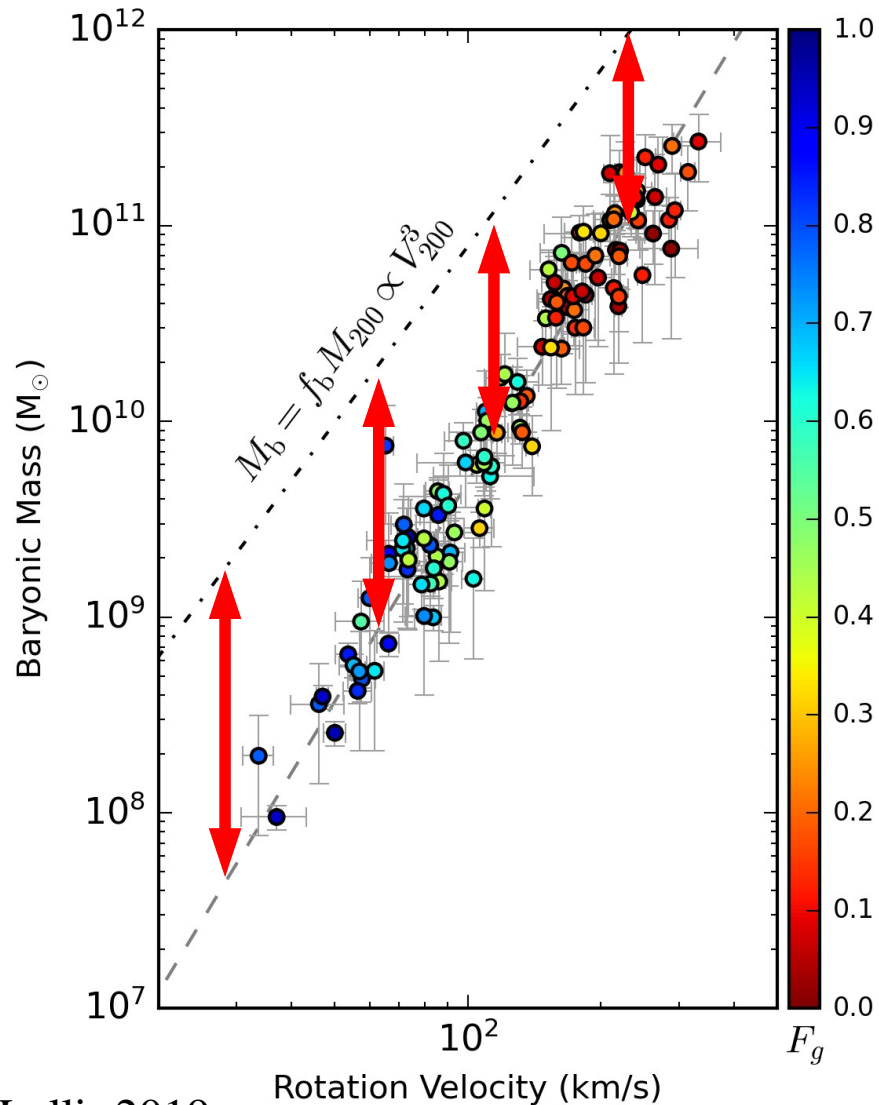
Introduce baryonic quantities:

$$f_b = M_b / M_{200} \simeq \Omega_b / \Omega_{DM} \quad (\text{from CMB})$$

$$f_V = V_f / V_{200} \simeq O(1) \quad (\text{for realistic halos})$$

$$\Rightarrow M_b \propto f_b V_f^3$$

1st Law - Baryonic Tully-Fisher Relation (BTFR)



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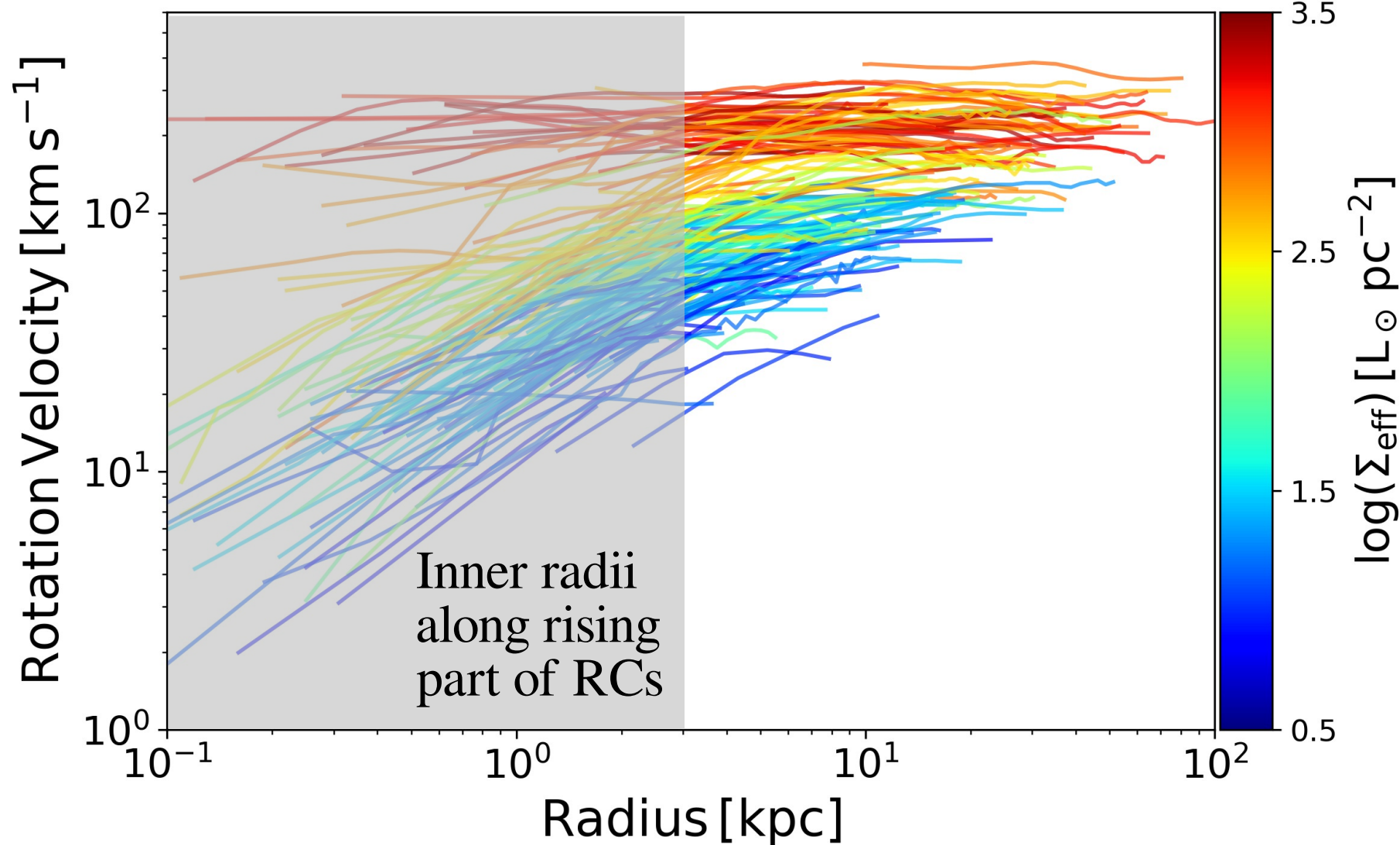
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$$\Rightarrow M_b \propto f_b V_f^3 \Rightarrow f_b \propto V_f \quad \text{Fine tuning!}$$

Three Dynamical Laws



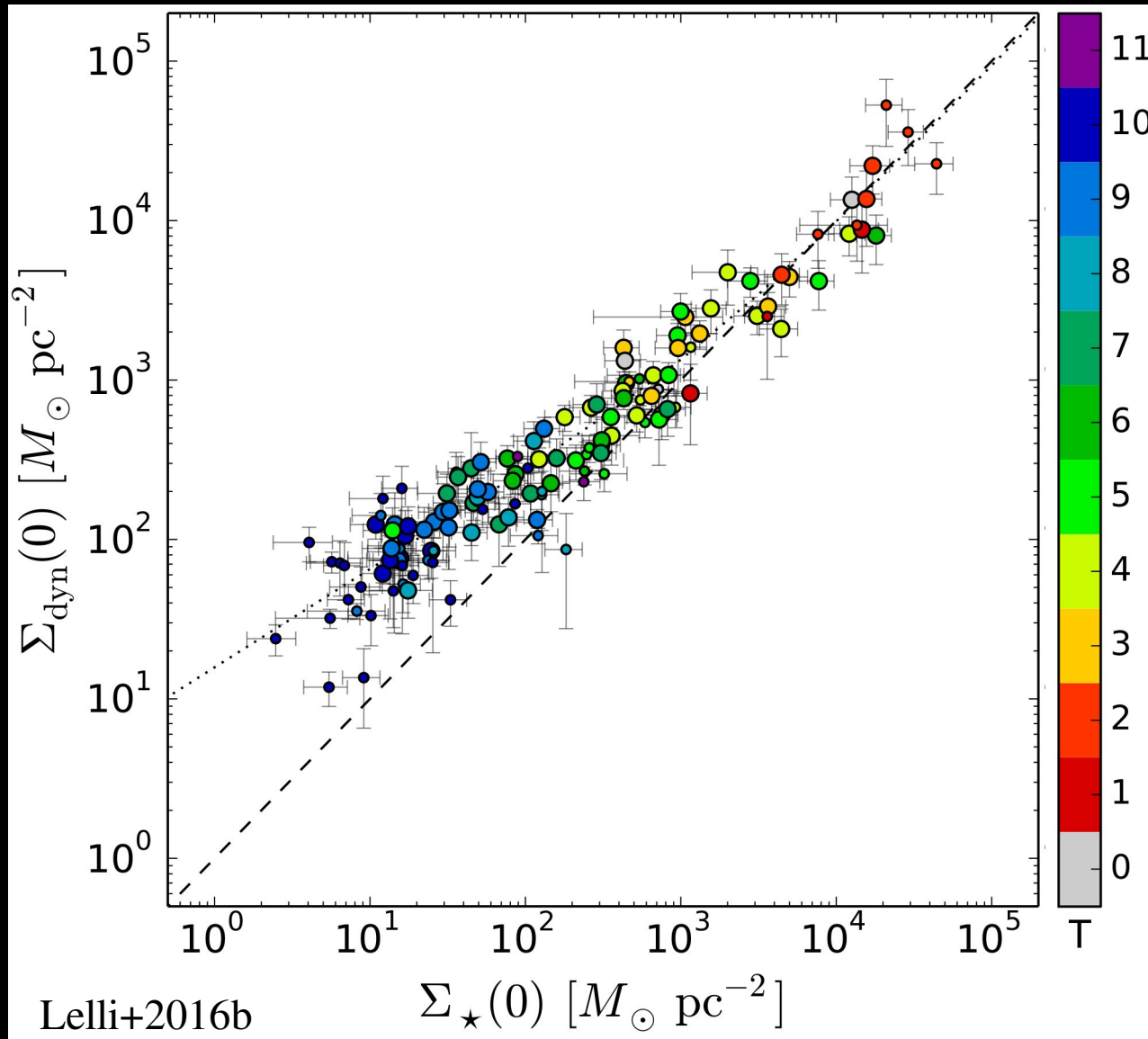
1st Law: $R \rightarrow \infty$

$$V_{\text{flat}} \leftrightarrow M_{\text{bar}}$$

2nd Law: $R \rightarrow 0$

$$\Sigma_{\text{dyn}, 0} \leftrightarrow \Sigma_{\text{bar}, 0}$$

2nd Law – Central Density Relation (CDR)

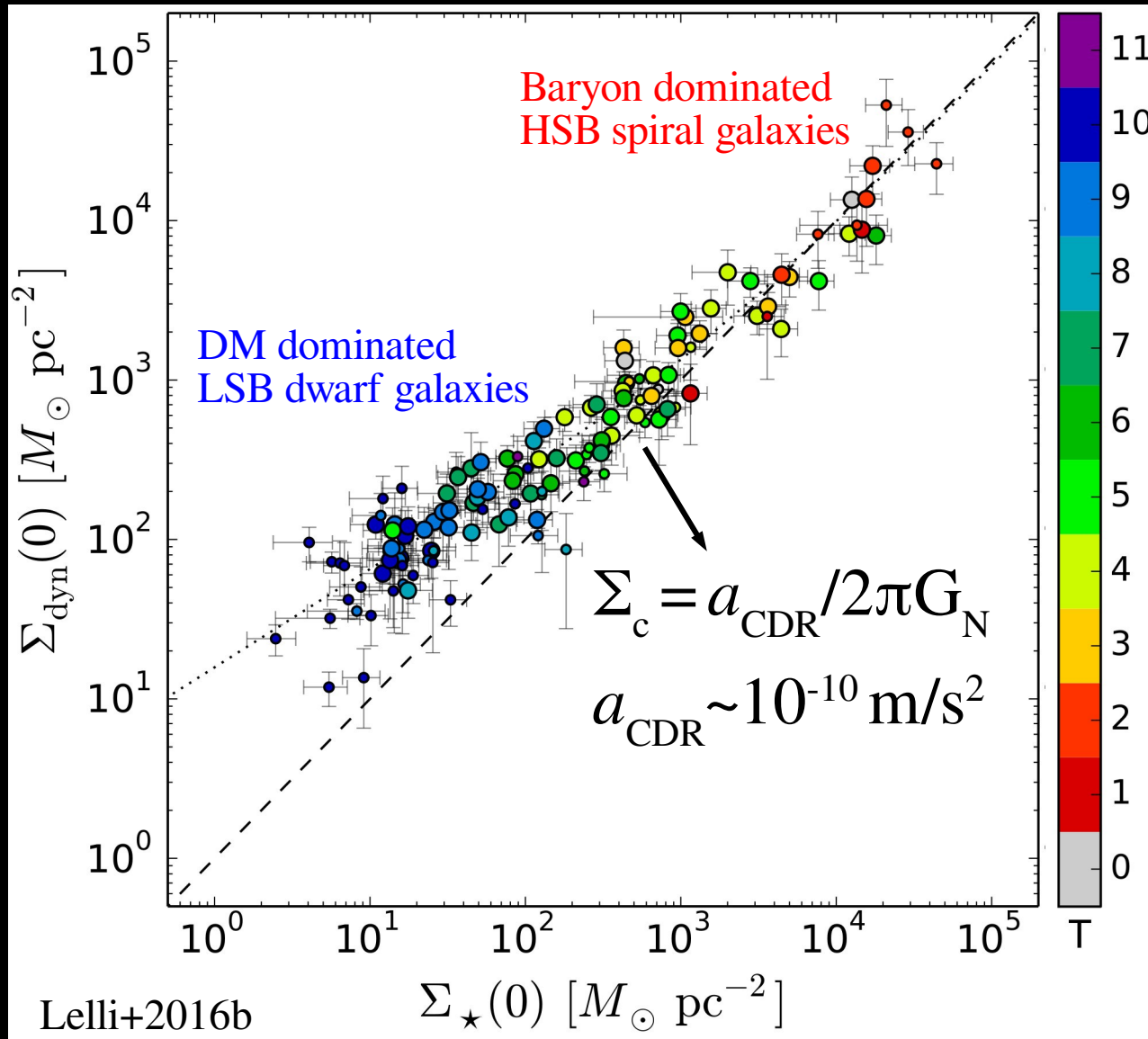


Observables:

$$\Sigma_{\text{dyn}}(0) = \frac{1}{2\pi G} \int_0^{\infty} \frac{V^2}{R^2} dR \quad \text{Toomre (1963)}$$

$\Sigma_{\text{bar}}(0) \rightarrow$ surface density of stars

2nd Law – Central Density Relation (CDR)



Observables:

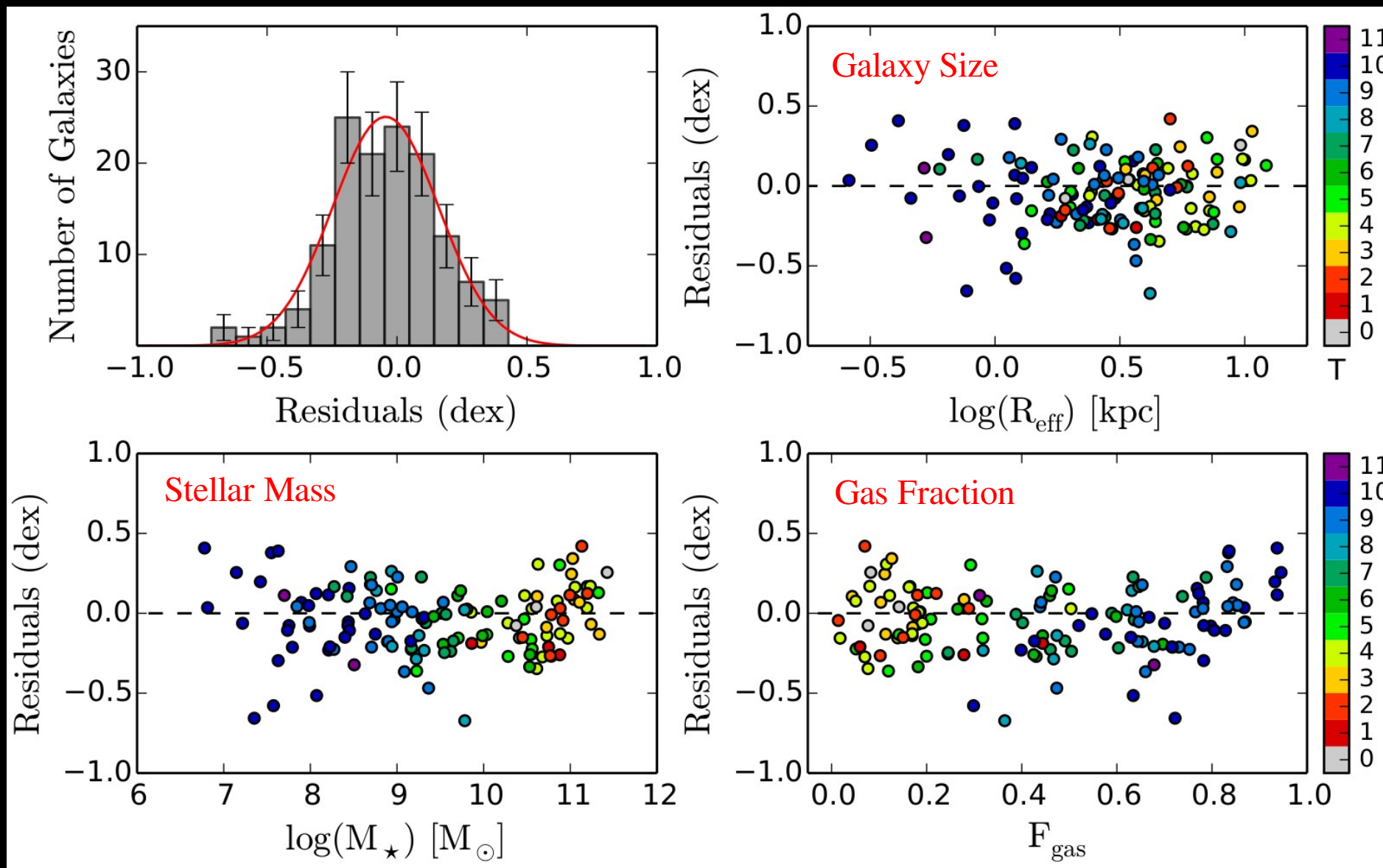
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$\Sigma_{\text{bar}}(0) \rightarrow$ surface density of stars

Key Properties:

- Non-linear relation $\rightarrow \Sigma_c$ critical density
 - $\Sigma_{\text{bar}}(0) > \Sigma_c \rightarrow$ baryons domination
 - $\Sigma_{\text{bar}}(0) < \Sigma_c \rightarrow$ DM domination
- Scatter is small (consistent with errors)

No Residual Correlations across the CDR

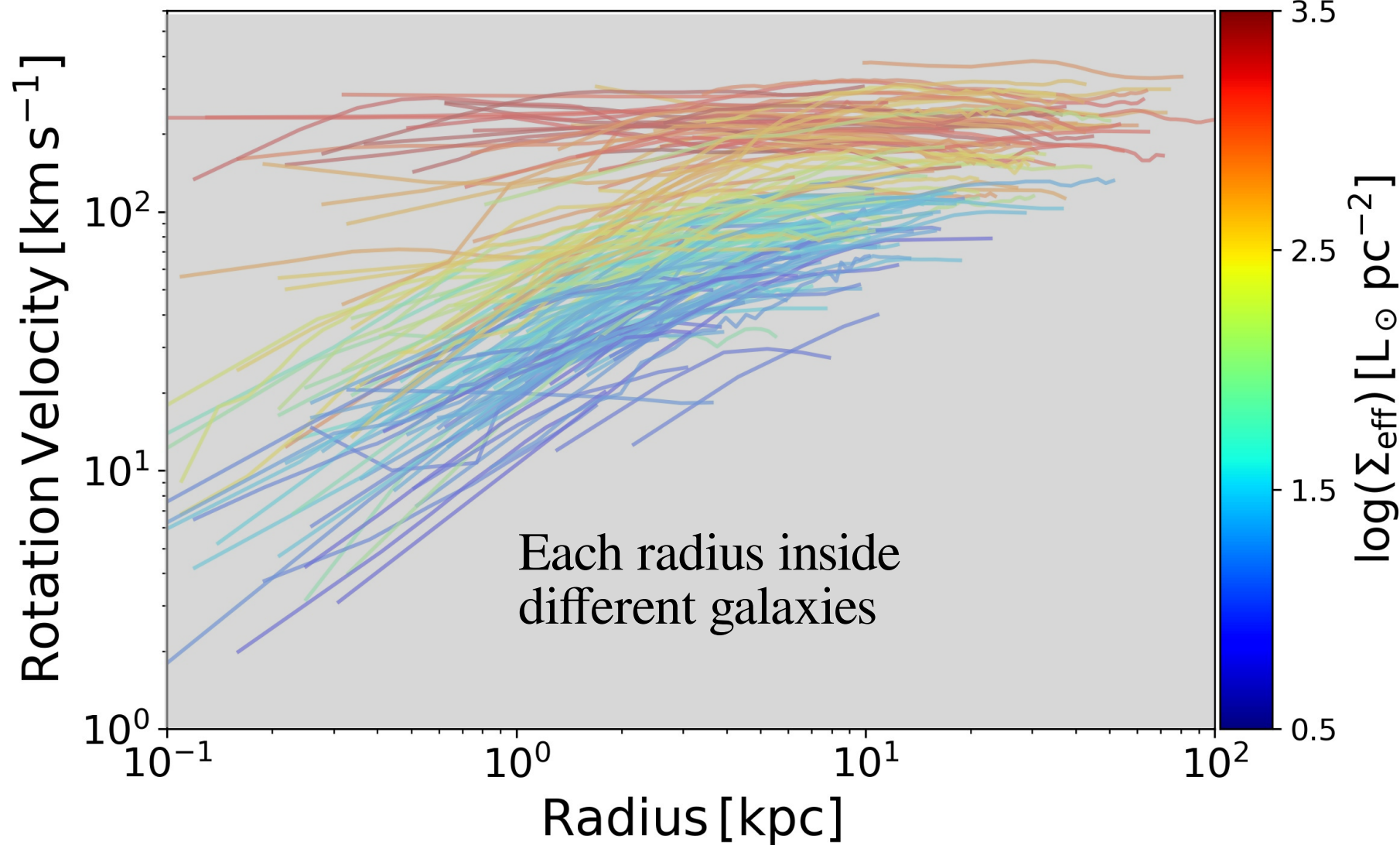


Newton's shell theorem
does NOT apply in disks!

$\Sigma_{\text{dyn}}(0)$ depends on the
mass distribution at all R .

We'd expect secondary
correlations with galaxy
mass or size, but none is
observed... problem!

Three Dynamical Laws



1st Law: $R \rightarrow \infty$

$$V_{\text{flat}} \leftrightarrow M_{\text{bar}}$$

2nd Law: $R \rightarrow 0$

$$\Sigma_{\text{dyn}, 0} \leftrightarrow \Sigma_{\text{bar}, 0}$$

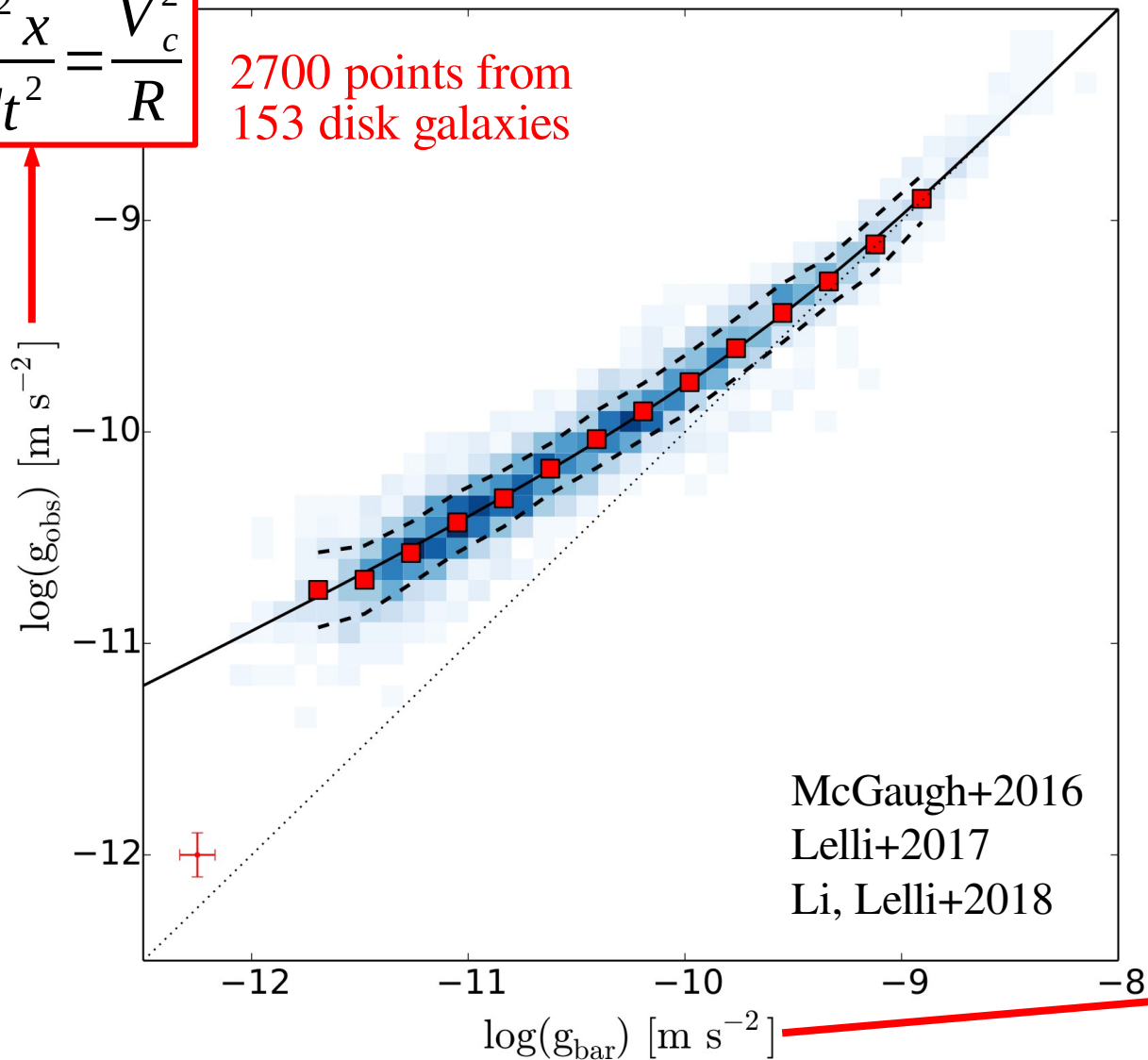
3rd Law: $\forall R$

$$g_{\text{obs}} \leftrightarrow g_{\text{bar}}$$

3rd Law – Radial Acceleration Relation (RAR)

$$\frac{d^2 x}{dt^2} = \frac{V_c^2}{R}$$

2700 points from
153 disk galaxies



Observables:

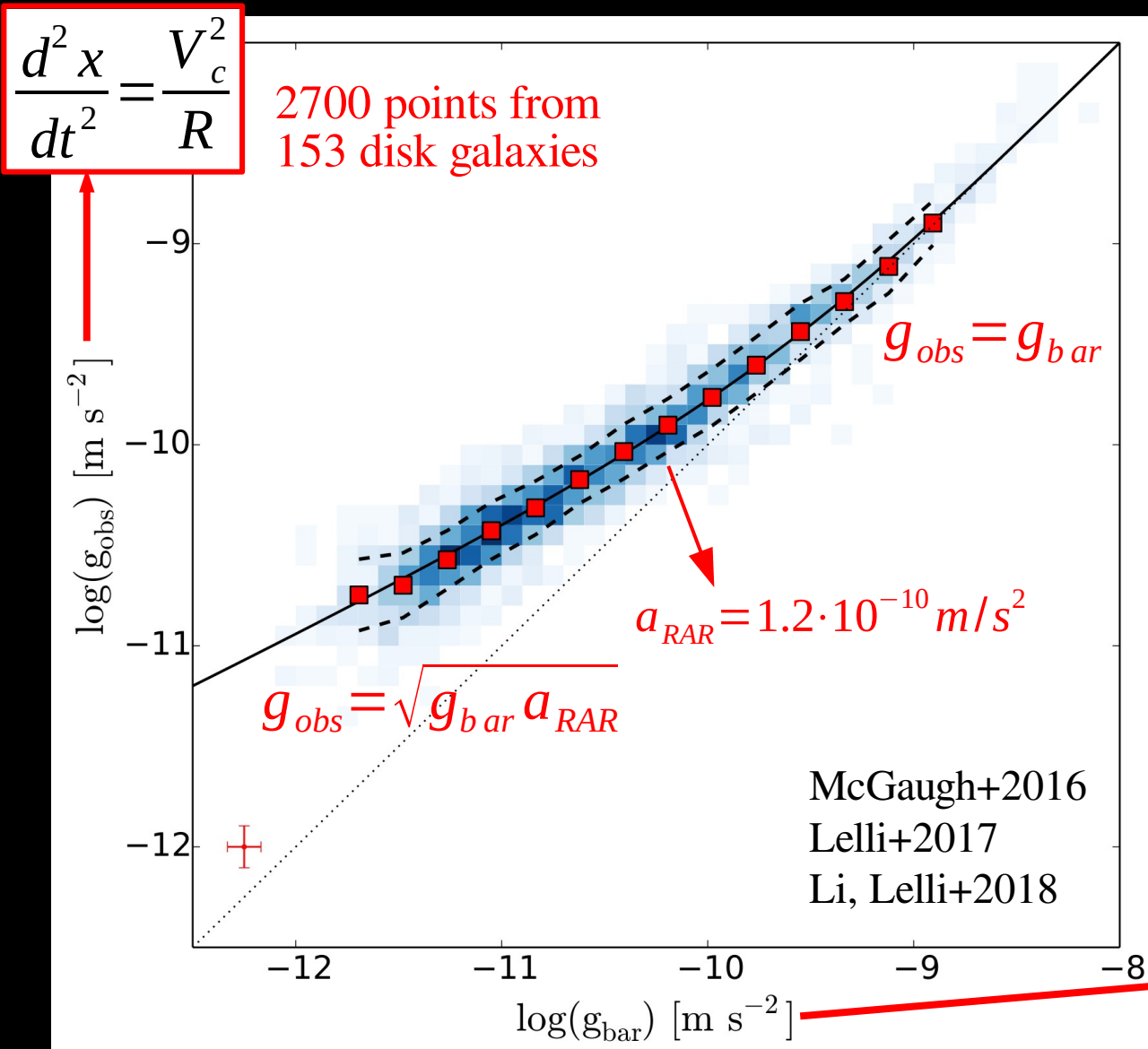
g_{obs} → centripetal acceleration from RCs

g_{bar} → gravitational field from baryons

$$g_b = -\nabla \Phi_b$$

$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

3rd Law – Radial Acceleration Relation (RAR)



Observables:

g_{obs} → centripetal acceleration from RCs

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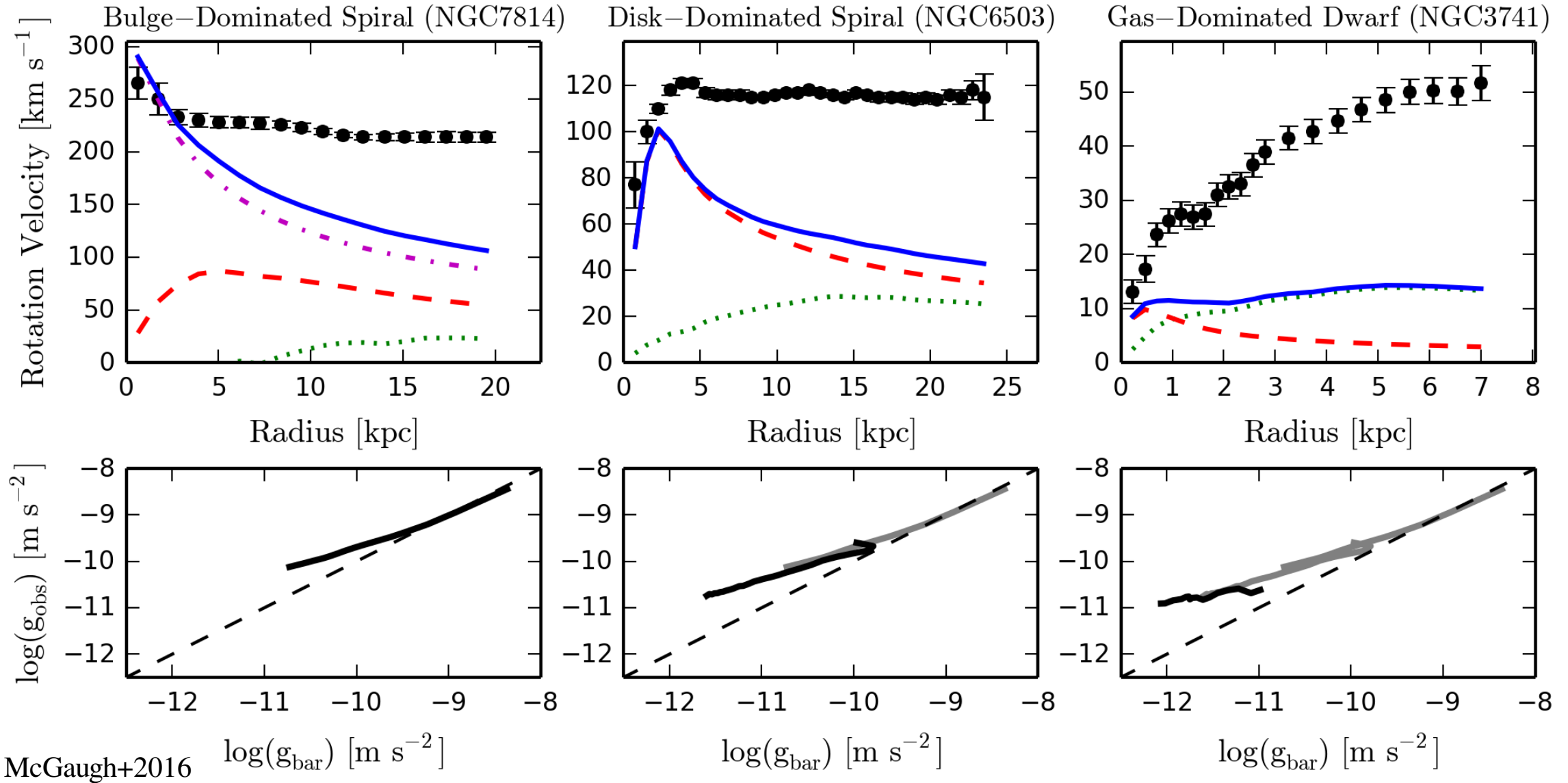
Key Properties:

- Acceleration scale $a_{RAR} \sim 10^{-10} m/s^2$
- Small scatter (consistent with obs. errors)
- No residual dependencies (radius, etc.)
- Baryon distribution ↔ Rotation Curve

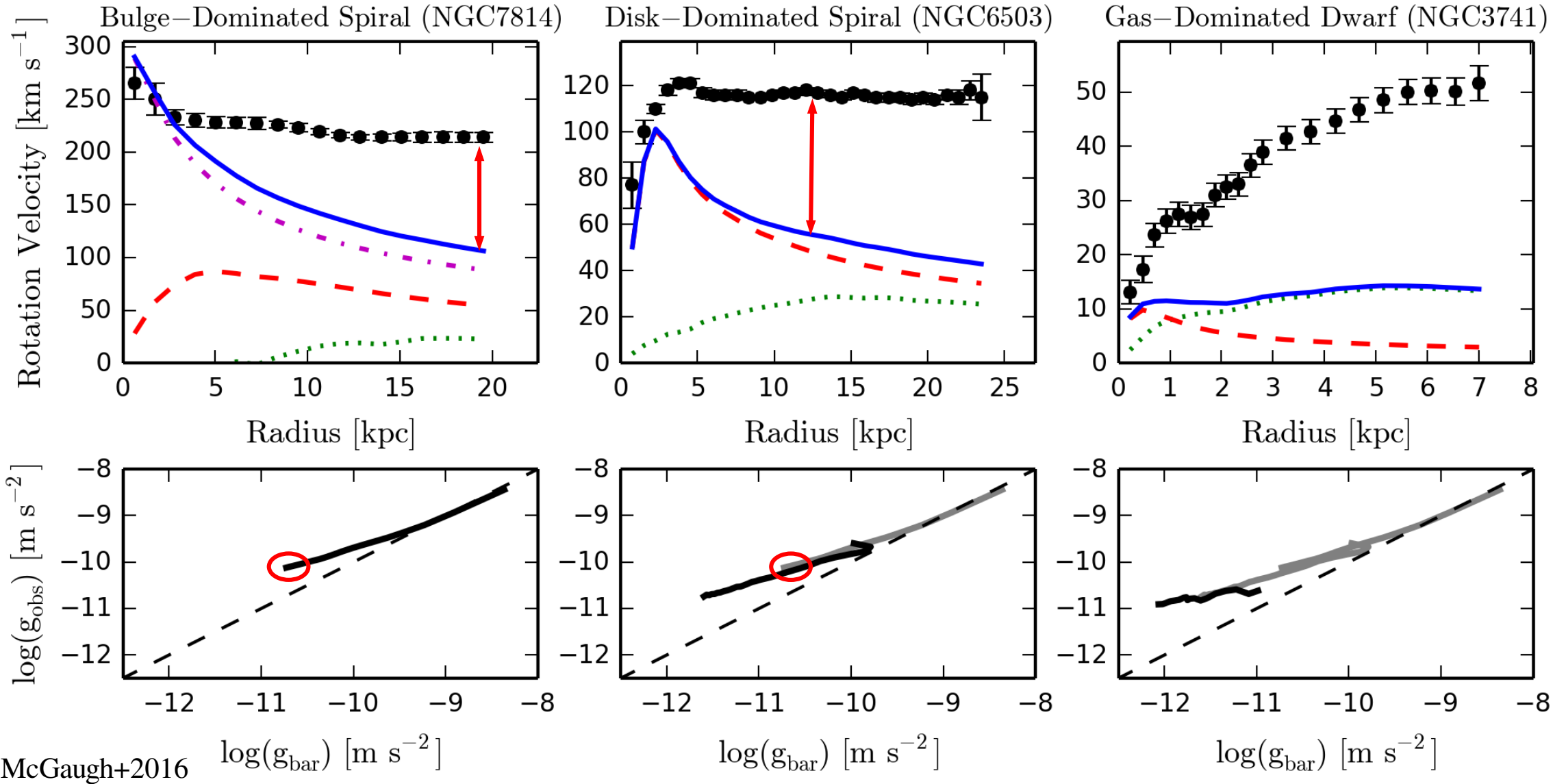
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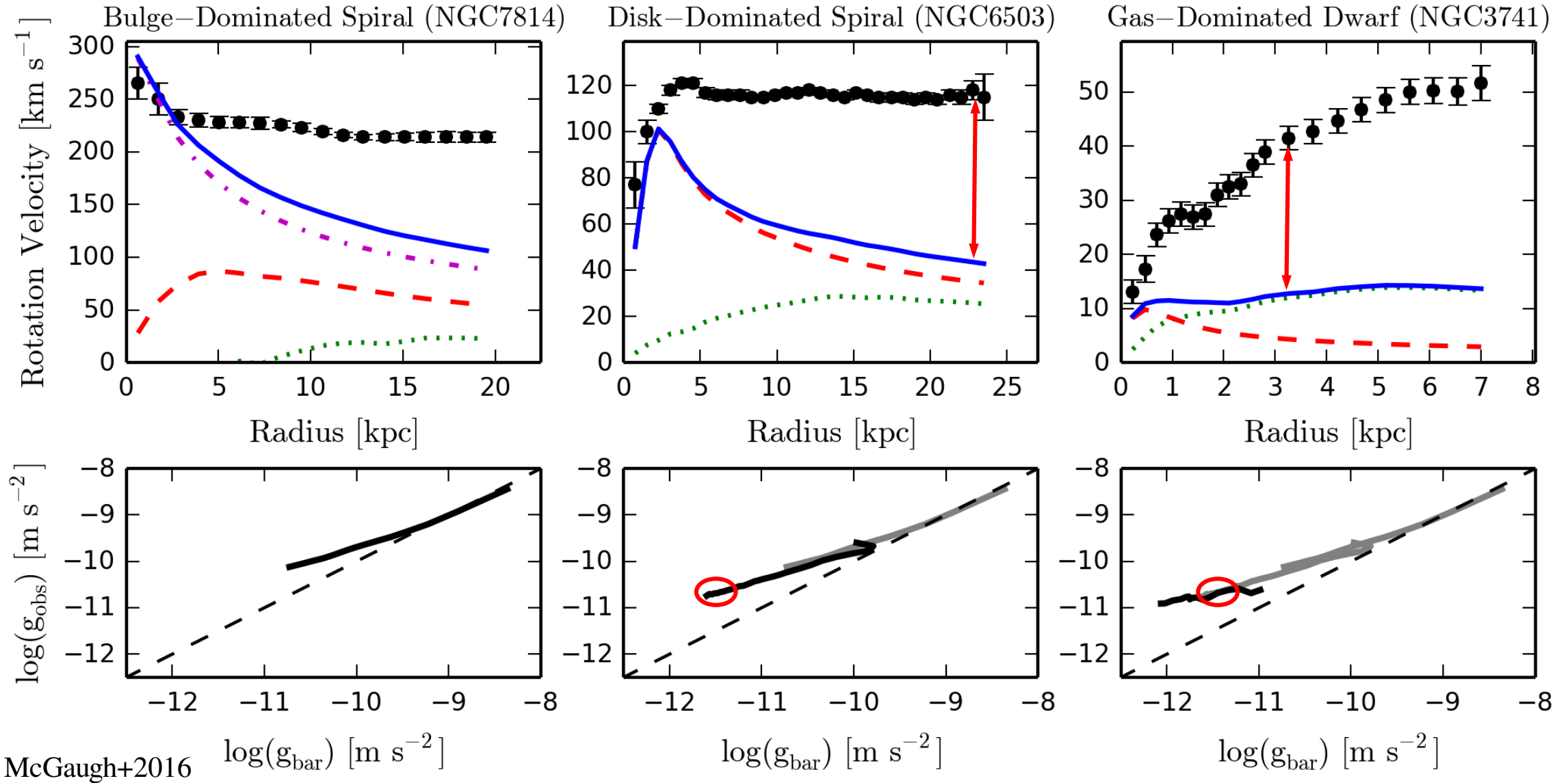
Very different galaxies on the same RAR



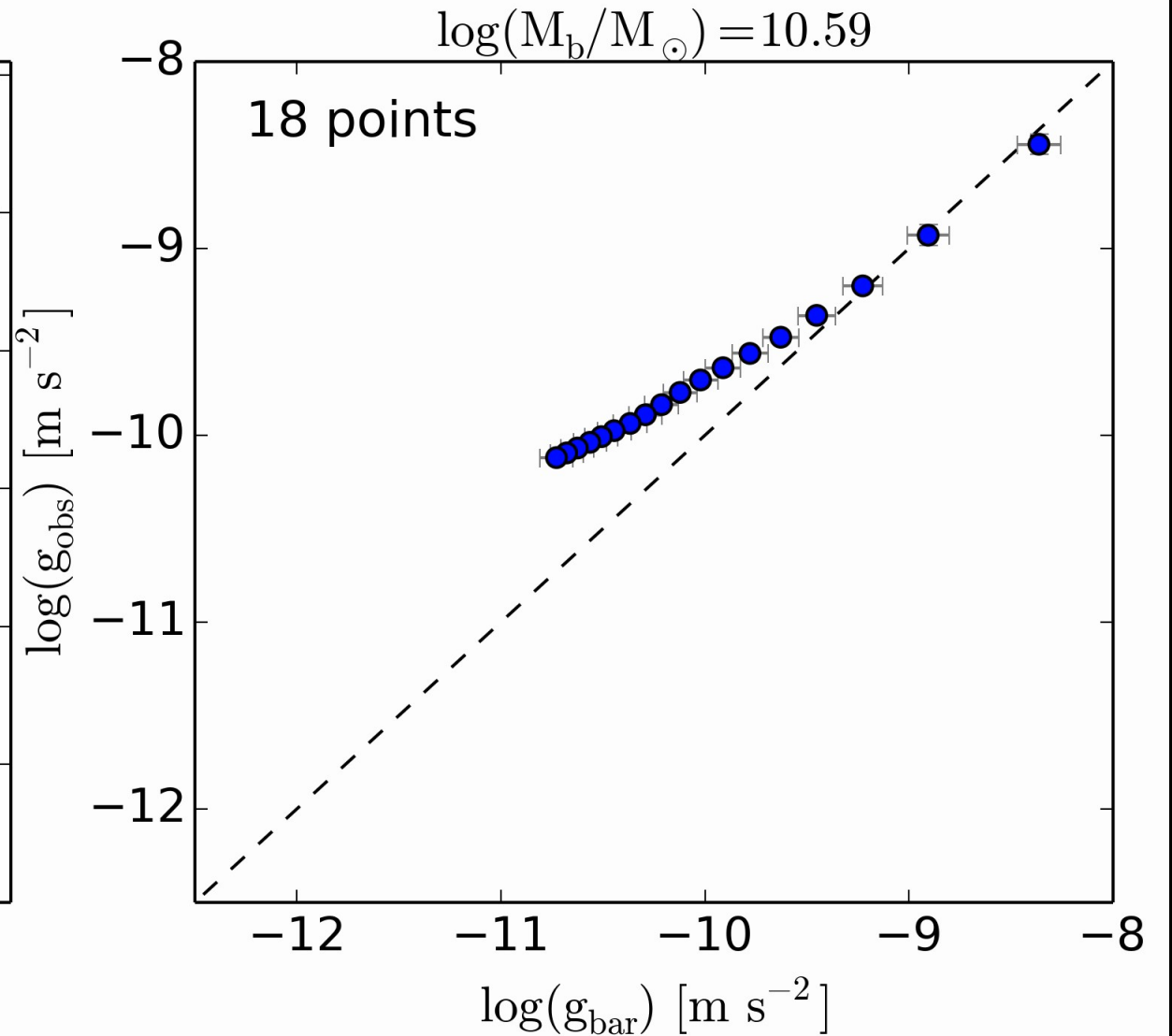
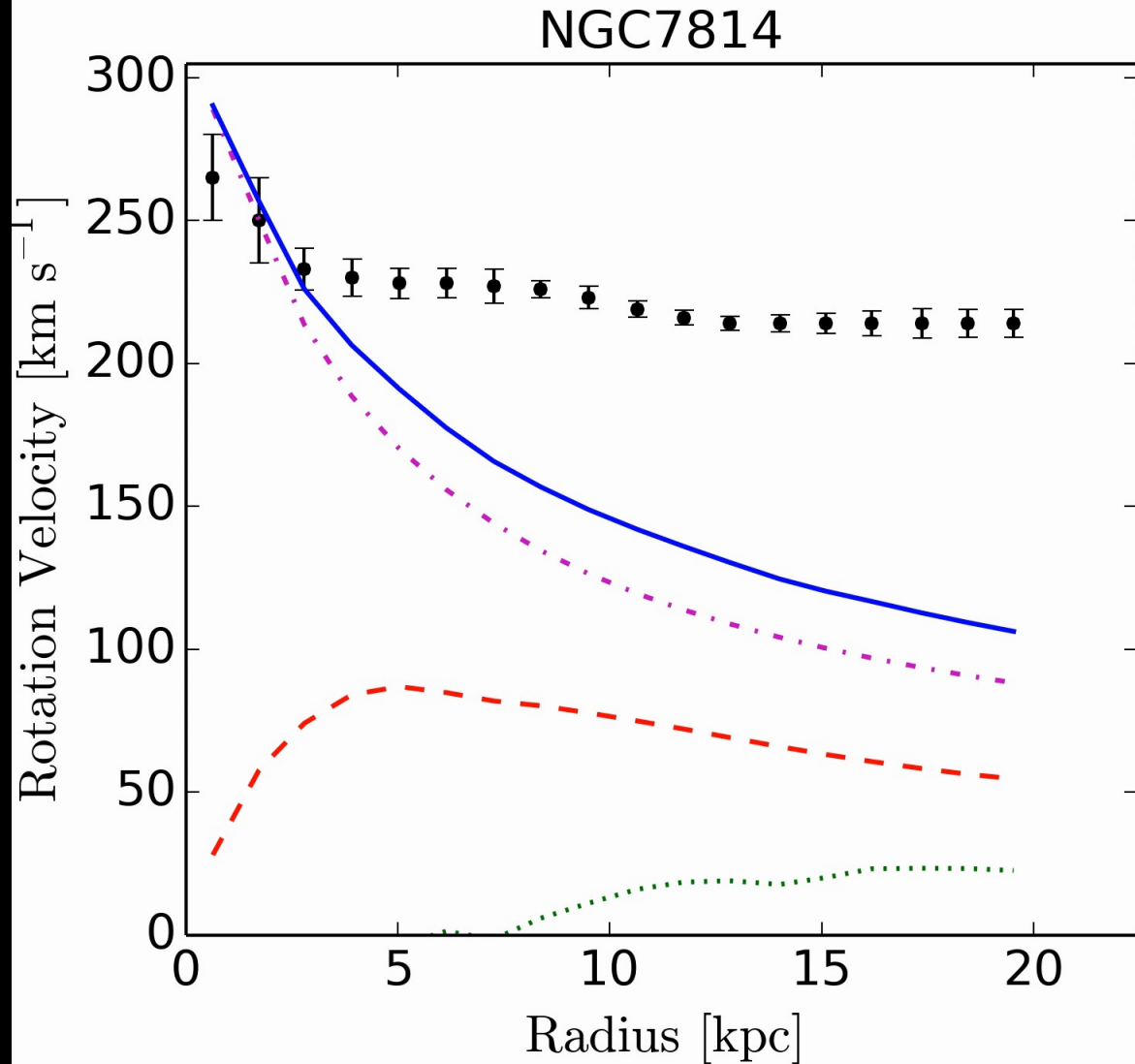
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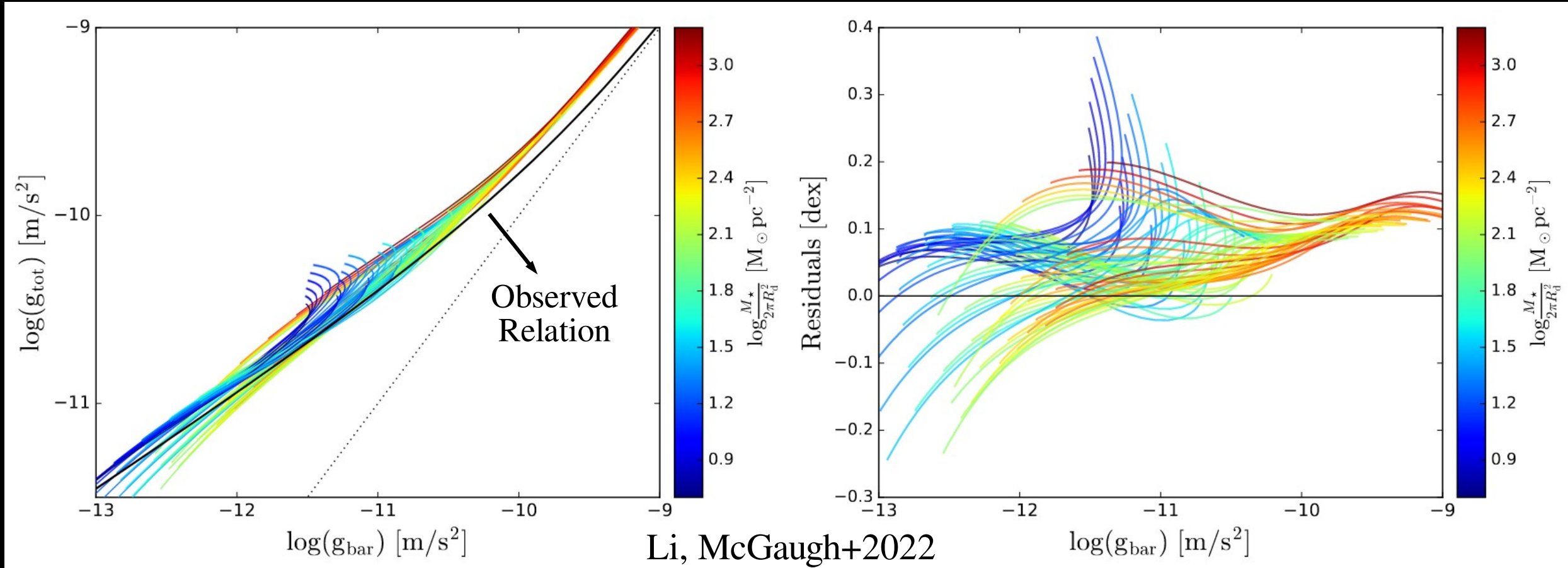
Very different galaxies on the same RAR



Building up the RAR (watch video here)



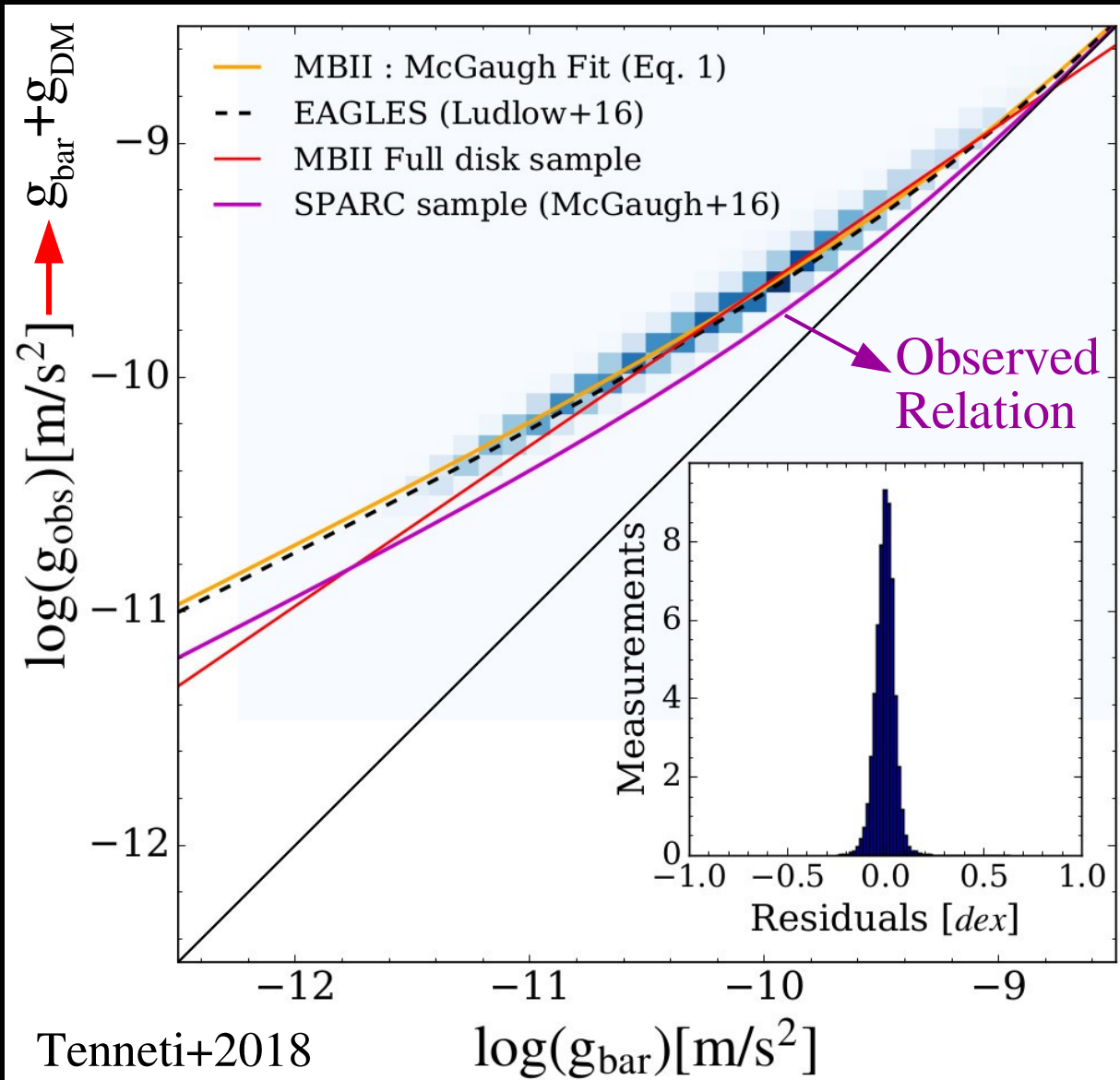
RAR from semi-empirical Λ CDM models



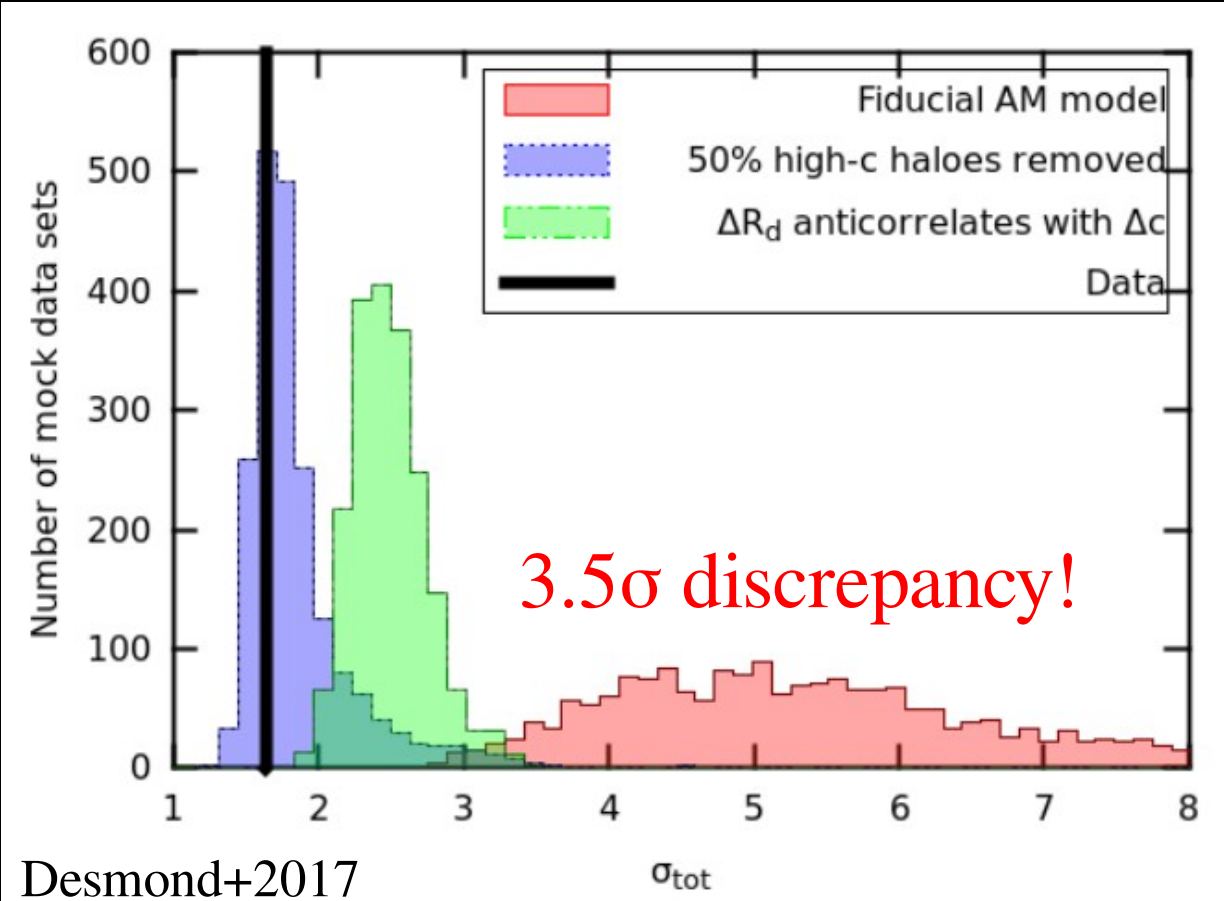
Basic model: Exponential disks + De Vaucouleurs Bulge + NFW halo

Basic physics: Gravity \rightarrow NFW halo adiabatically contracts as baryons fall in

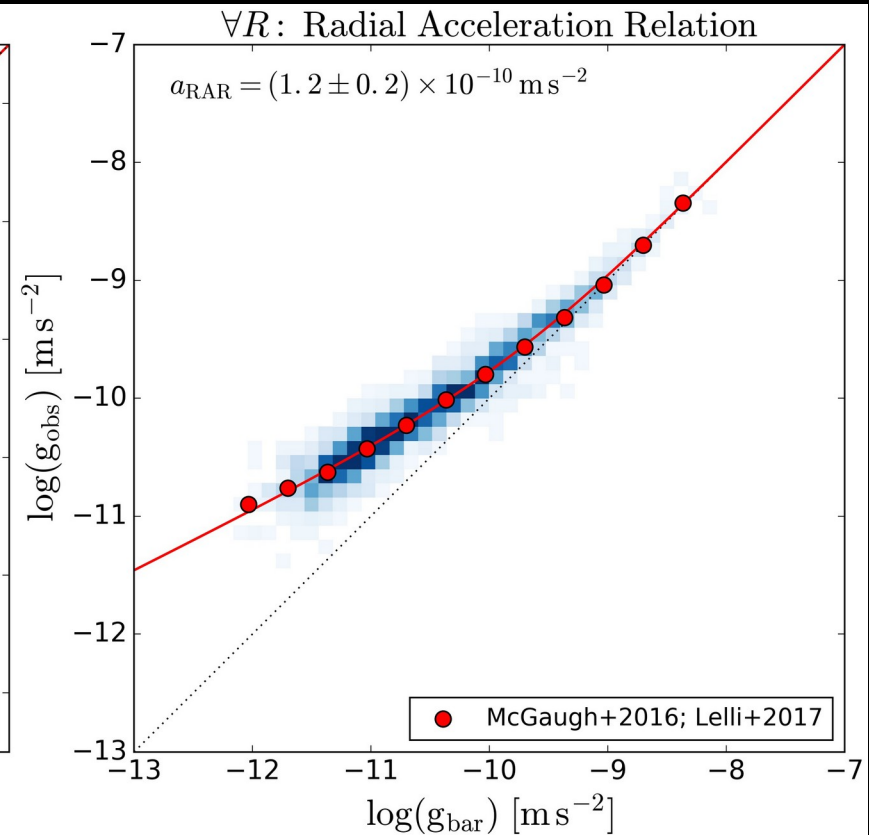
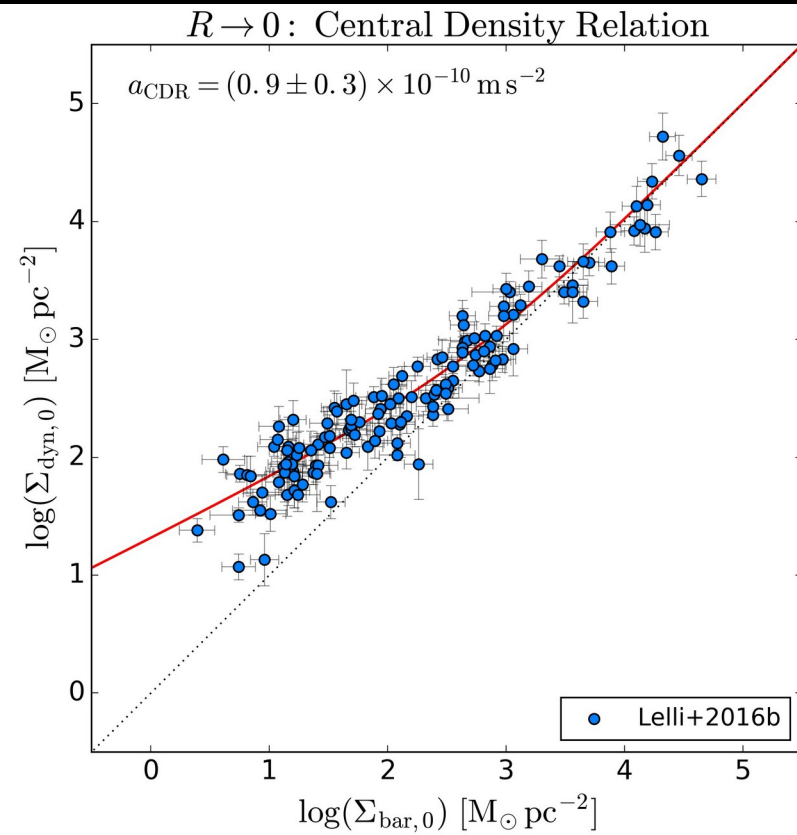
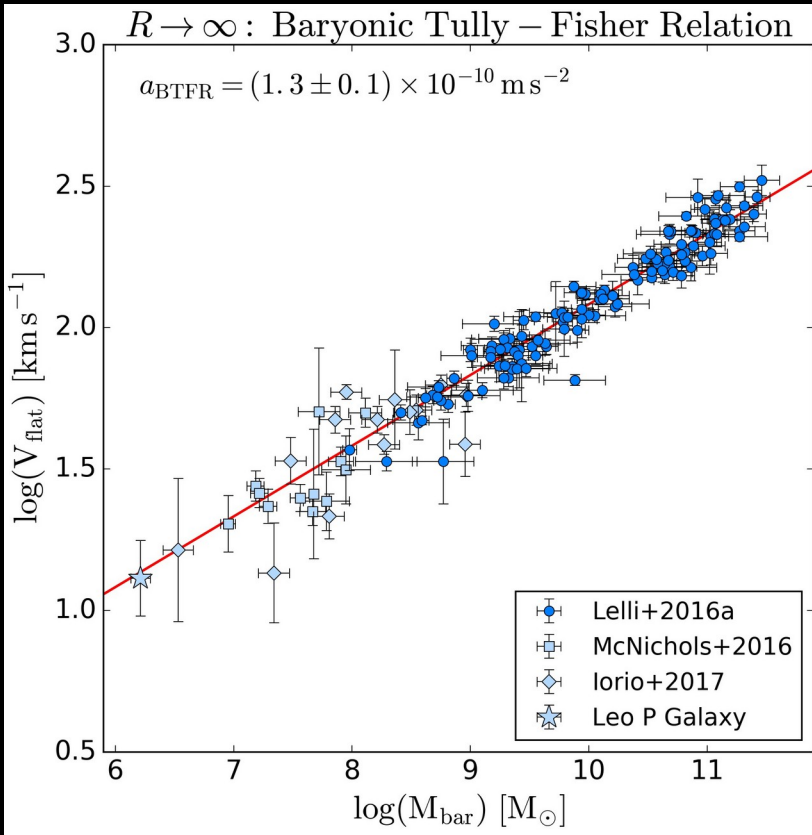
RAR from Λ CDM numerical simulations



Shape: too much DM at all radii
Scatter: too high \rightarrow stochasticity



Three Laws → Three Acceleration Scales

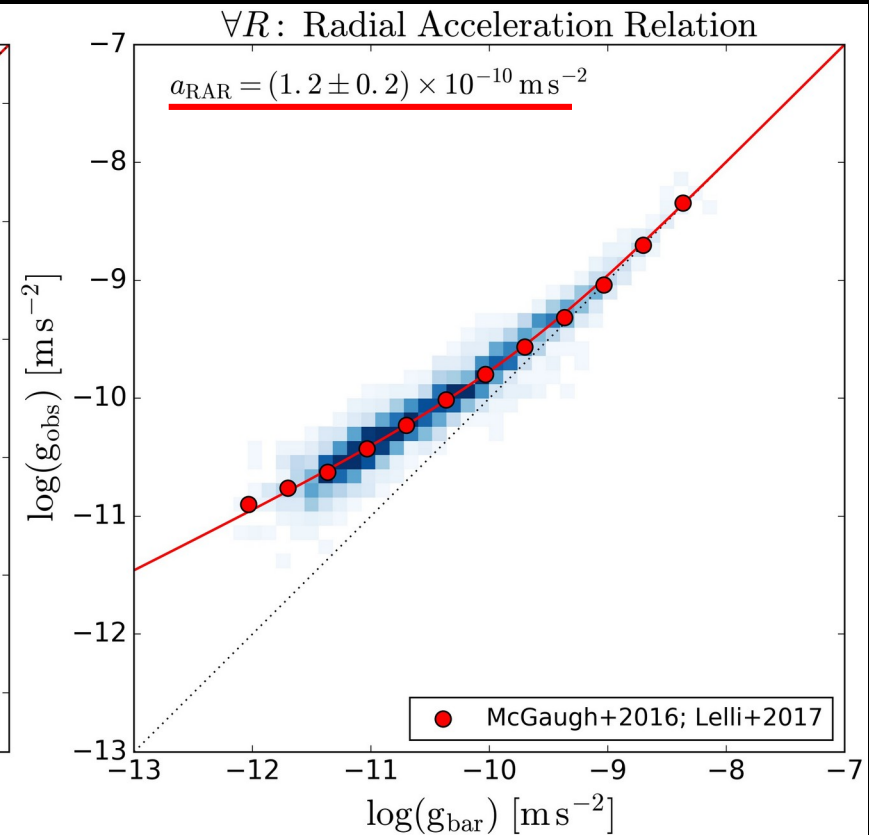
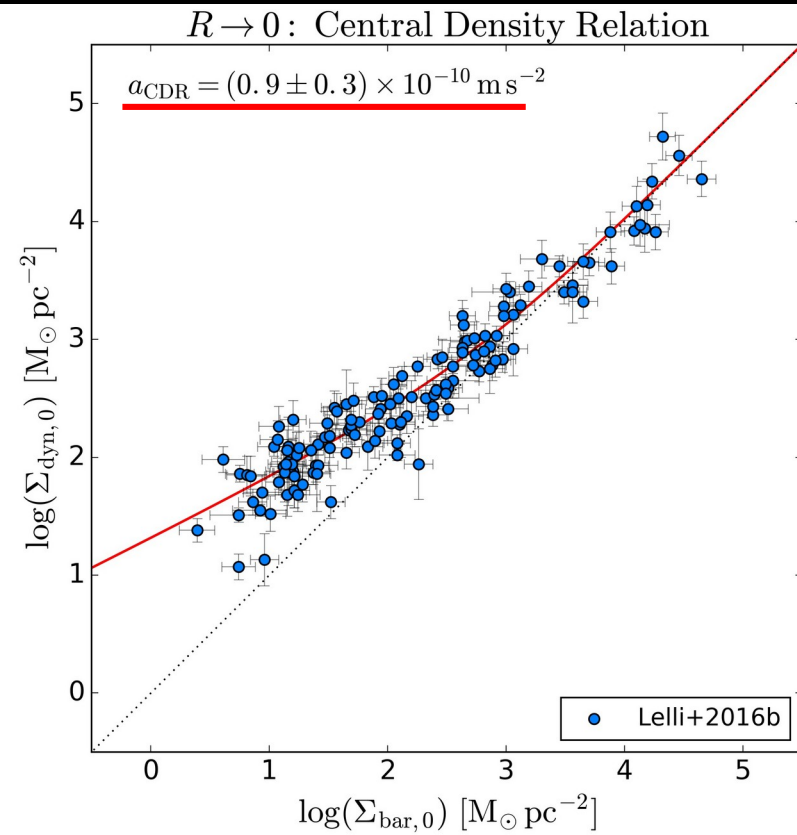
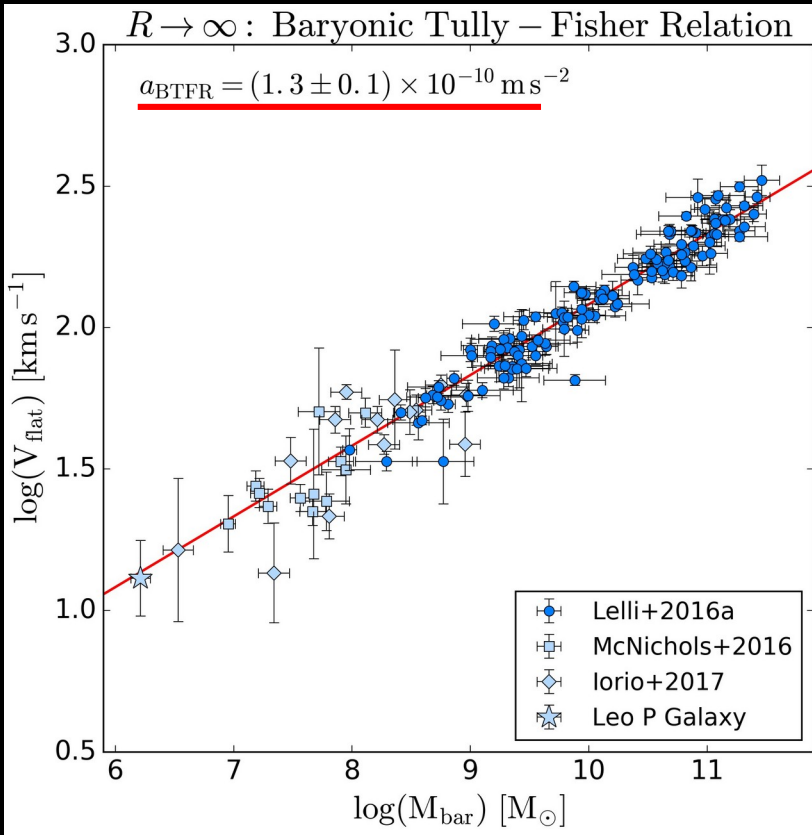


$a_{\text{BTFR}} \rightarrow$ Normalization BTFR
 \rightarrow Global baryon-to-DM ratio across galaxies

$a_{\text{CDR}} \rightarrow$ Critical Surface Density
 \rightarrow Transition baryon to DM dominated galaxies at $R=0$

$a_{\text{RAR}} \rightarrow$ Acceleration Scale
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A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES¹

M. MILGROM

Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study

Received 1982 February 4; accepted 1982 December 28

ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density Σ falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of Σ , the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant, a_0 , determined in a few independent ways is approximately $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$, which is of the order of $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$.

The main predictions are:

1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

2. The $V_\infty^4 = a_0 GM$ relation should hold exactly.

3. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing r in a predictable way.



A-priori MOND predictions in 1983:

✓ Baryonic TF Relation

✓ Central Density Relation

✓ Radial Acceleration Relation

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Cool manga in 1983:



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