MOND: An Alternative to Particle Dark Matter

Federico Lelli

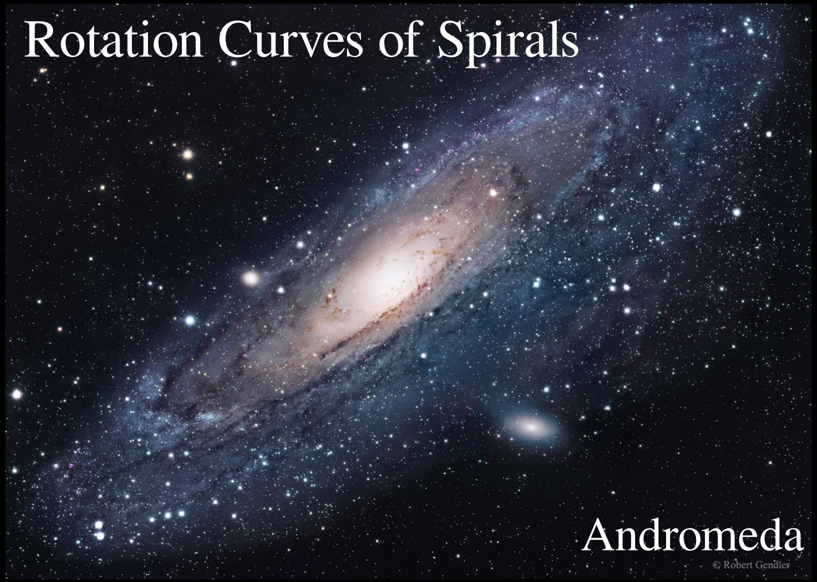
INAF – Arcetri Astrophysical Observatory

INAF
ISTITUTO NAZIONALE
DI ASTROFISICA

OSSERVATORIO ASTROFISICO DI ARCETRI

Evidence of Dark Matter at Various Scales

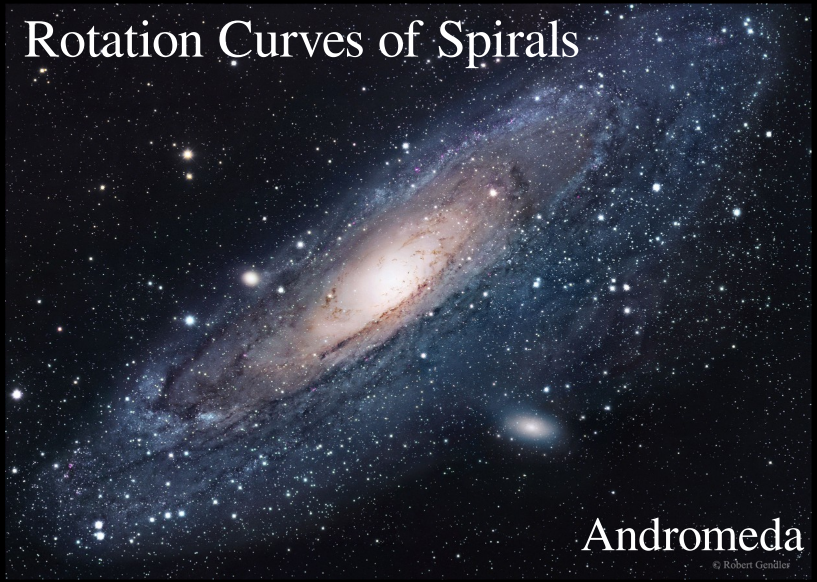
Galaxy Scales (~1-100 kpc)



Evidence of Dark Matter at Various Scales

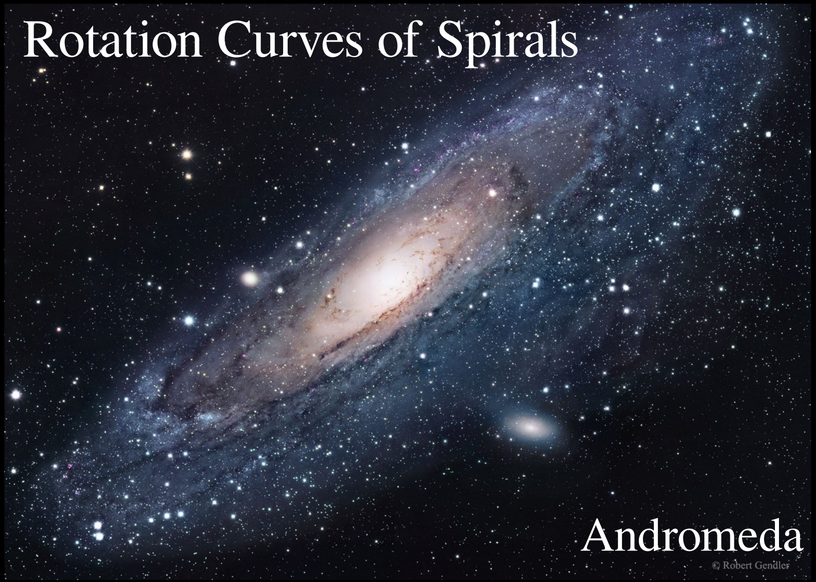
Galaxy Scales (~1-100 kpc)

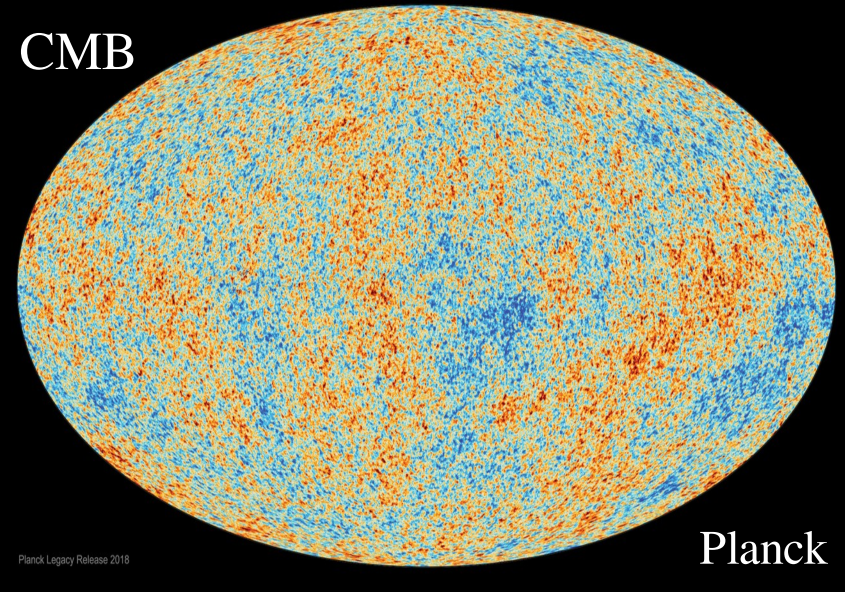
Groups/Clusters Scales (~1-5 Mpc)

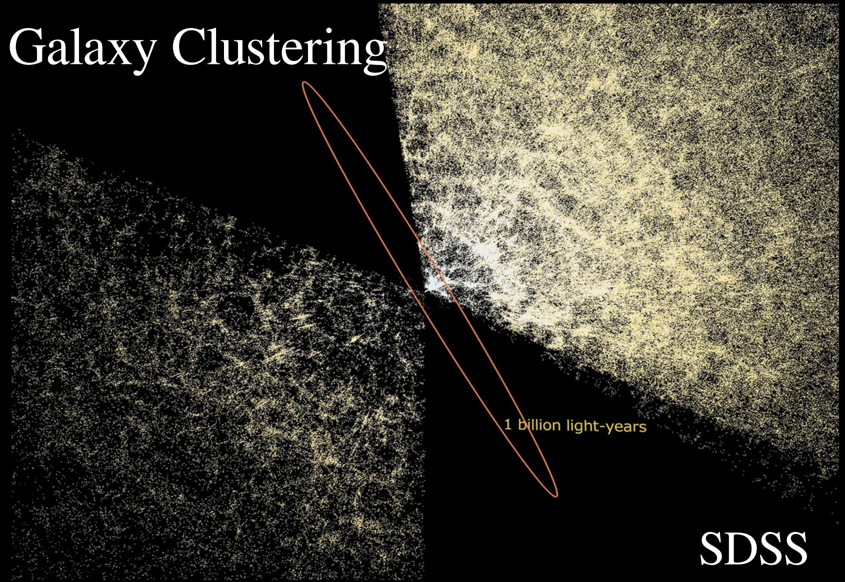


Evidence of Dark Matter at Various Scales

Galaxy Scales (~1-100 kpc) | **Groups/Clusters Scales (~1-5 Mpc)** | **Cosmological Scales (>100 Mpc)**

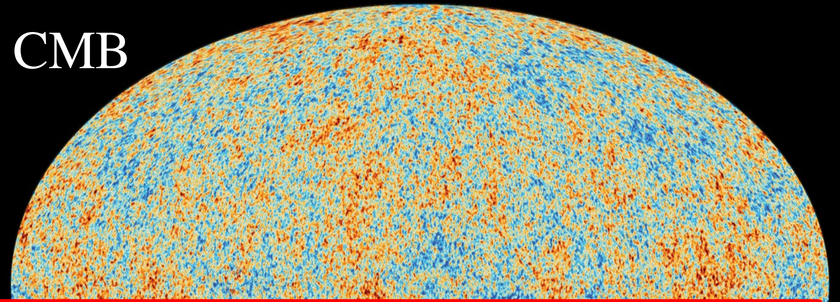






~~Evidence of Dark Matter at Various Scales~~

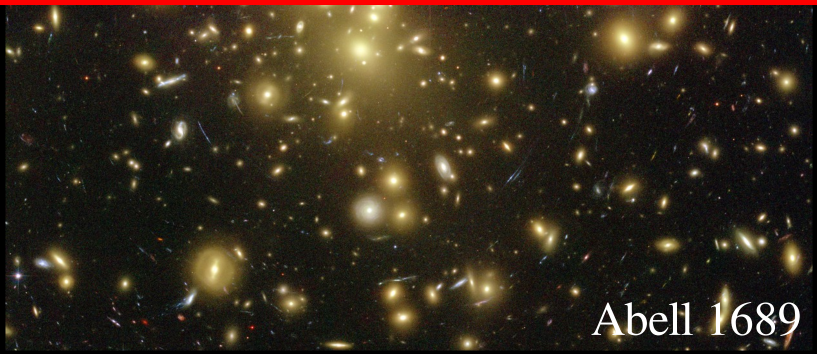
Galaxy Scales (~1-100 kpc) | **Groups/Clusters Scales (~1-5 Mpc)** | **Cosmological Scales (>100 Mpc)**

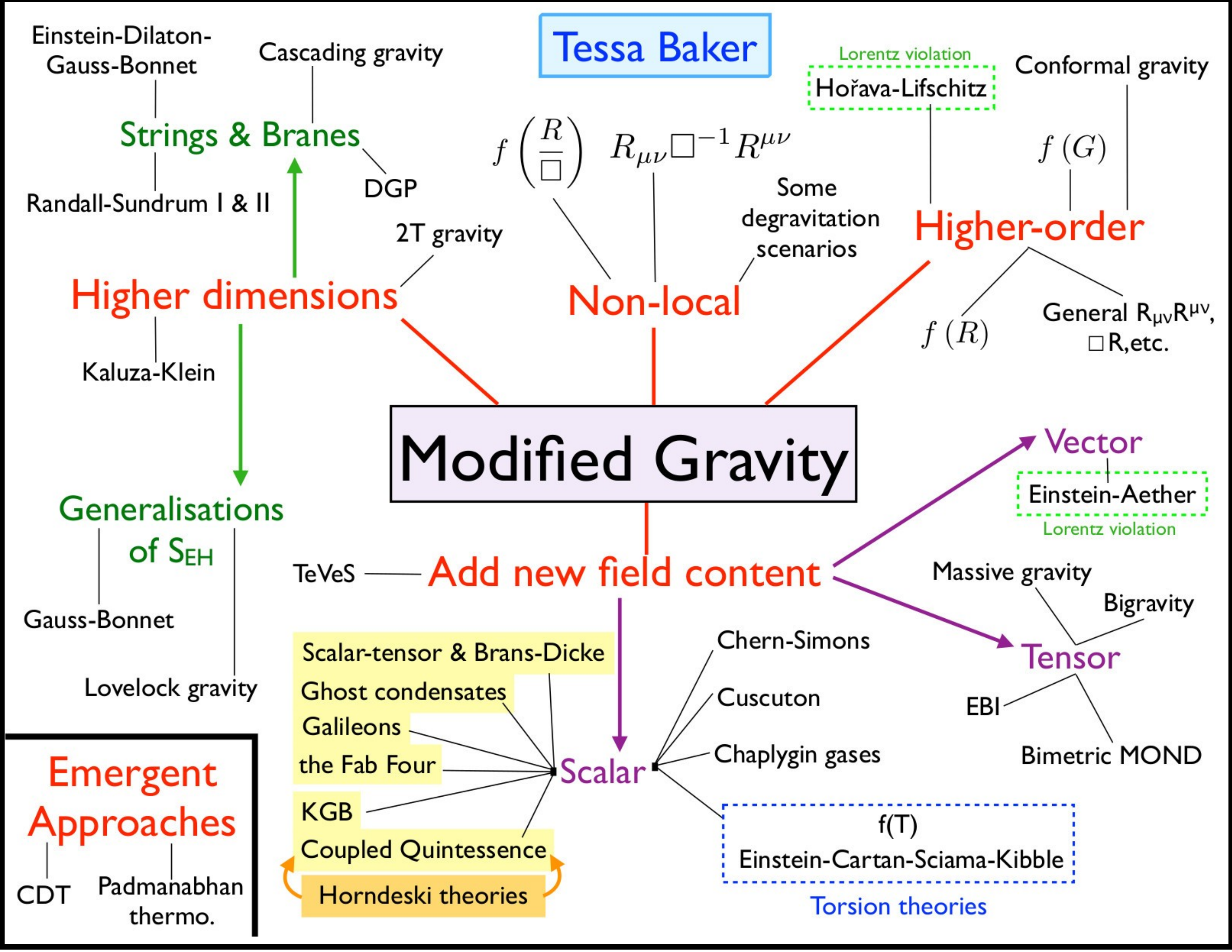


This is *__not__* direct evidence for *__particle__* dark matter!

Standard Laws of Gravity (Einstein & Newton) +

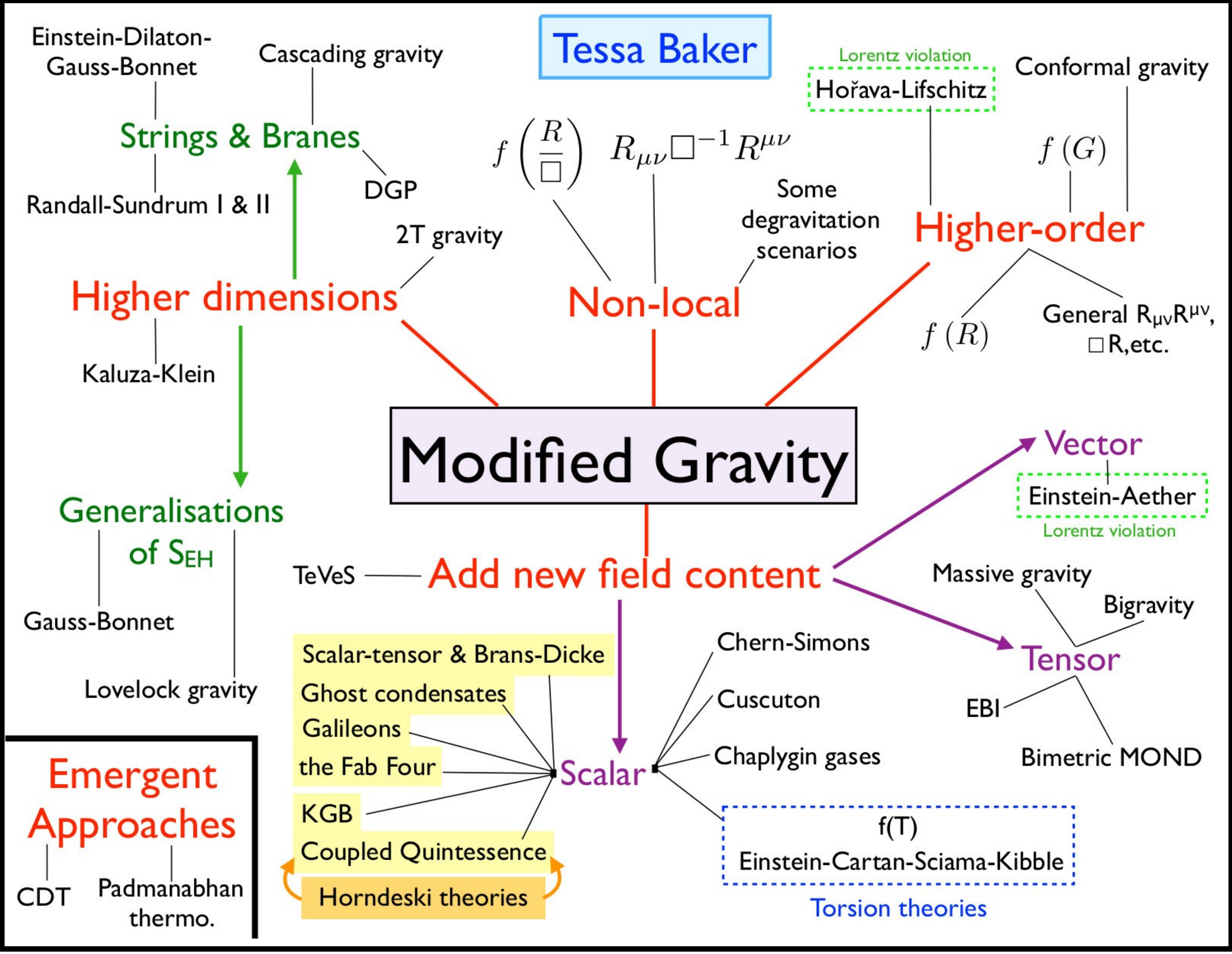
Standard Model of Particle Physics = Do NOT work





Many versions of Modified Gravity to explain DM or DE (each one with its own problems).

This lecture will not (cannot) cover all this.



Many versions of Modified Gravity to explain DM or DE (each one with its own problems).

This lecture will not (cannot) cover all this.

I will focus on Milgromian Dynamics (aka MOND). Empirically motivated alternative to CDM

<u>Tentative</u> Roadmap of the Lecture:

I. The general MOND paradigm

→ Results on galaxy scales

II. Non-relativistic Lagrangian MOND theories

→ Results on galaxy scales & galaxy cluster scales

III. Relativistic Lagrangian MOND theories

→ Results on cosmological scales

I. The general MOND paradigm

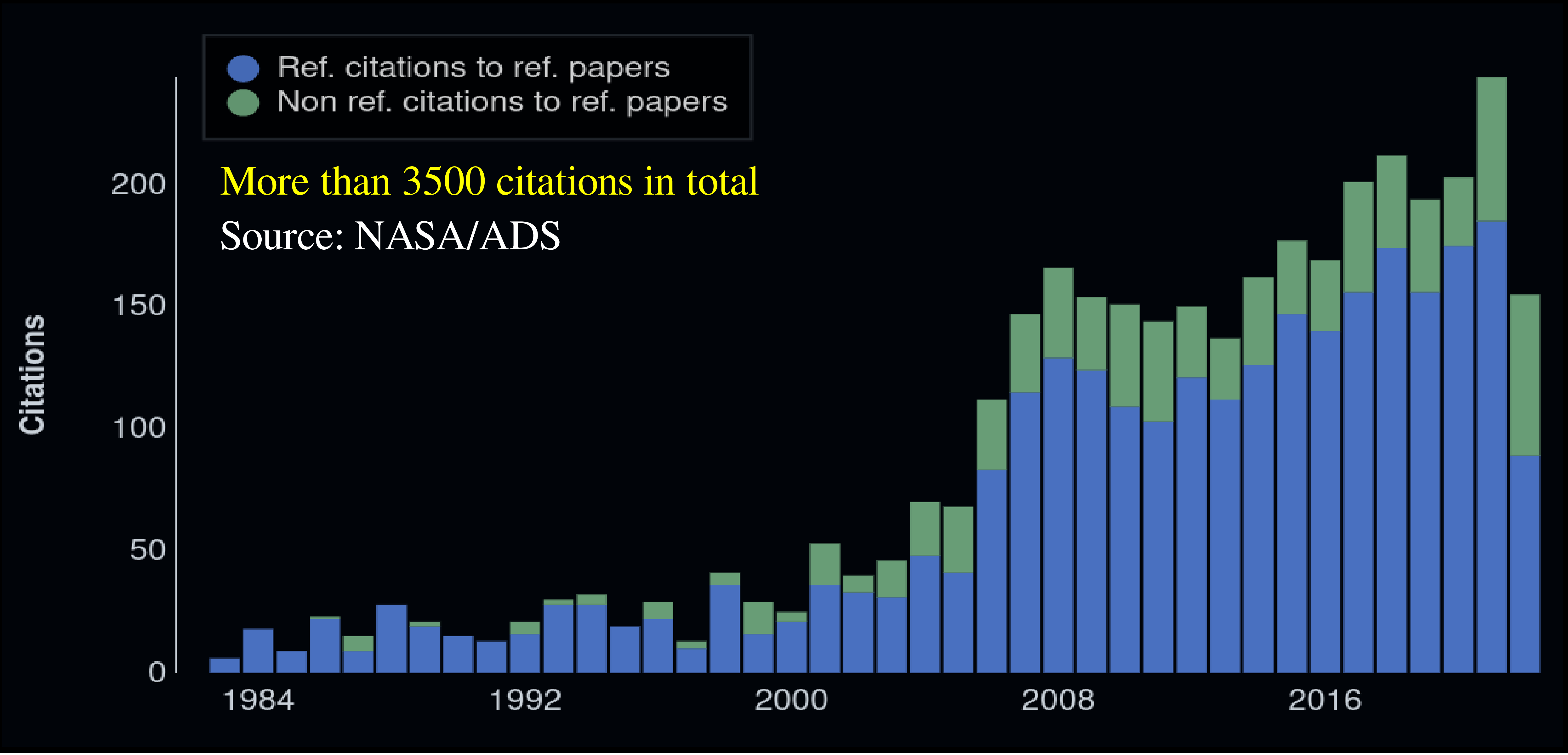
MOND = Modified Newtonian Dynamics

or MilgrOmiaN Dynamics

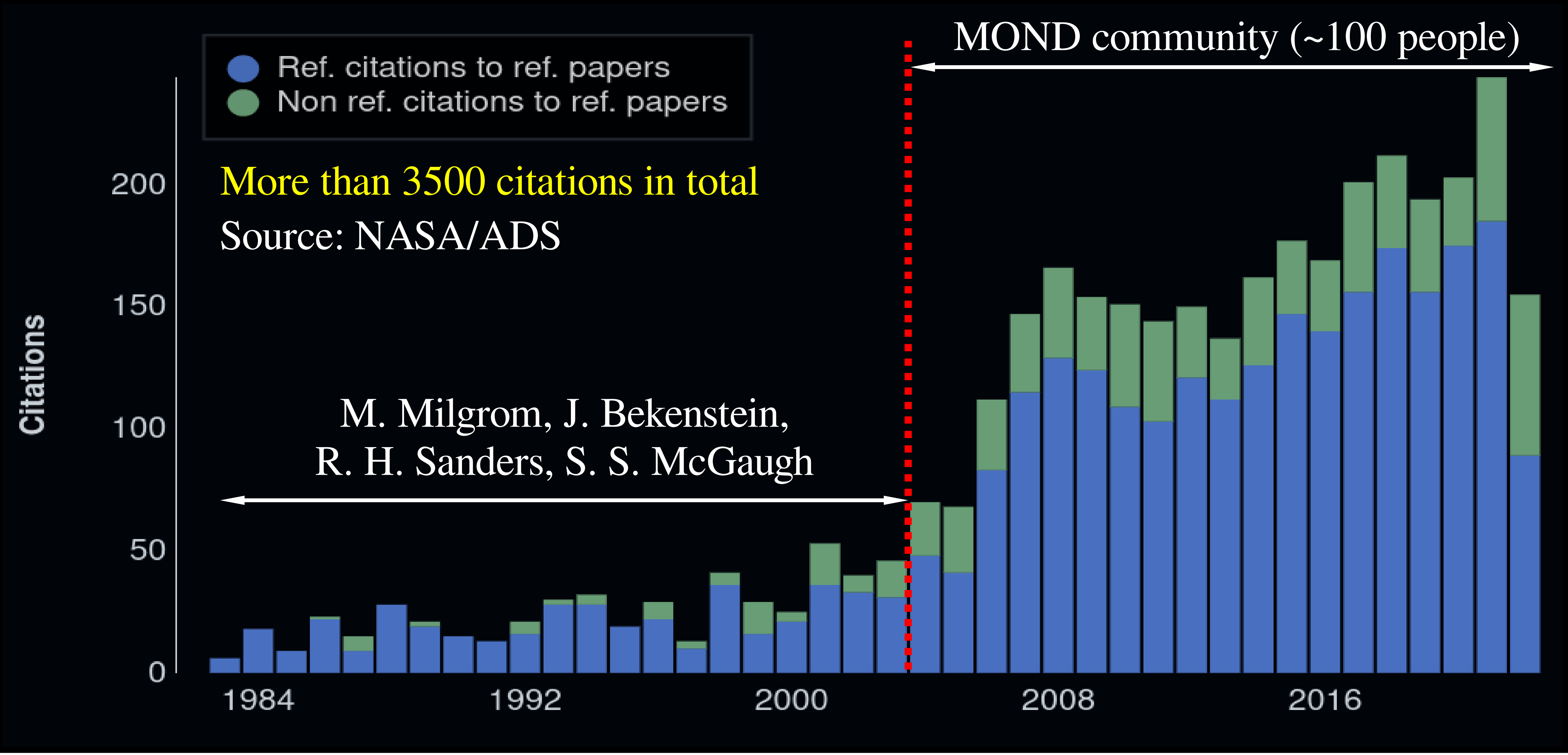
Proposed by Moderhai Milgrom (1983a, b, c, ApJ).

MOND is a general paradigm that includes different theories at the non-relativistic & relativistic level.

Citations to the original MOND trilogy (Milgrom 1983a, b, c)



Citations to the original MOND trilogy (Milgrom 1983a, b, c)



General MOND postulates (at the non-relativistic level)

1) New constant of Physics: $a_0 \sim 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

General MOND postulates (at the non-relativistic level)

1) New constant of Physics: $a_0 \sim 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow \bar{a} = \bar{g}_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

General MOND postulates (at the non-relativistic level)

1) New constant of Physics: $a_0 \sim 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow \bar{a} = \bar{g}_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow$ scale invariance (Milgrom 2009, ApJ): $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$

$$a = \sqrt{g_N a_0}$$

General MOND postulates (at the non-relativistic level)

1) New constant of Physics: $a_0 \sim 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow \bar{a} = \bar{g}_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla} \phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow$ scale invariance (Milgrom 2009, ApJ): $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$

$$a = \sqrt{g_N a_0} \quad \Longrightarrow \quad \frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$$

Circular orbit at large R

General MOND postulates (at the non-relativistic level)

1) New constant of Physics: $a_0 \sim 10^{-10}$ m/s^2

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) For $a \gg a_0 \rightarrow \bar{a} = \bar{g}_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2\vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

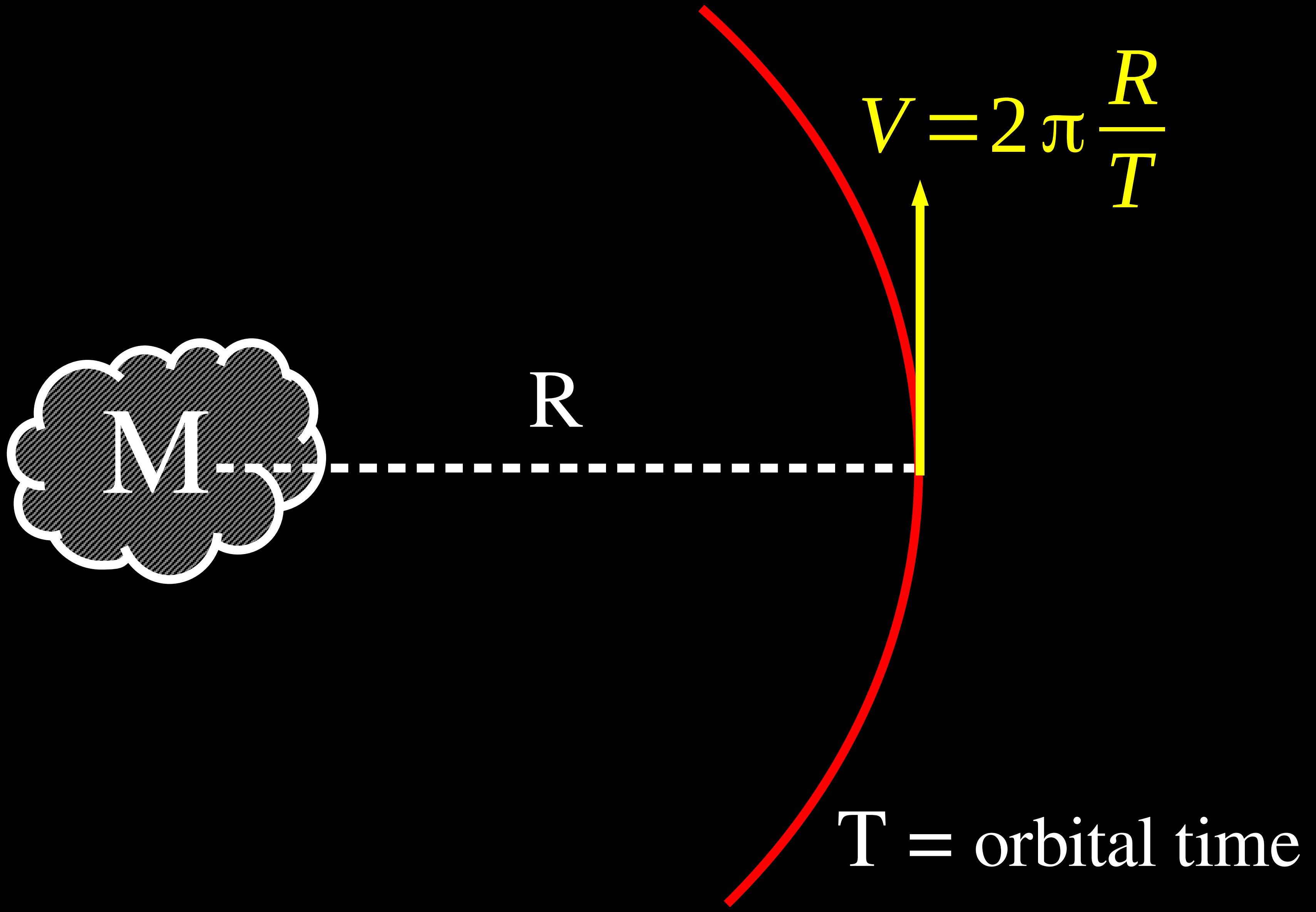
$\vec{g}_N = -\vec{\nabla}\phi_N$ Newtonian gravitational field (from the Poisson's equation)

3) For $a \ll a_0 \rightarrow$ scale invariance (Milgrom 2009, ApJ): $(\vec{x}, t) \rightarrow (\lambda\vec{x}, \lambda t)$

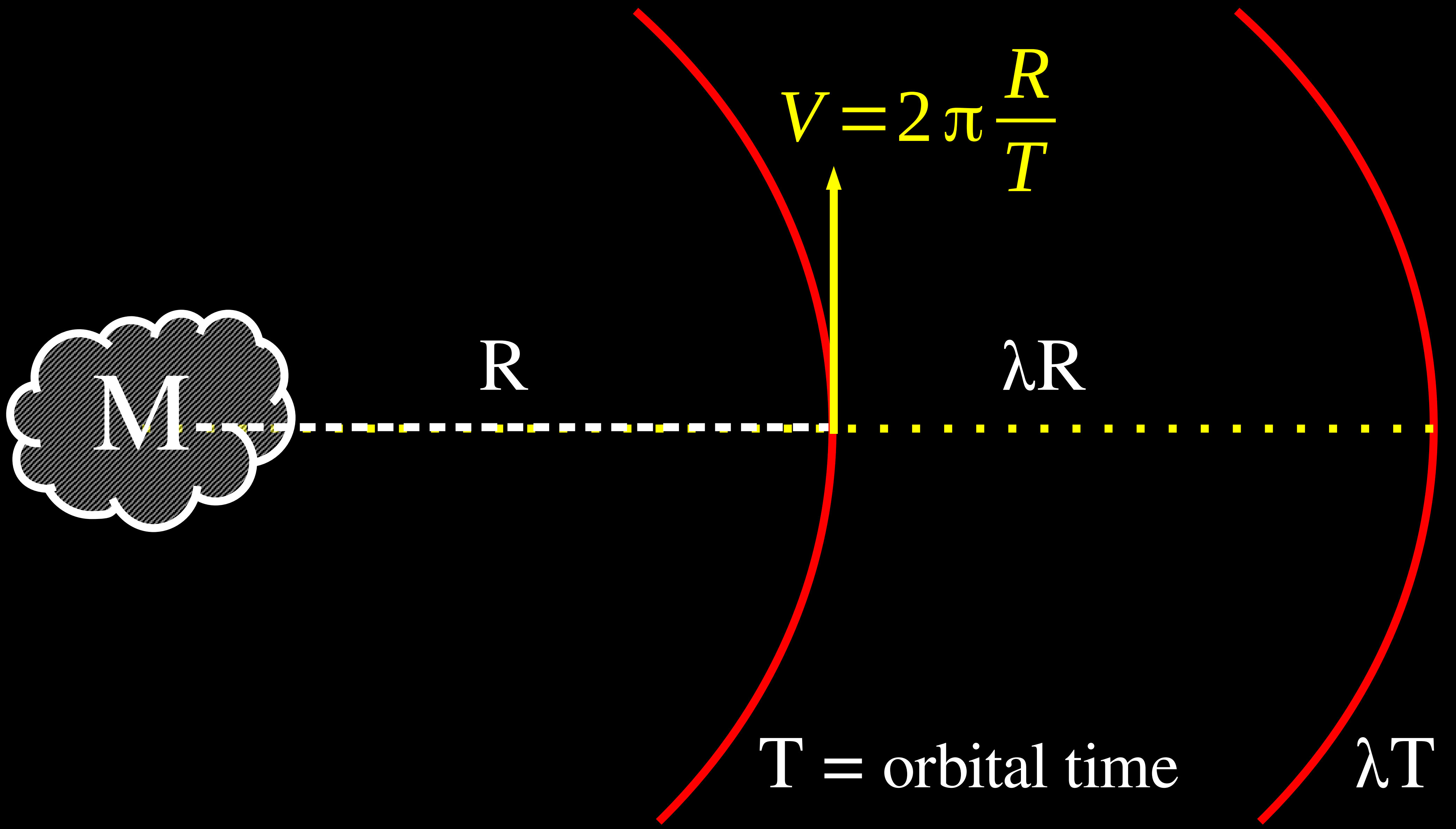
$a = \sqrt{g_N a_0}$ $\Rightarrow$ $\frac{V^2}{R} = \sqrt{\frac{a_0 G M_b}{R^2}}$ Flat rotation curve!

Circular orbit at large R

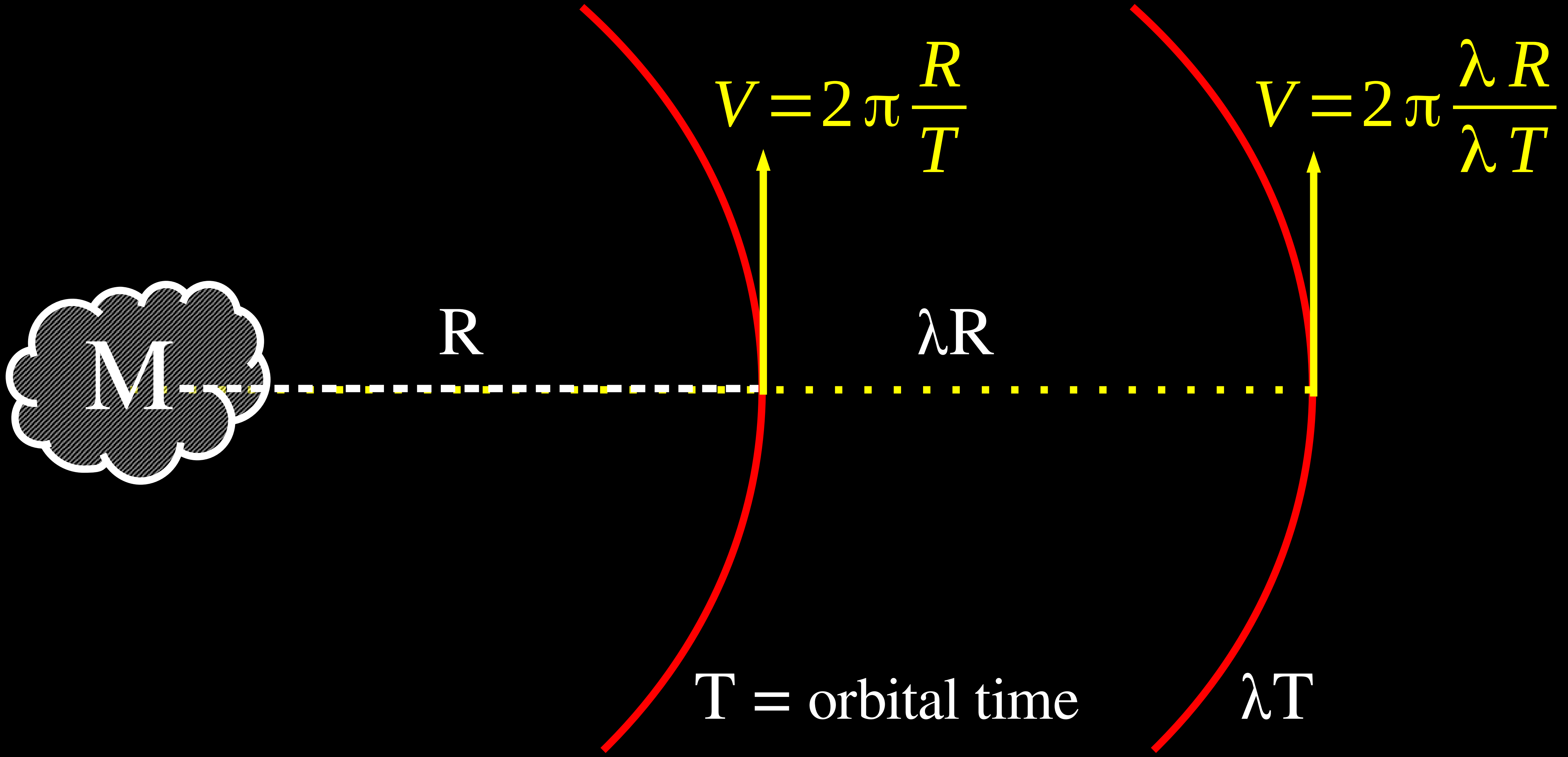
Intuitive Cartoon: Scale Invariance = Flat Rotation Curves



Intuitive Cartoon: Scale Invariance = Flat Rotation Curves



Intuitive Cartoon: Scale Invariance = Flat Rotation Curves



A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES[1]

M. MILGROM
Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study
Received 1982 February 4; accepted 1982 December 28

40 year ago!

ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density Σ falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of Σ, the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant, a_0, determined in a few independent ways is approximately $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$, which is of the order of $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$.

The main predictions are:

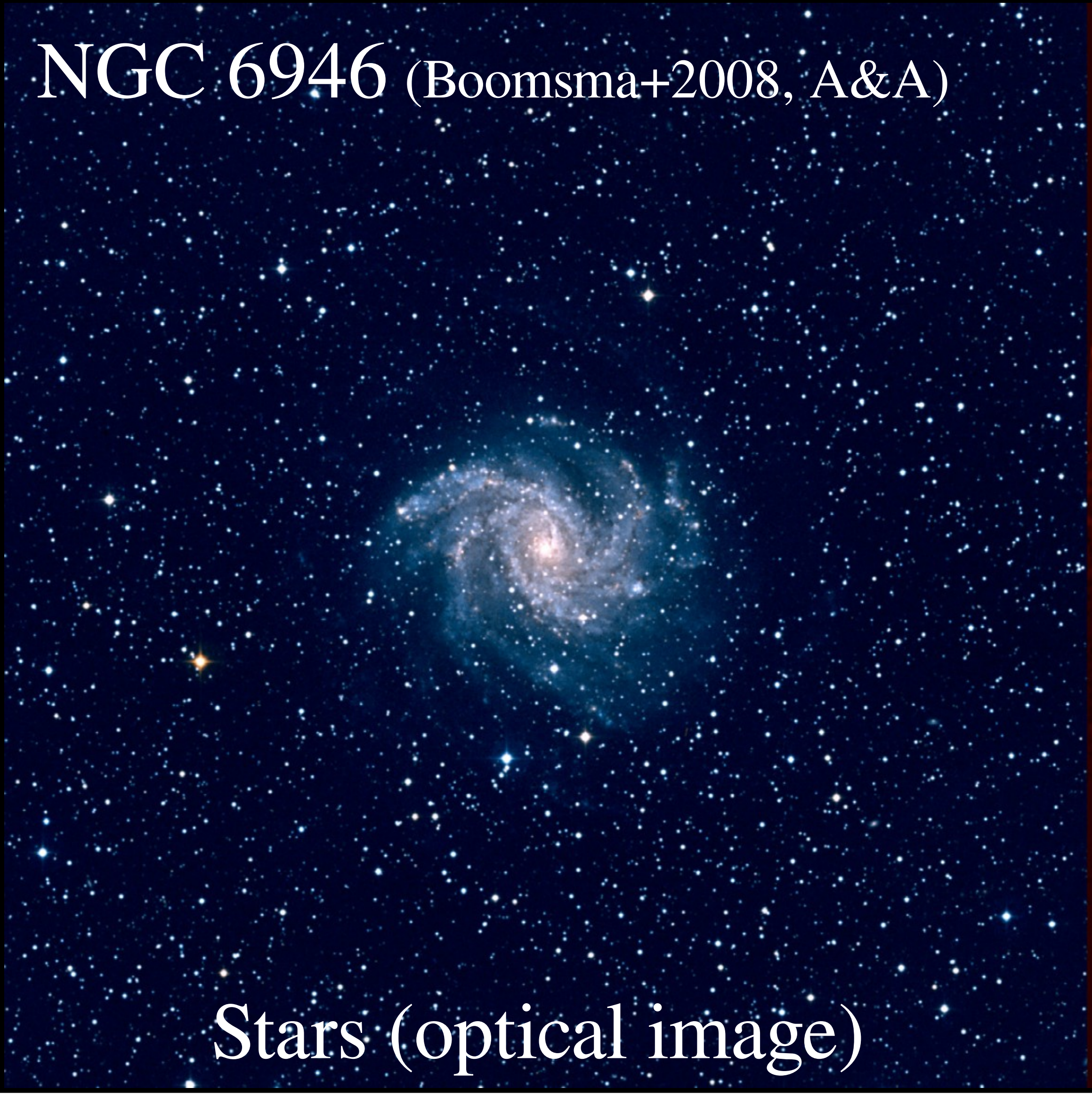
1. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

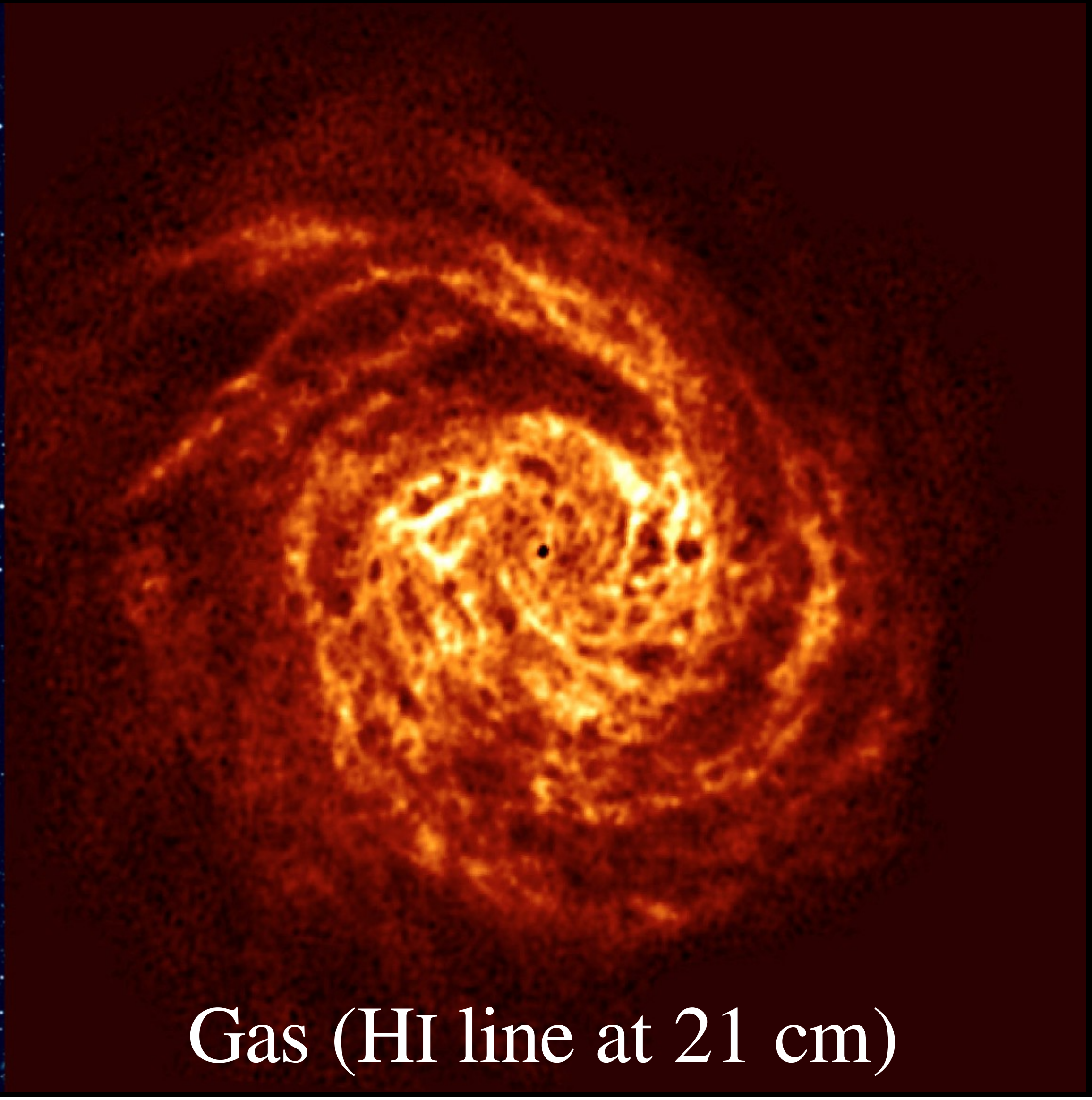
2. The $V_\infty^4 = a_0 GM$ relation should hold exactly.

3. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing r in a predictable way.

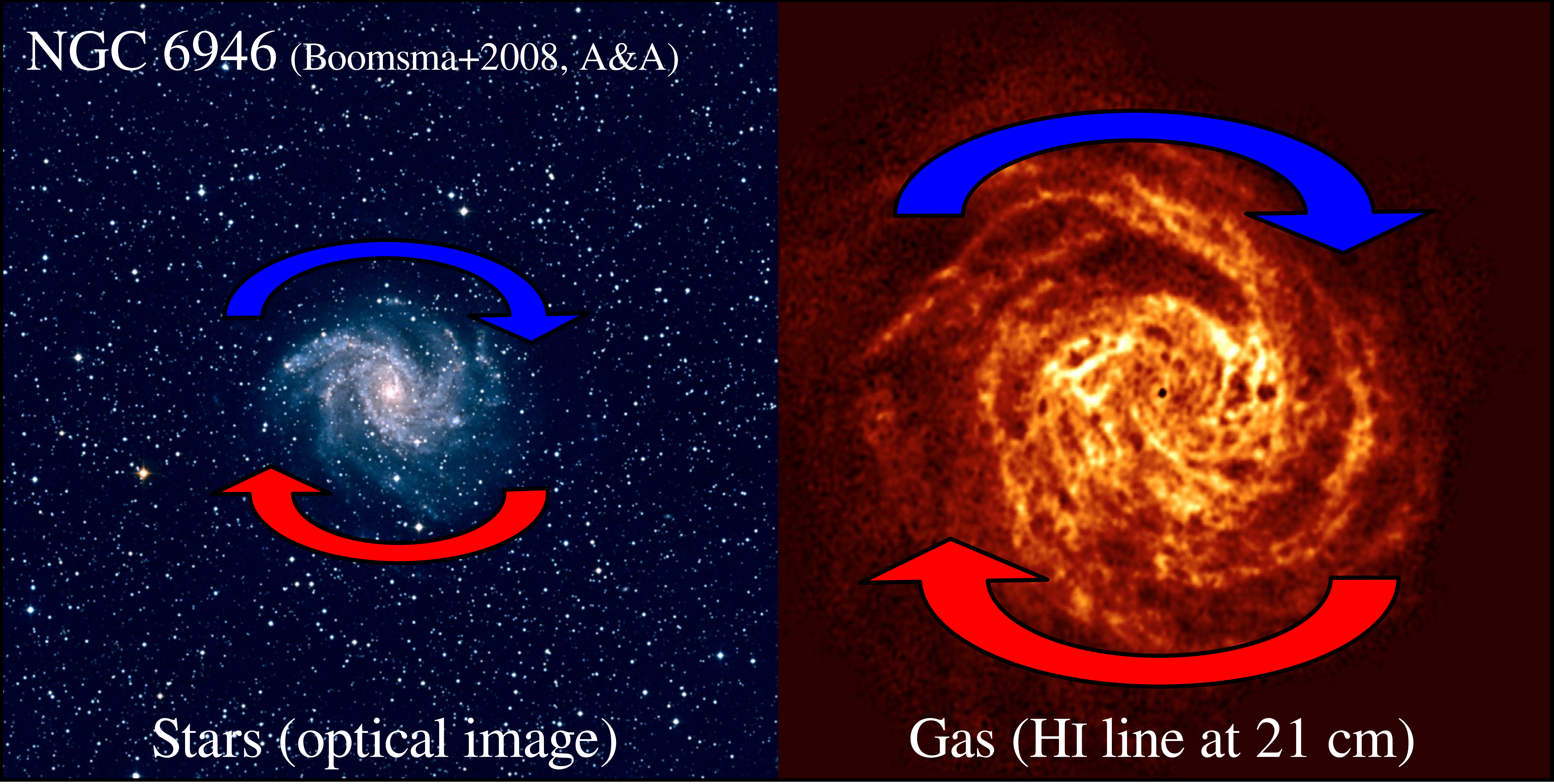
(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies

(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies

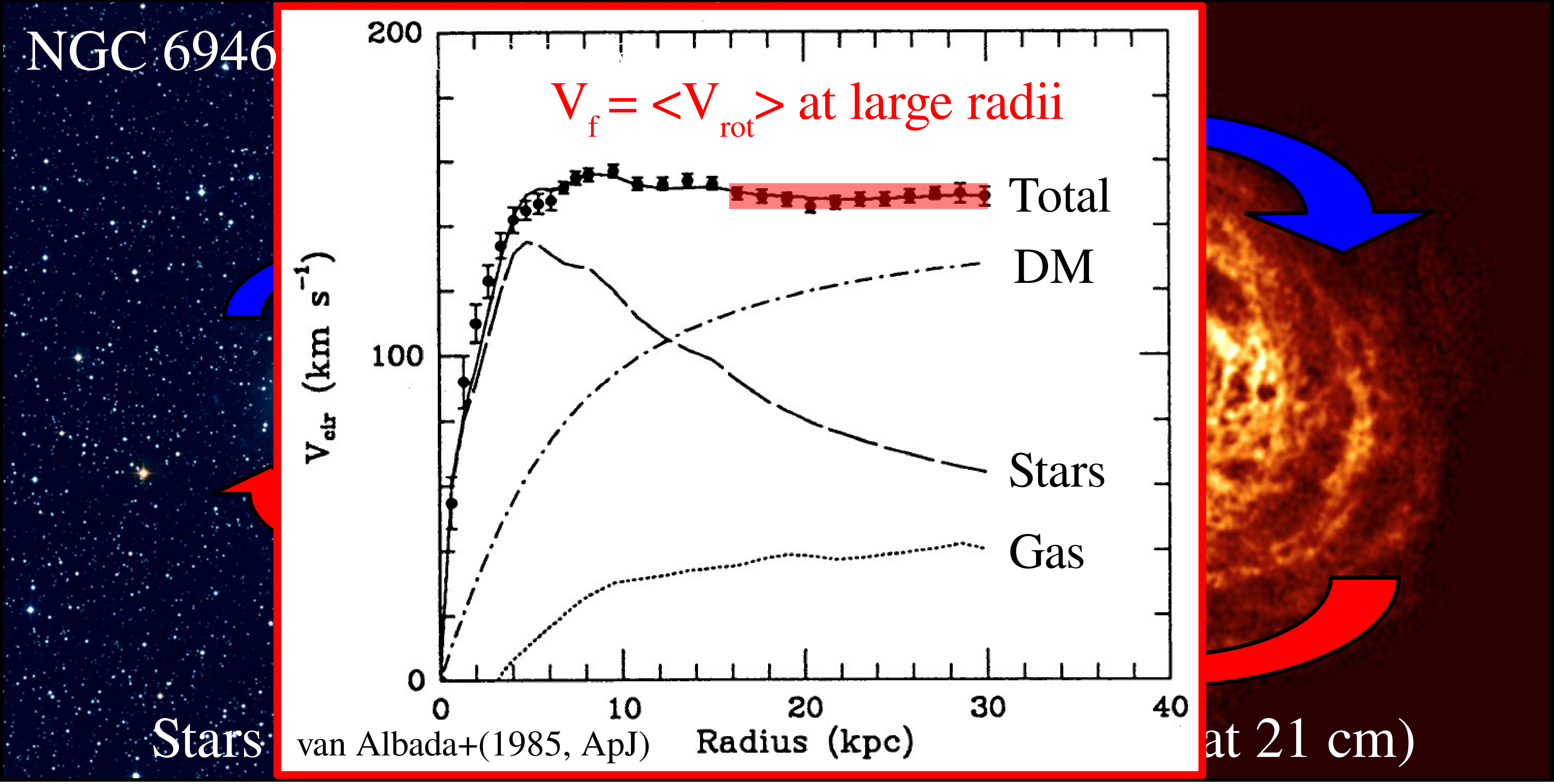




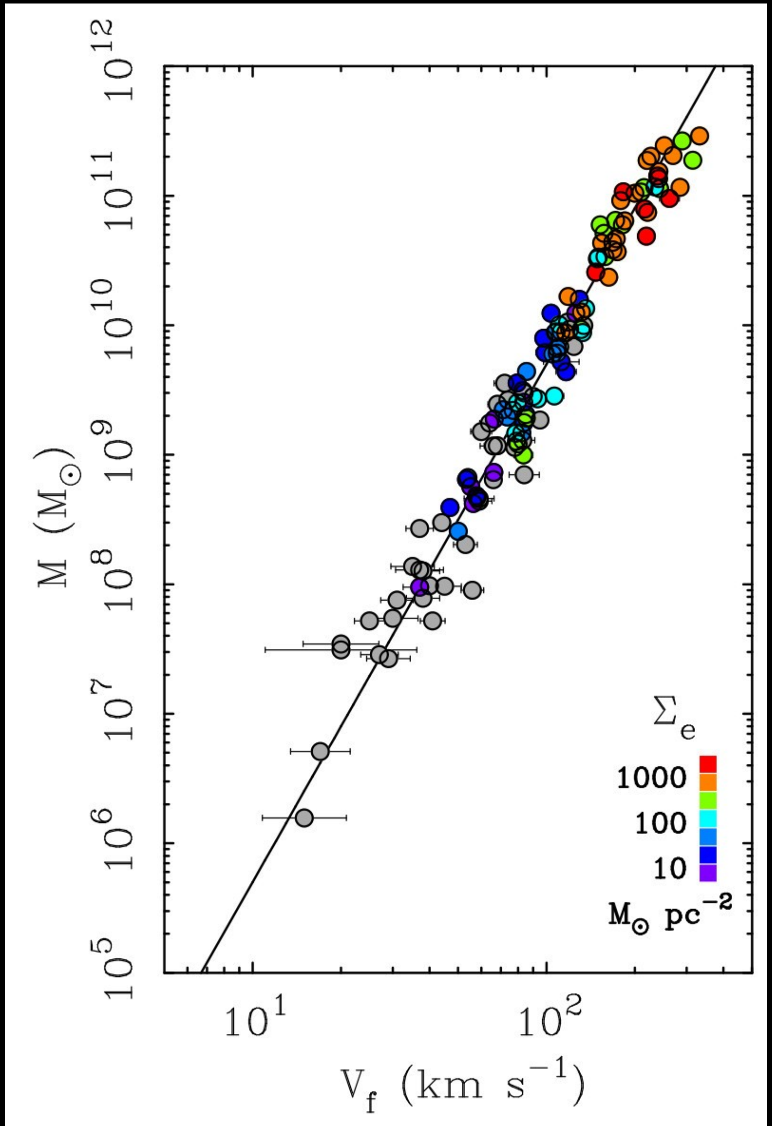
(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies



(1) $V_f^{\,4} = a_0 \, G \, M_b$ for circular orbits → rotation-supported galaxies



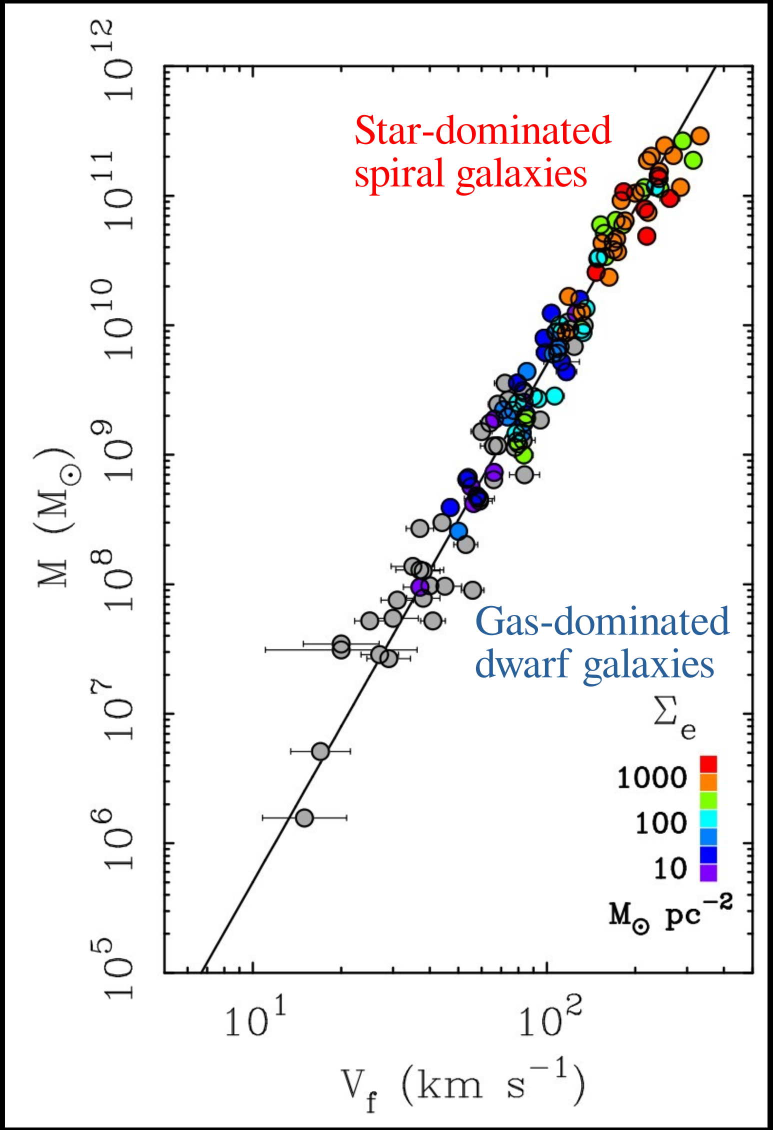
(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^{\,4} = a_0 G M_b$ for circular orbits → rotation-supported galaxies



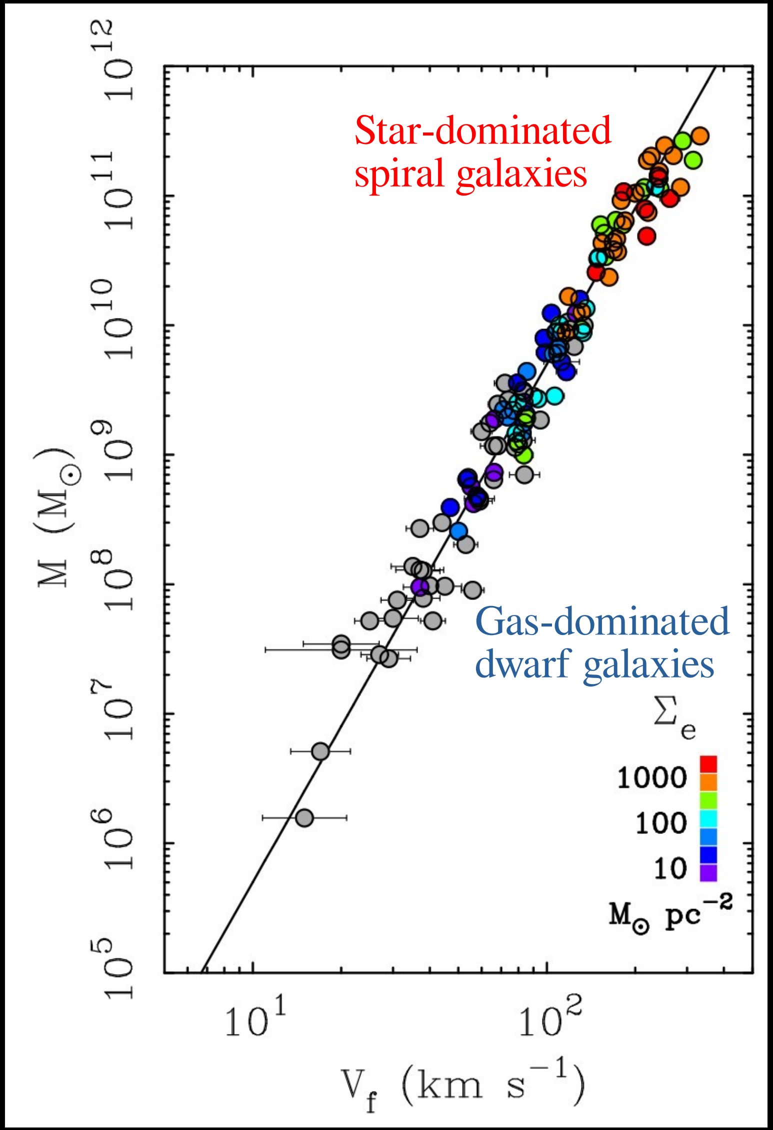
Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

Four a-priori independent predictions in one equation:

(i) The relevant quantities are M_b (stars+gas) and V_f → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

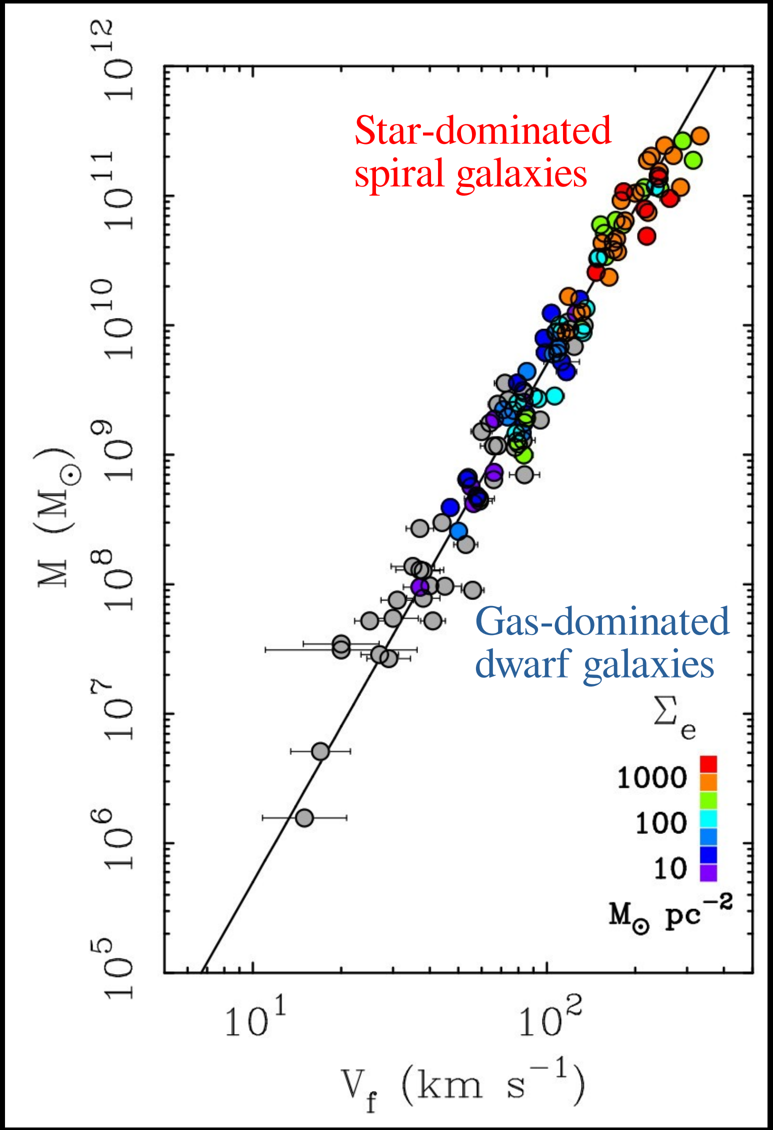
Four a-priori independent predictions in one equation:

(i) The relevant quantities are M_b (stars+gas) and V_f → OK

(ii) Slope should be exactly 4 → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

Four a-priori independent predictions in one equation:

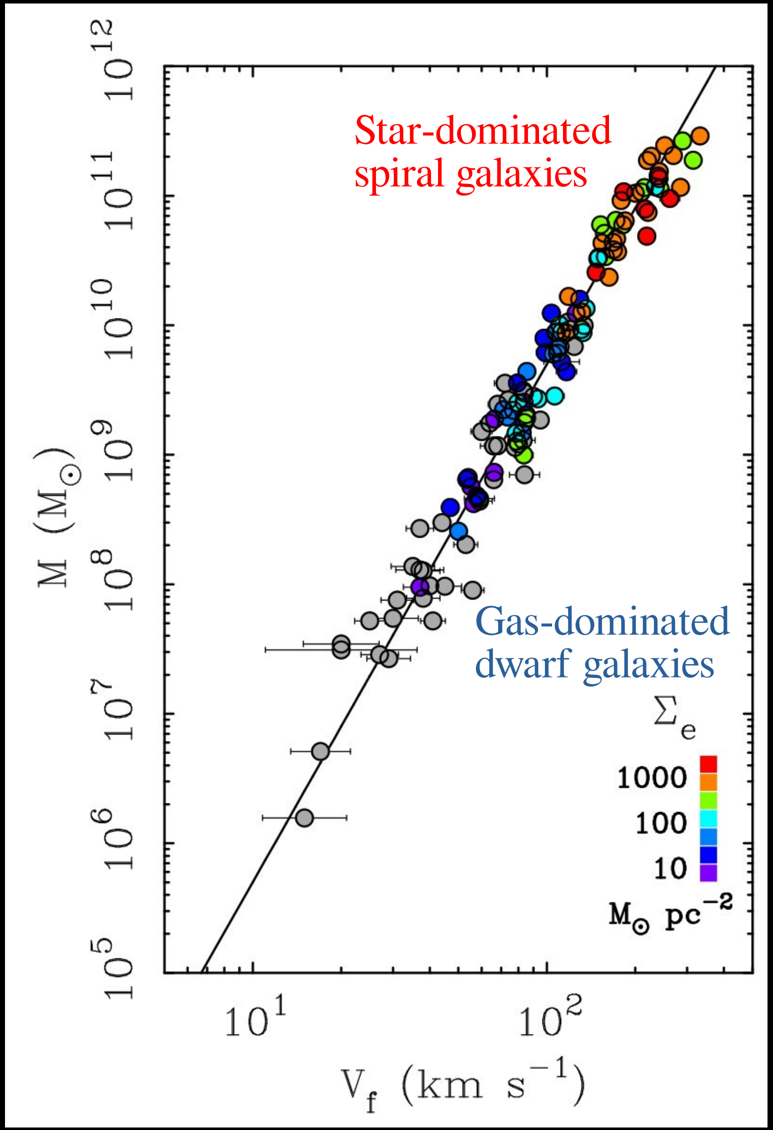
(i) The relevant quantities are M_b (stars+gas) and V_f → OK

(ii) Slope should be exactly 4 → OK

(iii) Normalization is a_0G → OK with other estimates

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^{\,4} = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

Four a-priori independent predictions in one equation:

(i) The relevant quantities are M_b (stars+gas) and V_f → OK

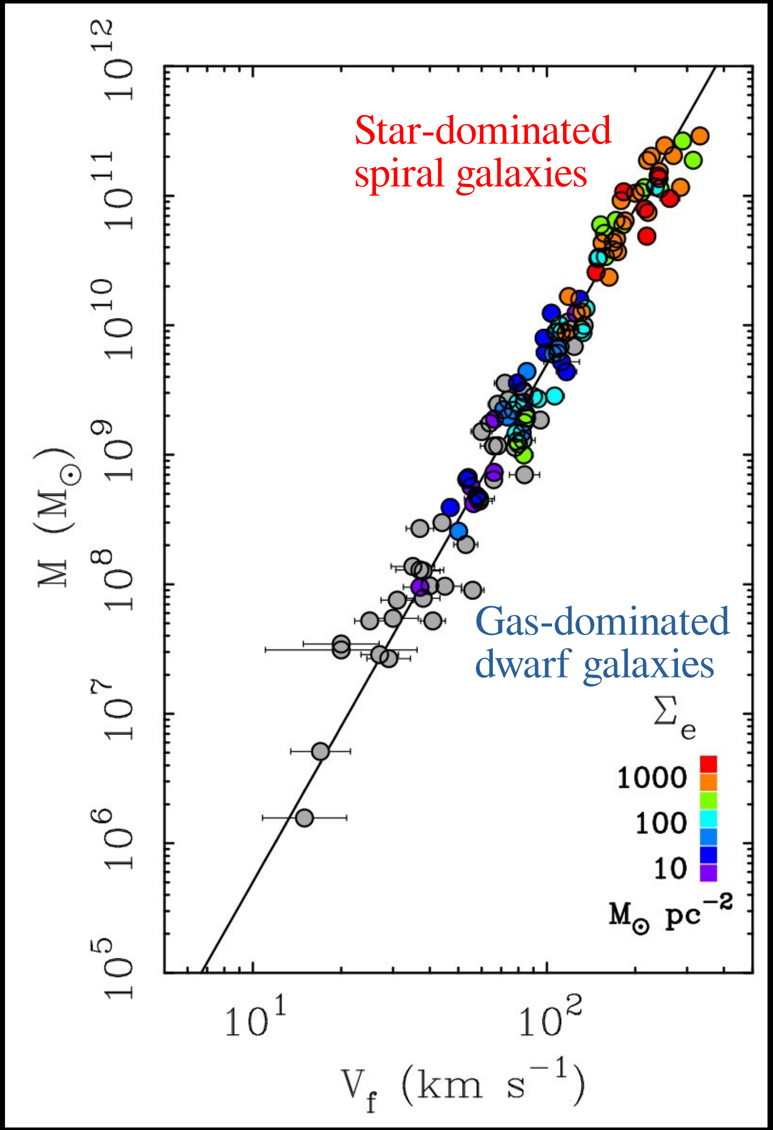
(ii) Slope should be exactly 4 → OK

(iii) Normalization is a_0G → OK with other estimates

(iv) No dependence on other quantities → OK

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^4 = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

Four a-priori independent predictions in one equation:

(i) The relevant quantities are M_b (stars+gas) and V_f → OK

(ii) Slope should be exactly 4 → OK

(iii) Normalization is $a_0 G$ → OK with other estimates

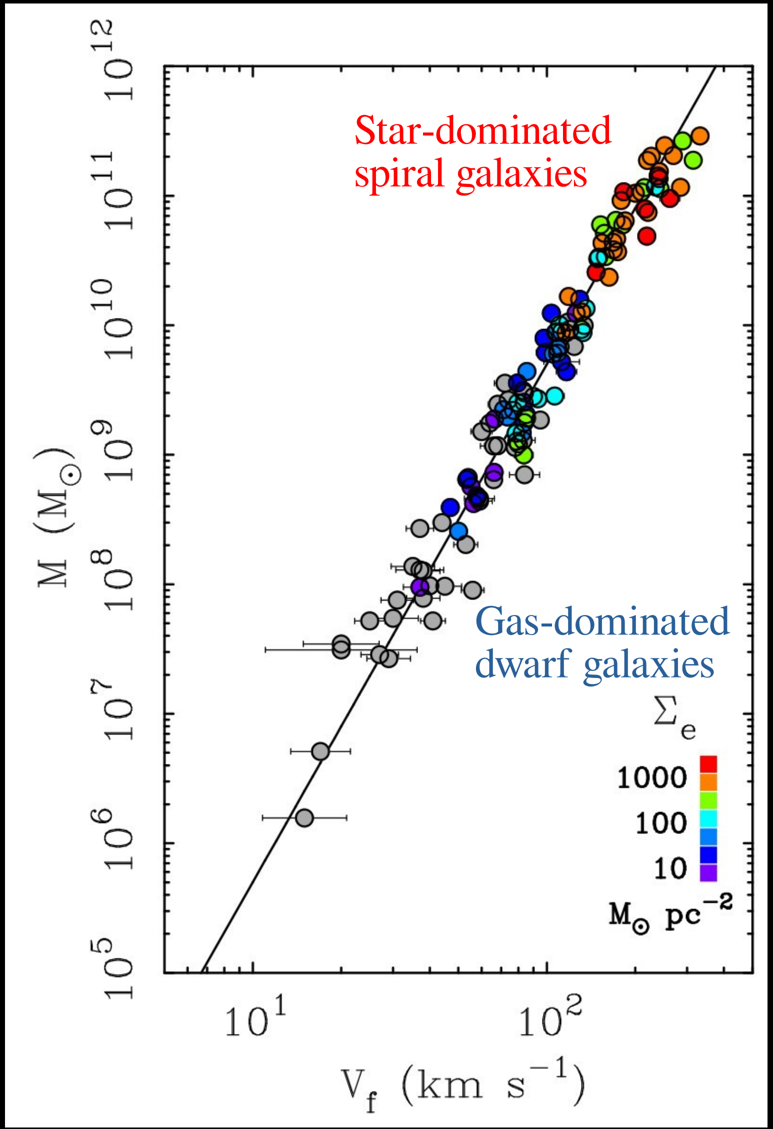
(iv) No dependence on other quantities → OK

This is weird in a Newtonian+DM context:

$$\frac{V^2}{R} = \frac{G M_{tot}}{R^2} \quad f_b = \frac{M_b}{M_{tot}} \quad \Sigma_b = \frac{M_b}{\pi R^2}$$

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

(1) $V_f^{\,4} = a_0 G M_b$ for circular orbits → rotation-supported galaxies



Tully-Fisher relation (1977, A&A): L_B *vs* HI linewidth

Four a-priori independent predictions in one equation:

(i) The relevant quantities are M_b (stars+gas) and V_f → OK

(ii) Slope should be exactly 4 → OK

(iii) Normalization is $a_0 G$ → OK with other estimates

(iv) No dependence on other quantities → OK

This is weird in a Newtonian+DM context:

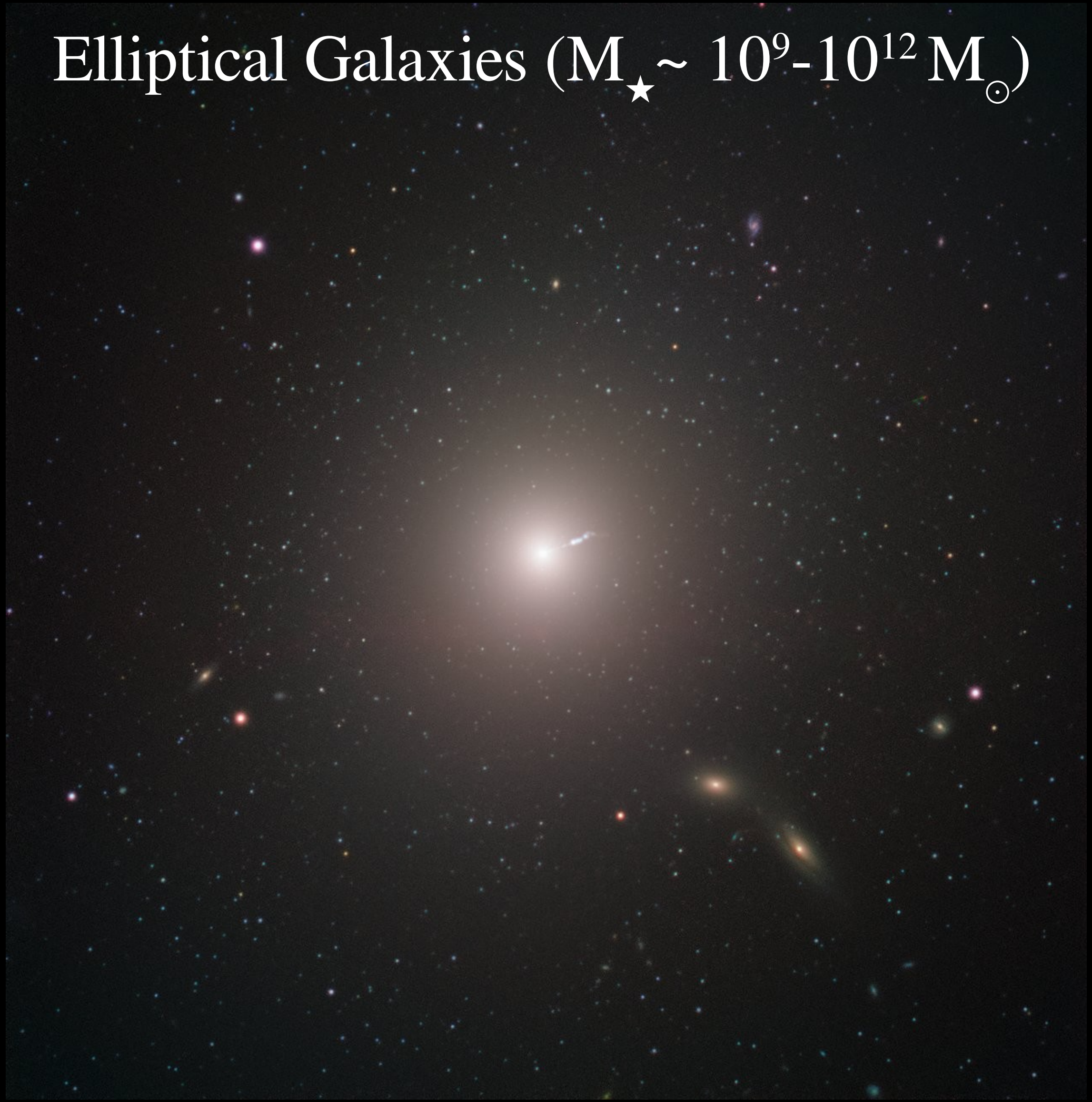
$$\frac{V^2}{R} = \frac{G M_{tot}}{R^2} \quad f_b = \frac{M_b}{M_{tot}} \quad \Sigma_b = \frac{M_b}{\pi R^2} \Rightarrow V^4 = \frac{\pi G^2}{f_b^2} \Sigma_b M_b$$

McGaugh+(2000, 2005, 2010), Verheijen+2001, Lelli+(2016, 2019, 2022)

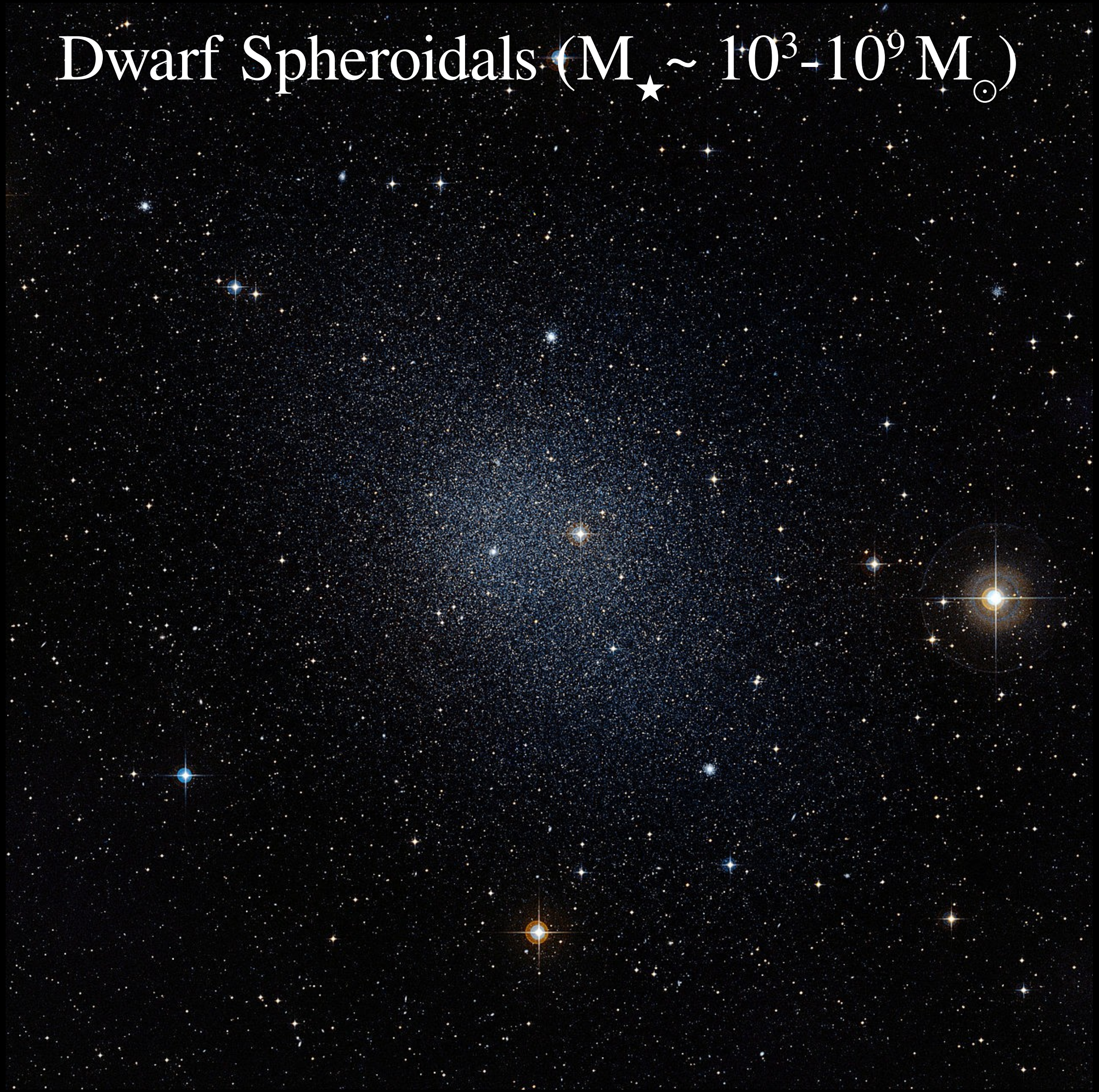
(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals

(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals

Elliptical Galaxies ($M_\star \sim 10^9$-10^{12} $M_\odot$)

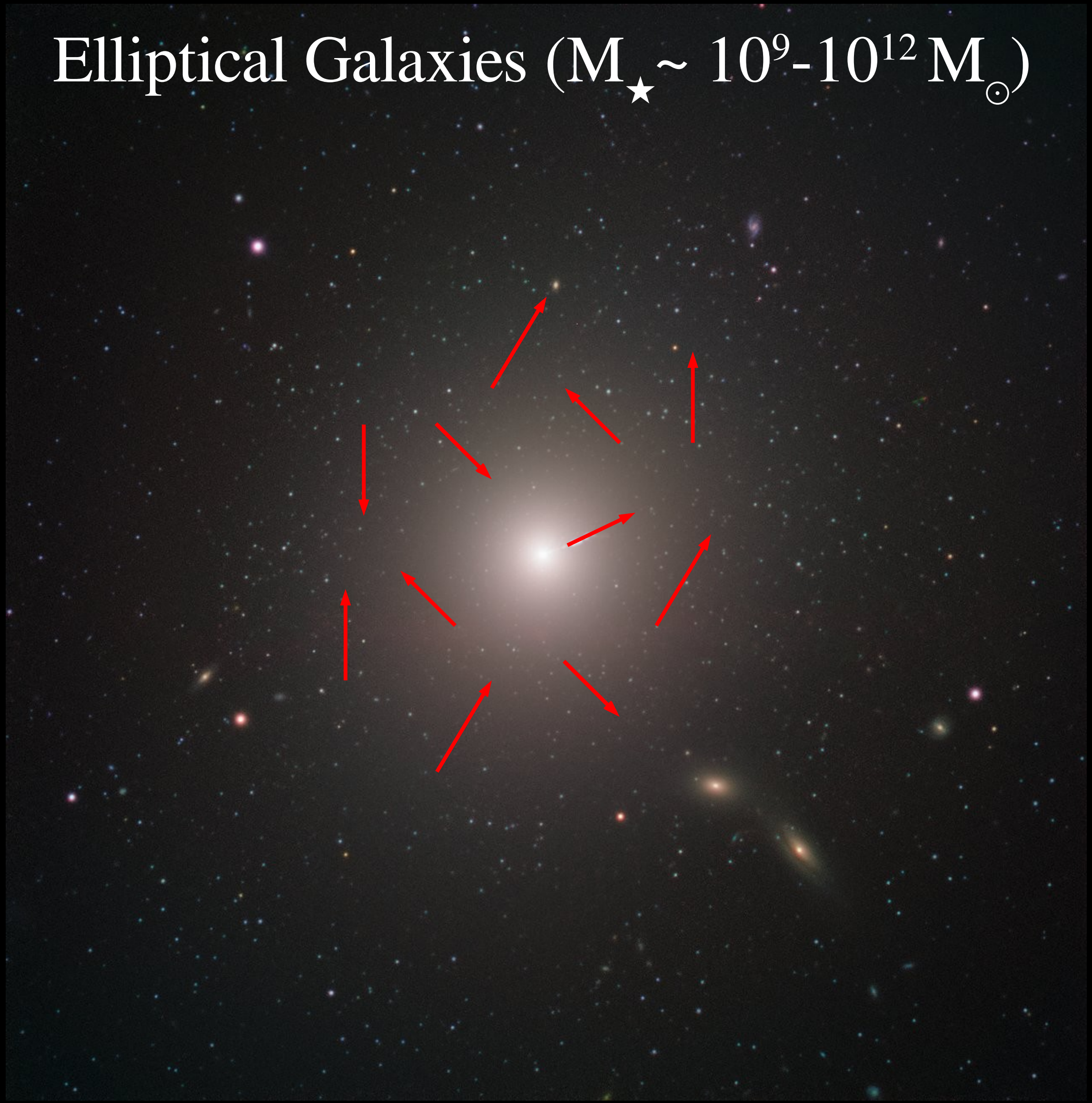


Dwarf Spheroidals ($M_\star \sim 10^3$-10^9 $M_\odot$)

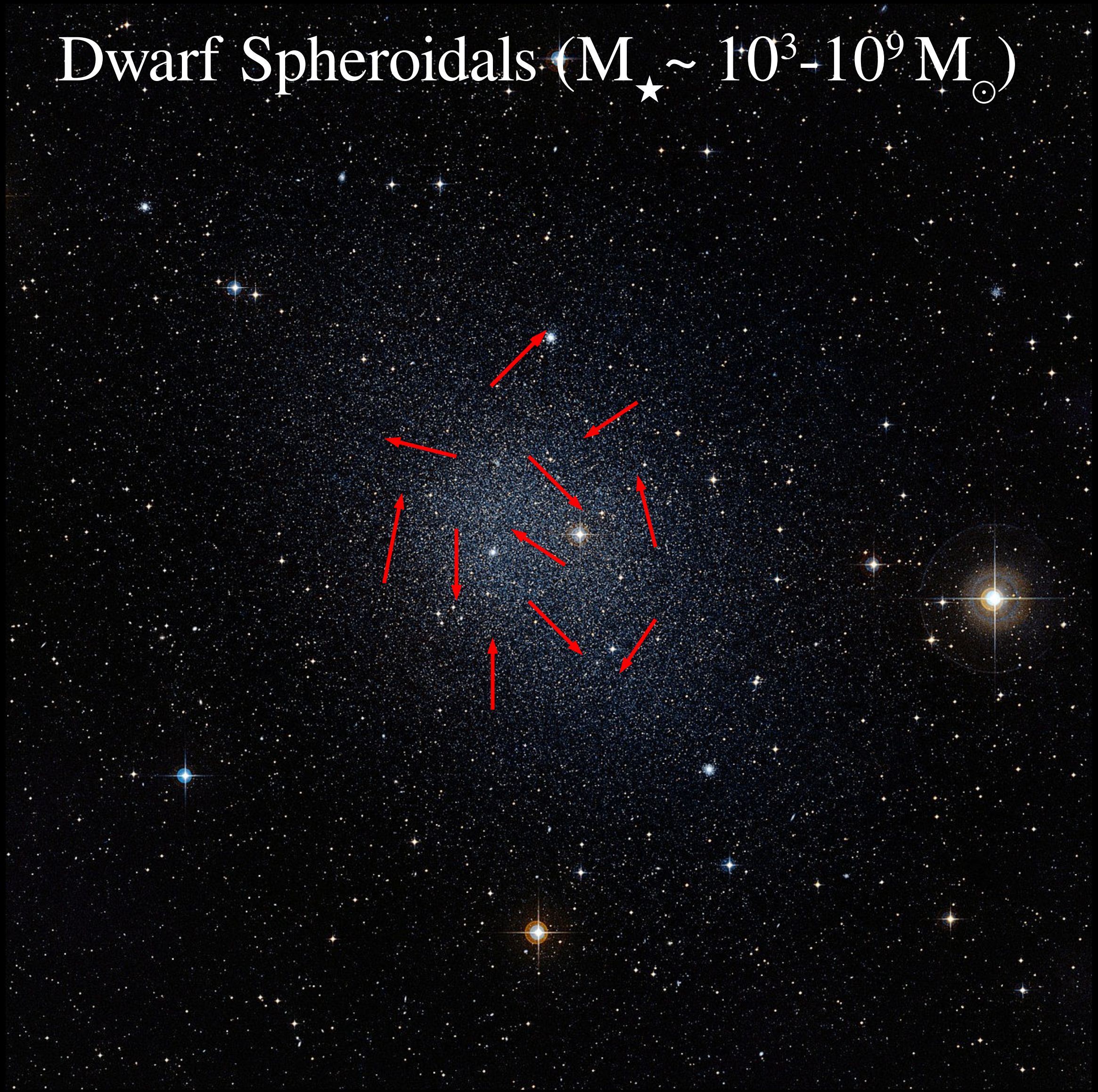


(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals

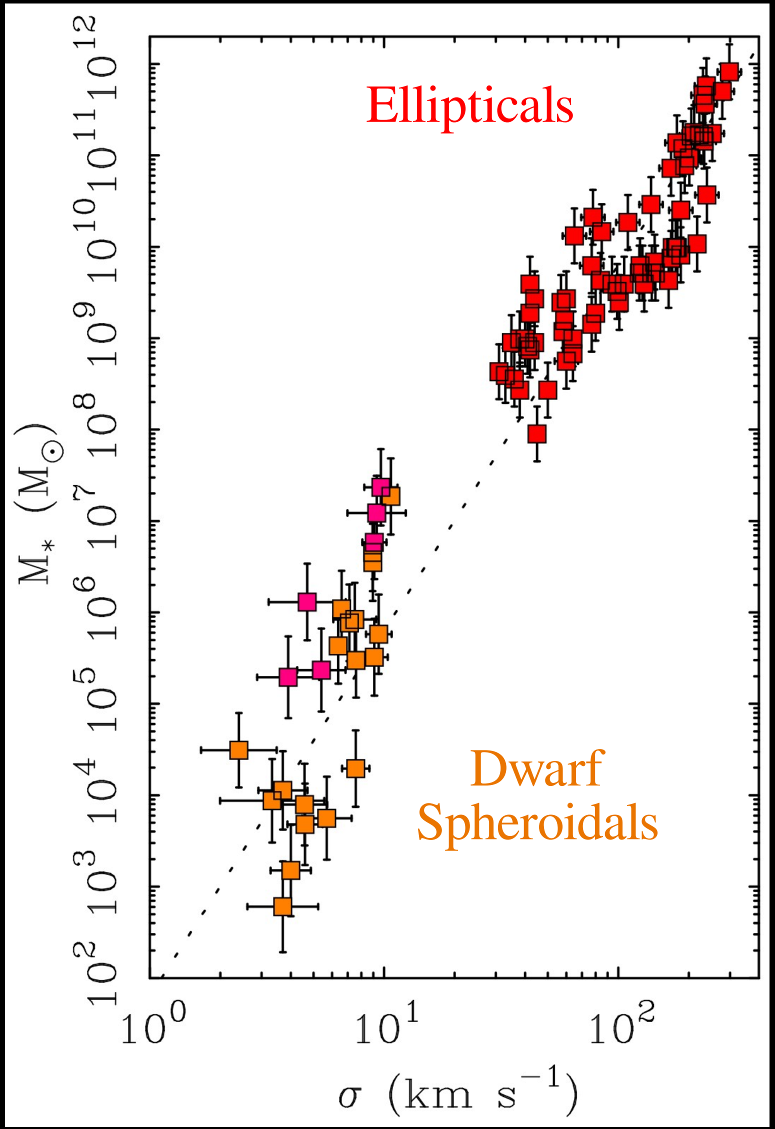
Elliptical Galaxies ($M_\star \sim 10^9$-$10^{12}\,M_\odot$)



Dwarf Spheroidals ($M_\star \sim 10^3$-$10^9\,M_\odot$)



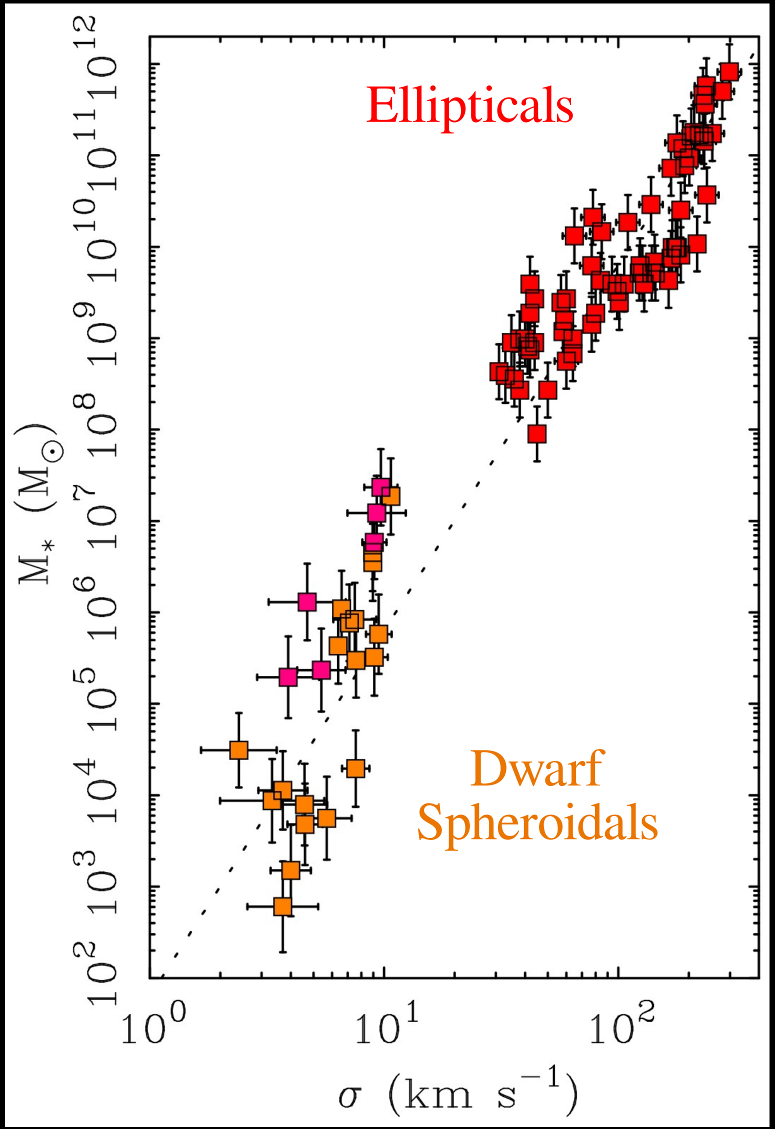
(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals



Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals

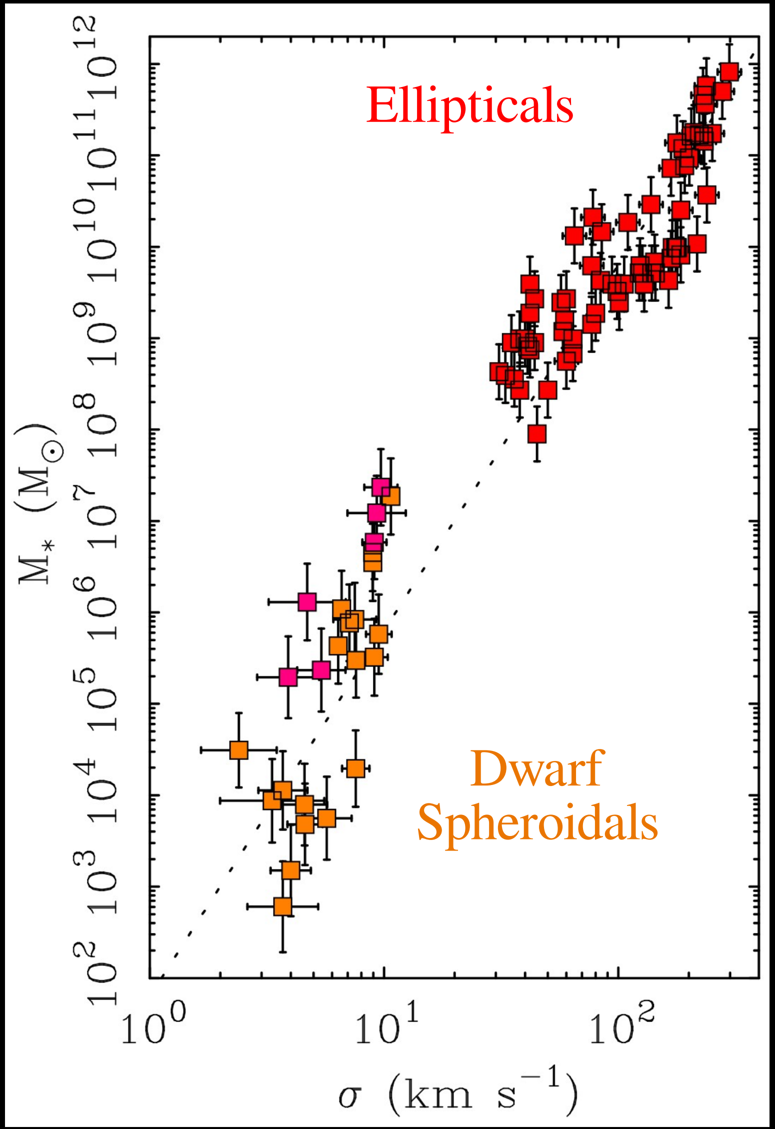


Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(2) $\sigma_V{}^4 = a_0\, G\, M_b$ for quasi-isothermal systems → pressure-supported gals



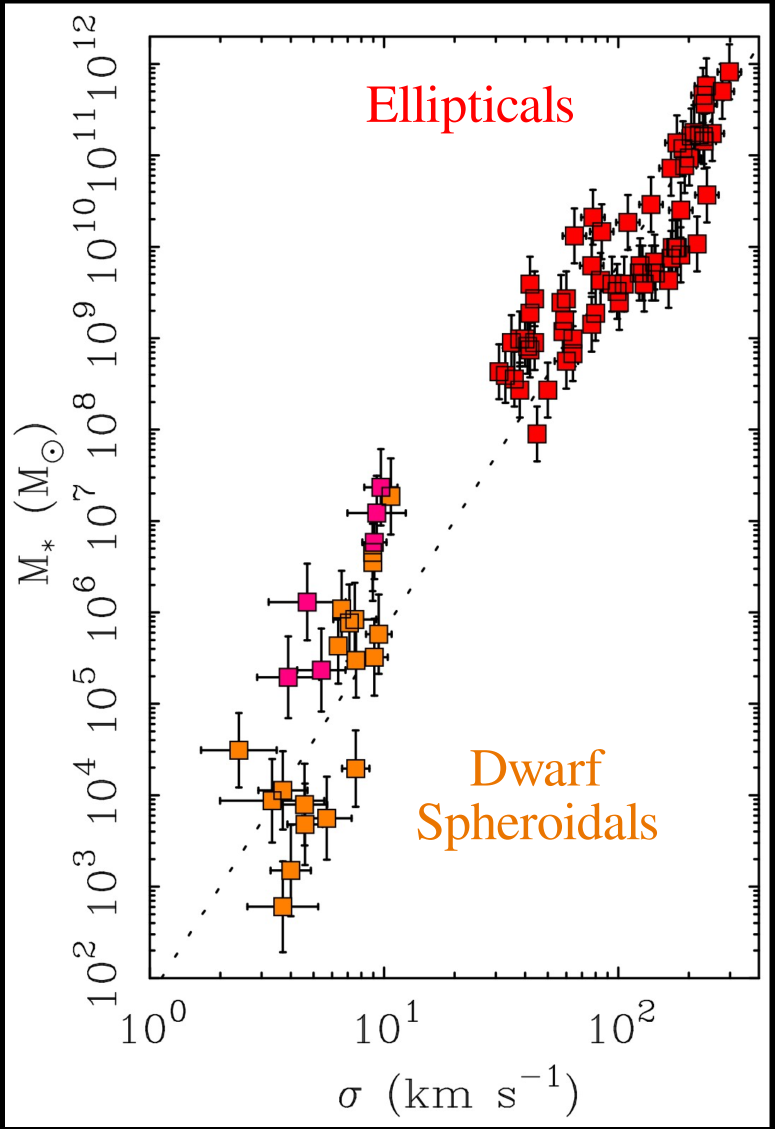
Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(ii) Normalization is a_0G → OK with BTFR estimate!

(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals



Faber-Jackson relation (1976, ApJ) for ellipticals

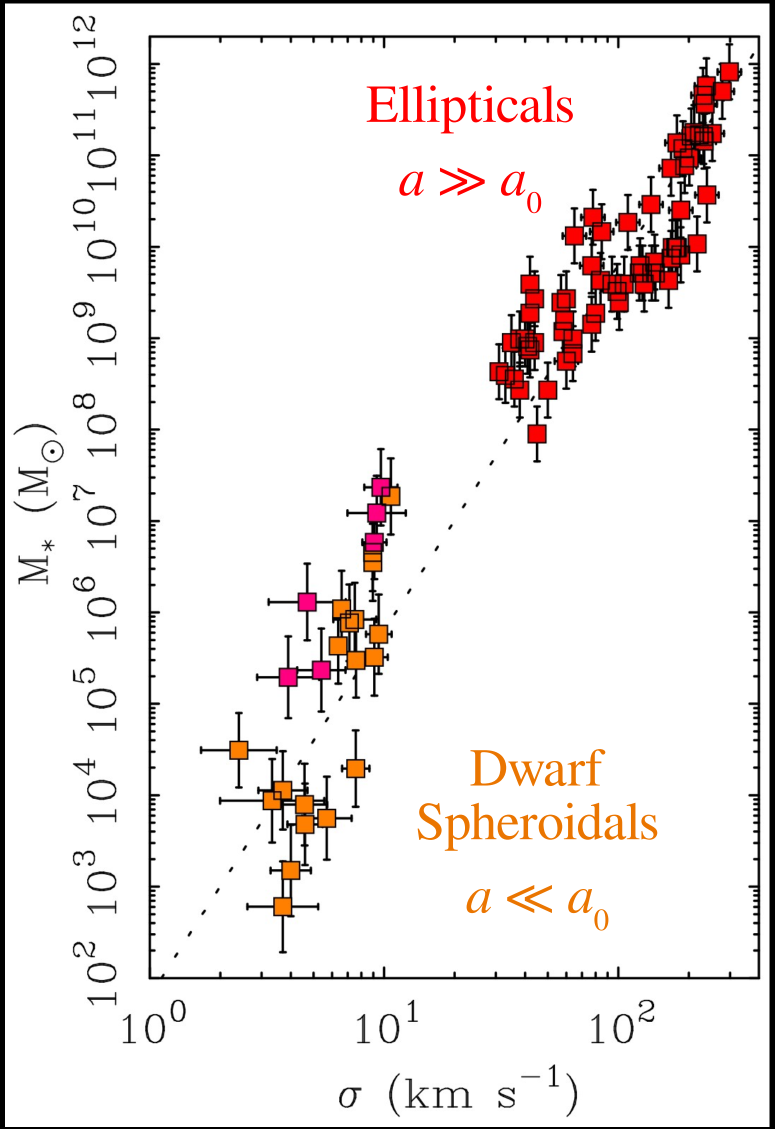
Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(ii) Normalization is a_0G → OK with BTFR estimate!

(iii) No dependence on other quantities IF $a \ll a_0$ → OK

(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals



Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(ii) Normalization is a_0G → OK with BTFR estimate!

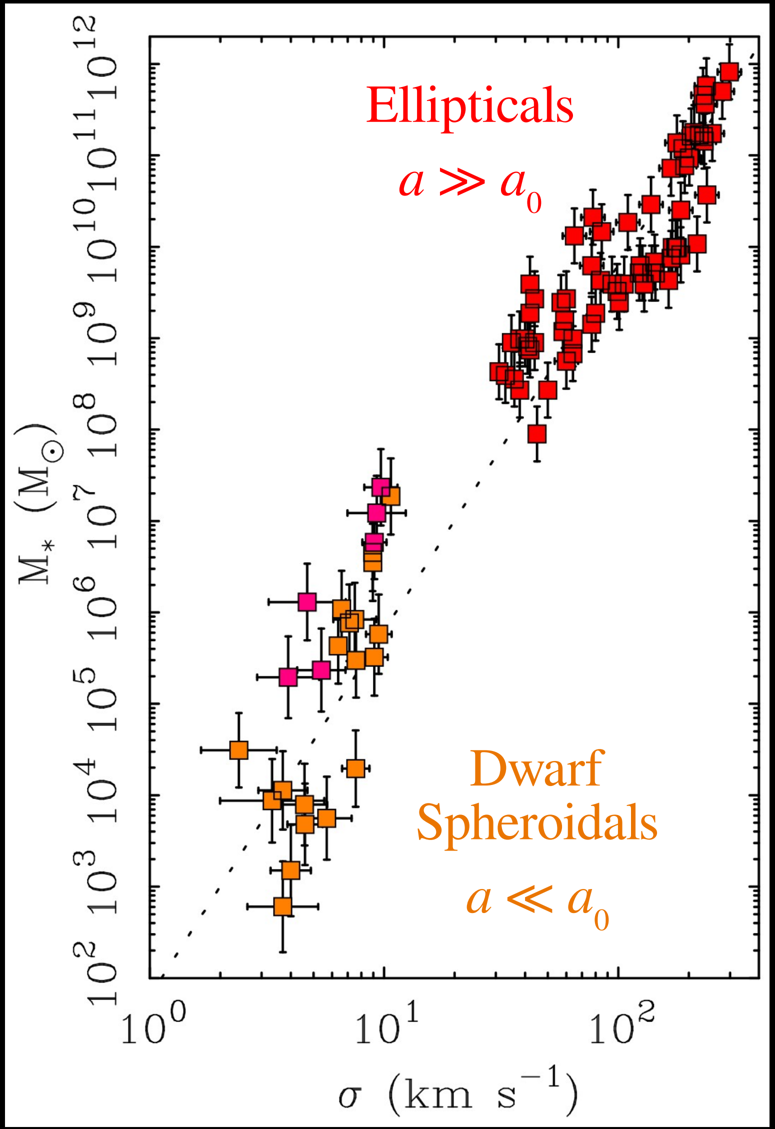
(iii) No dependence on other quantities IF $a \ll a_0$ → OK

σ_V is measured at R < R_e (containing half luminosity):

For dwarf spheroidals: $a \ll a_0$ at R<R_e → MOND regime

For giant ellipticals: $a \gg a_0$ at R<R_e → Newtonian regime

(2) $\sigma_V^4 = a_0 G M_b$ for quasi-isothermal systems → pressure-supported gals



Faber-Jackson relation (1976, ApJ) for ellipticals

Three a-priori independent predictions in one equation:

(i) Slope should be exactly 4 → OK

(ii) Normalization is a_0G → OK with BTFR estimate!

(iii) No dependence on other quantities IF $a \ll a_0$ → OK

σ_V is measured at R < R_e (containing half luminosity):

For dwarf spheroidals: $a \ll a_0$ at R<R_e → MOND regime

For giant ellipticals: $a \gg a_0$ at R<R_e → Newtonian regime

$$\frac{\sigma_V^2}{R} \sim \frac{G M}{R^2} \Rightarrow M \sim \sigma_V^2 R_e$$

Fundamental plane of ellipticals (Djorgovski & Davis 1987; Dressler 1987)

(3) Rotation curves can be predicted from the baryon distribution

(3) Rotation curves can be predicted from the baryon distribution

We introduce an interpolation function μ(x) with $x = a/a_0$:

$$a\mu(x) = g_N$$

(3) Rotation curves can be predicted from the baryon distribution

We introduce an interpolation function μ(x) with x = a/a_0:

$$a\mu(x) = g_N \begin{cases} \lim_{x\to\infty} \mu \to 1 \Rightarrow a = g_N & \text{Newtonian regime} \\ \lim_{x\to 0} \mu \to x \Rightarrow \dfrac{a^2}{a_0} = g_N \Rightarrow a = \sqrt{a_0 g_N} & \text{MOND regime} \end{cases}$$

(3) Rotation curves can be predicted from the baryon distribution

We introduce an interpolation function $\mu(x)$ with $x = a/a_0$:

$$a\mu(x) = g_N \begin{cases} \lim_{x \to \infty} \mu \to 1 \quad \Rightarrow \quad a = g_N & \text{Newtonian regime} \\ \lim_{x \to 0} \mu \to x \quad \Rightarrow \quad \frac{a^2}{a_0} = g_N \quad \Rightarrow \quad a = \sqrt{a_0 g_N} & \text{MOND regime} \end{cases}$$

Interpolation functions are *common* in Physics. Examples:

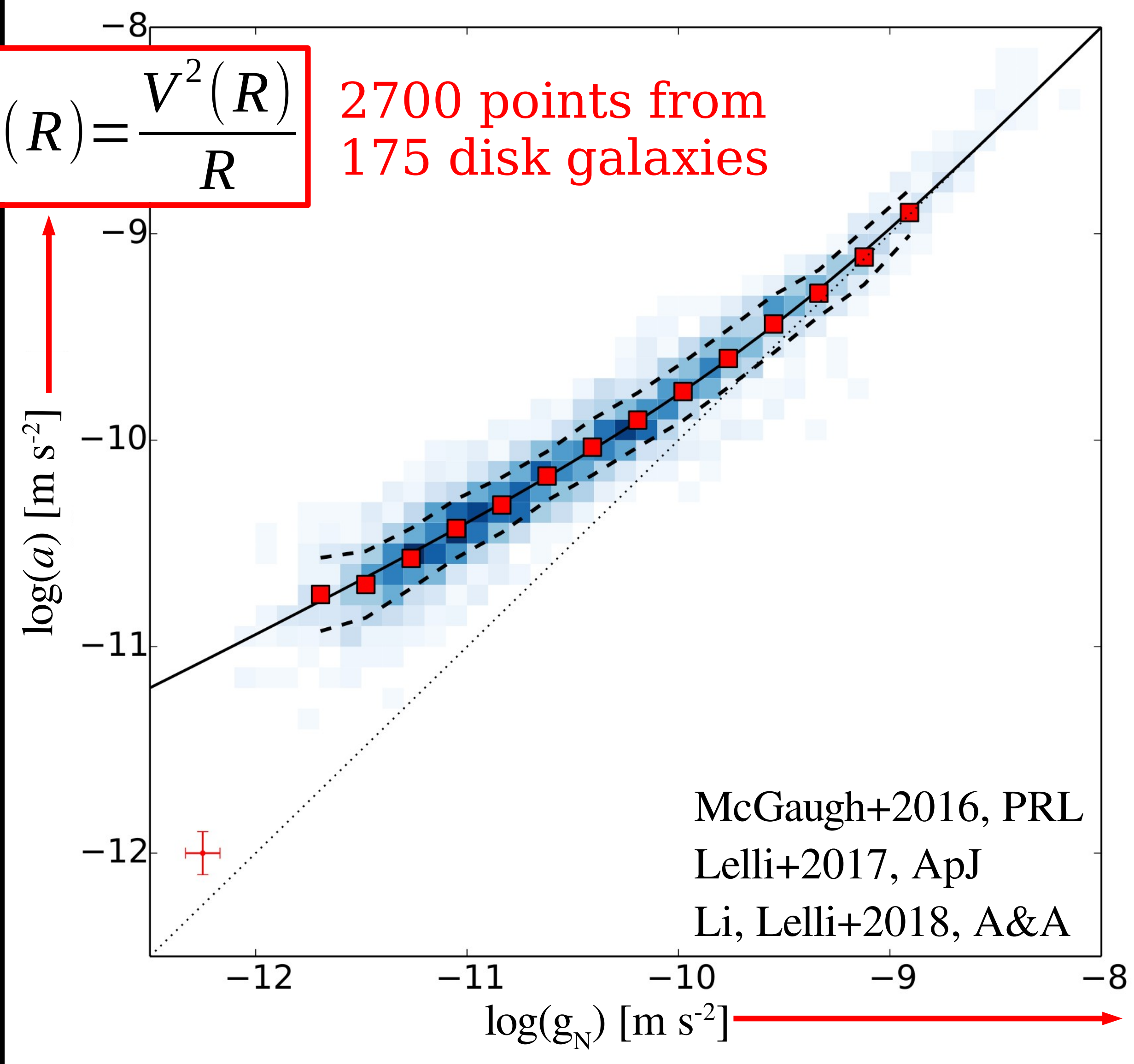
- Lorentz factor γ (via c): Newton's second law ↔ special relativity
- Planck's law for the blackbody radiation (via $\hbar$): Rayleigh-Jeans ↔ Wein regimes
- Probability for quantum tunnelling (via $\hbar$): classical mechanics ↔ quantum theory

MOND postulates specify only asymptotic limits of μ. Which function to choose?

(3) Rotation curves can be predicted from the baryon distribution

$$a(R) = \frac{V^2(R)}{R}$$

2700 points from
175 disk galaxies

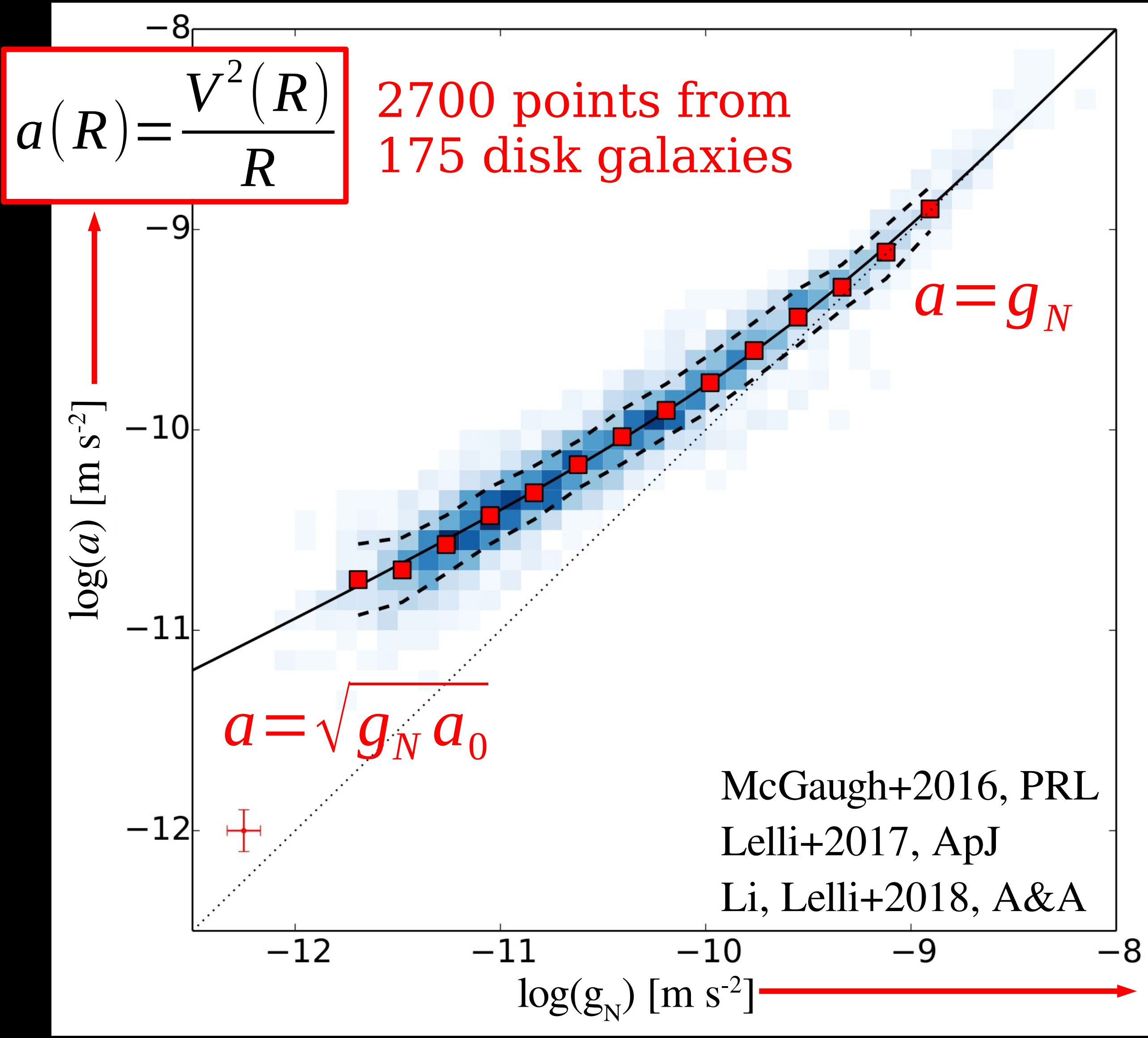


Radial Acceleration Relation (RAR)

- Fully empirical - independent of MOND

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution

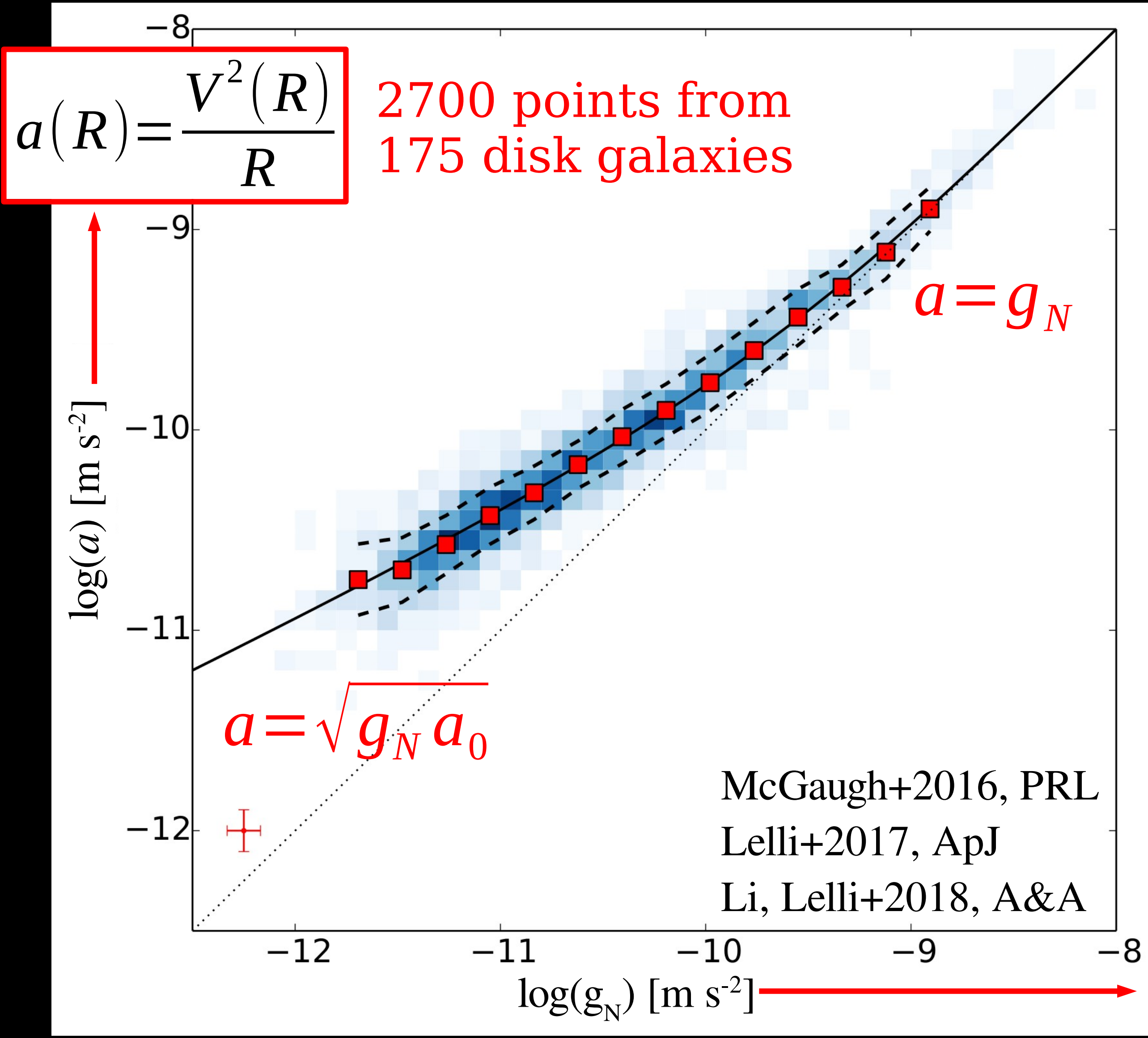


Radial Acceleration Relation (RAR)

- Fully empirical - independent of MOND
- Asymptotic limits consistent with MOND

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

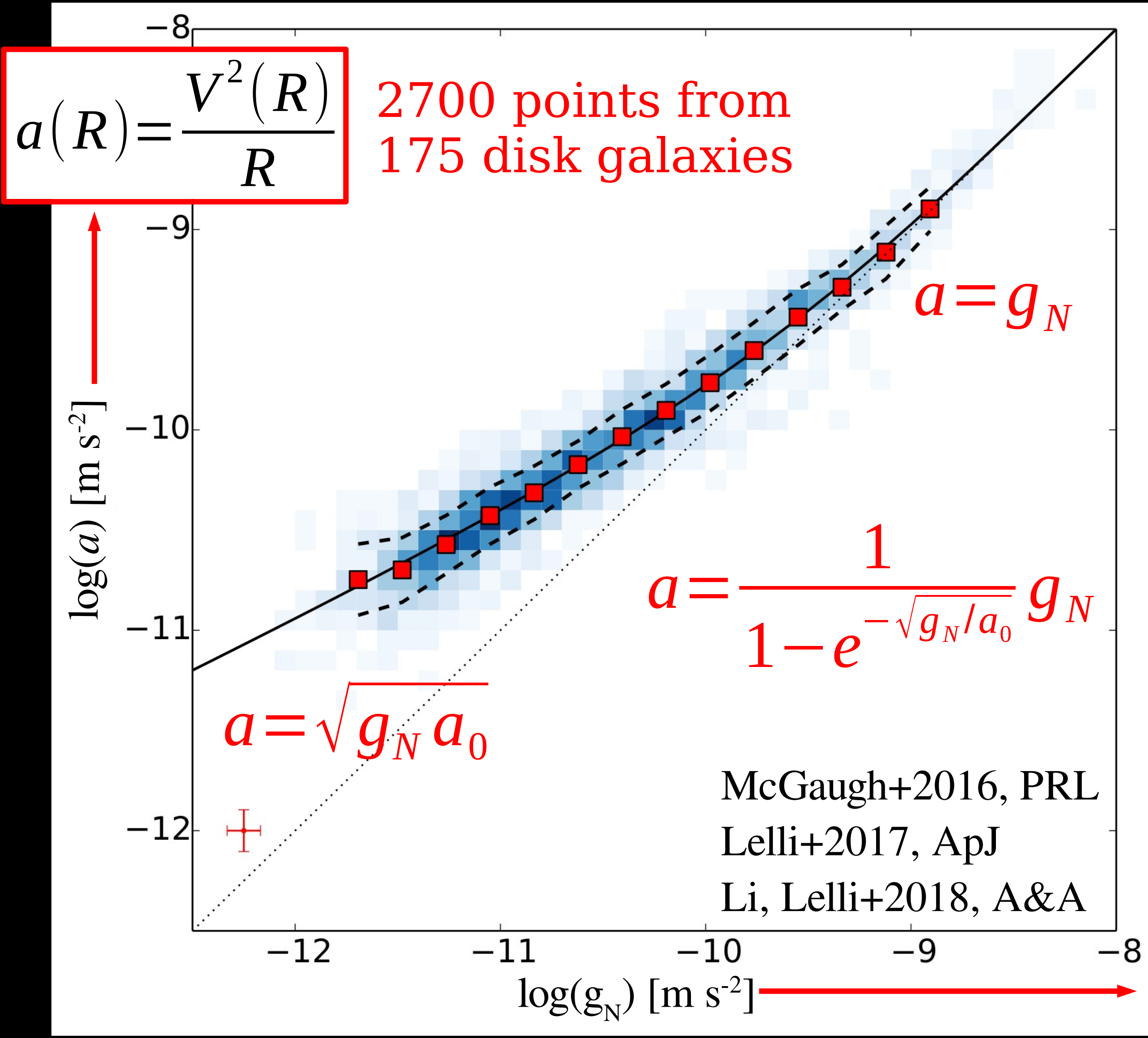
- Fully empirical - independent of MOND
- Asymptotic limits consistent with MOND
- RAR shape specifies the form of μ(x)

$$a\,\mu\left(\frac{a}{a_0}\right) = g_N \iff a = \nu\left(\frac{g_N}{a_0}\right) g_N$$

$$\nu = \mu^{-1}$$

$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

- Fully empirical - independent of MOND
- Asymptotic limits consistent with MOND
- RAR shape specifies the form of $\mu(x)$

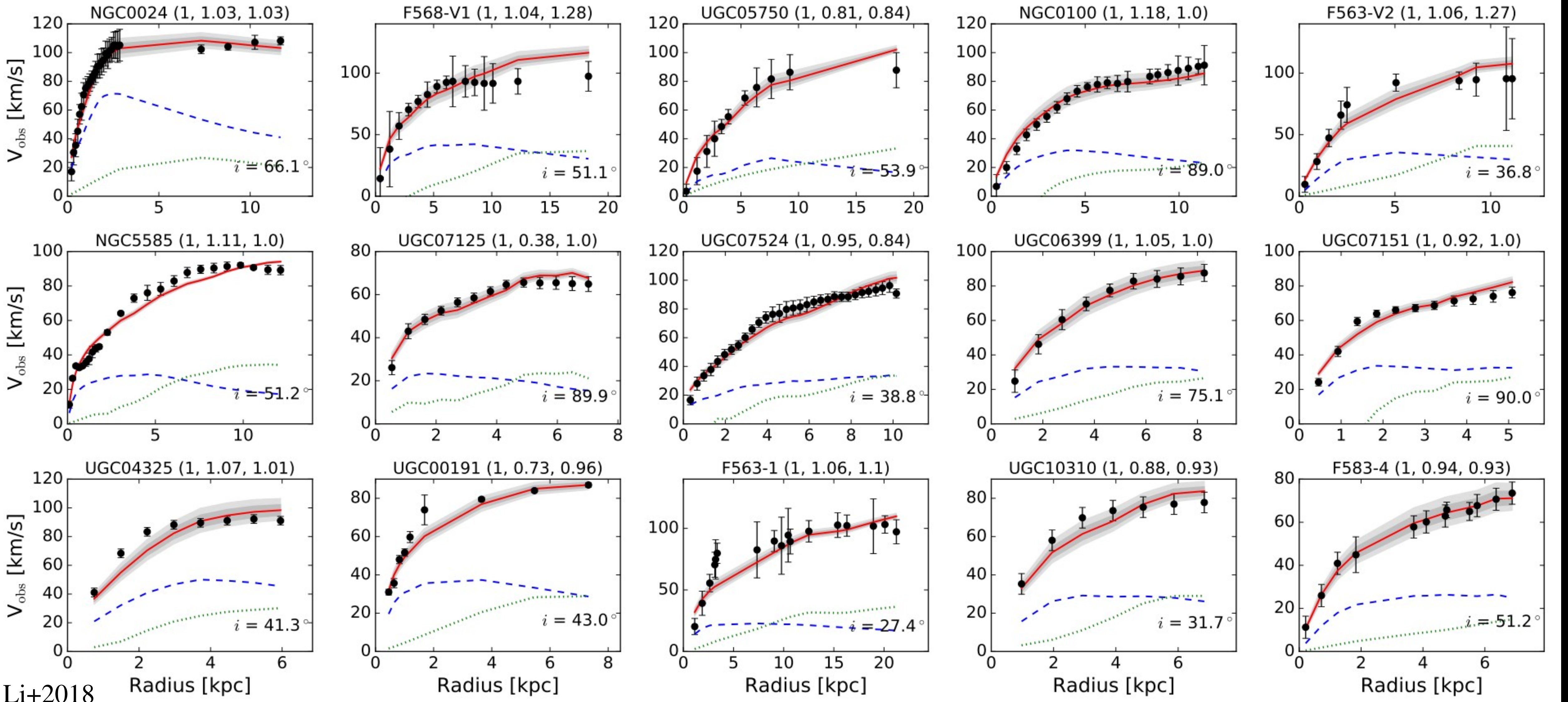
$$a\mu\left(\frac{a}{a_0}\right) = g_N \iff a = \nu\left(\frac{g_N}{a_0}\right) g_N$$

$$\nu = \mu^{-1}$$

We can now assume $\nu(g_N/a_0)$ and predict rotation curves given ρ_b (within the errors)

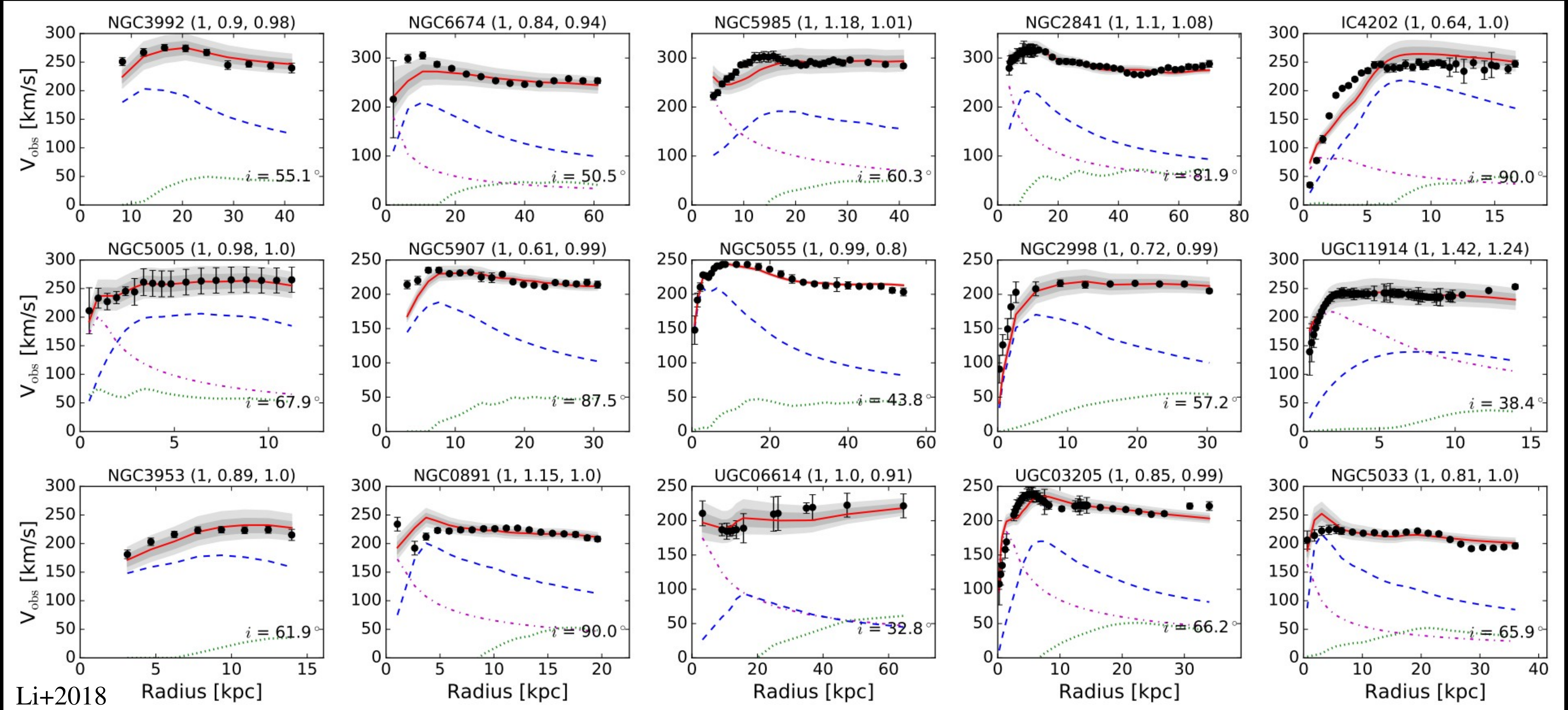
$$\nabla^2 \Phi_N(R,z) = 4\pi G \rho_b(R,z)$$
$$g_N(R,z=0) = -\nabla \Phi_N(R,z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



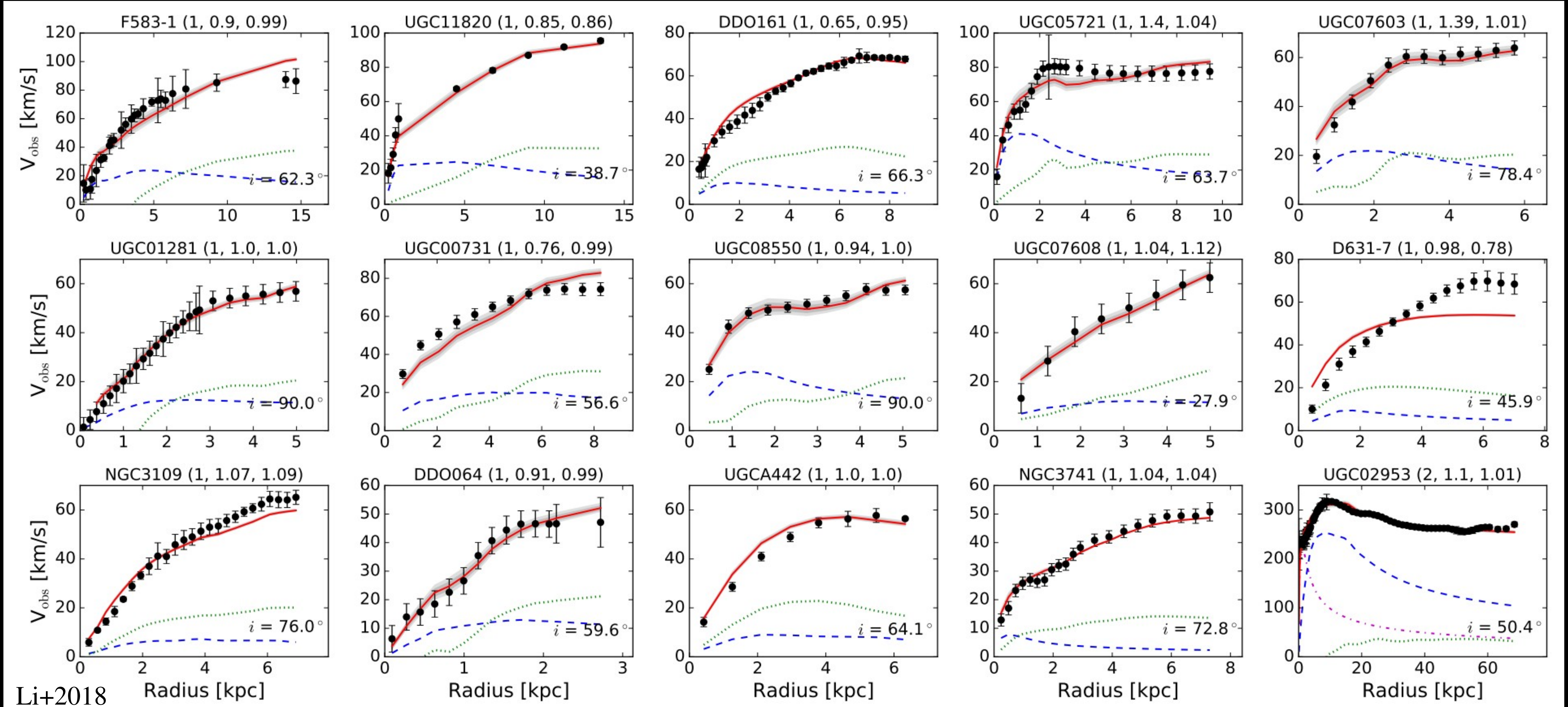
Li+2018

(3) Rotation curves can be predicted from the baryon distribution



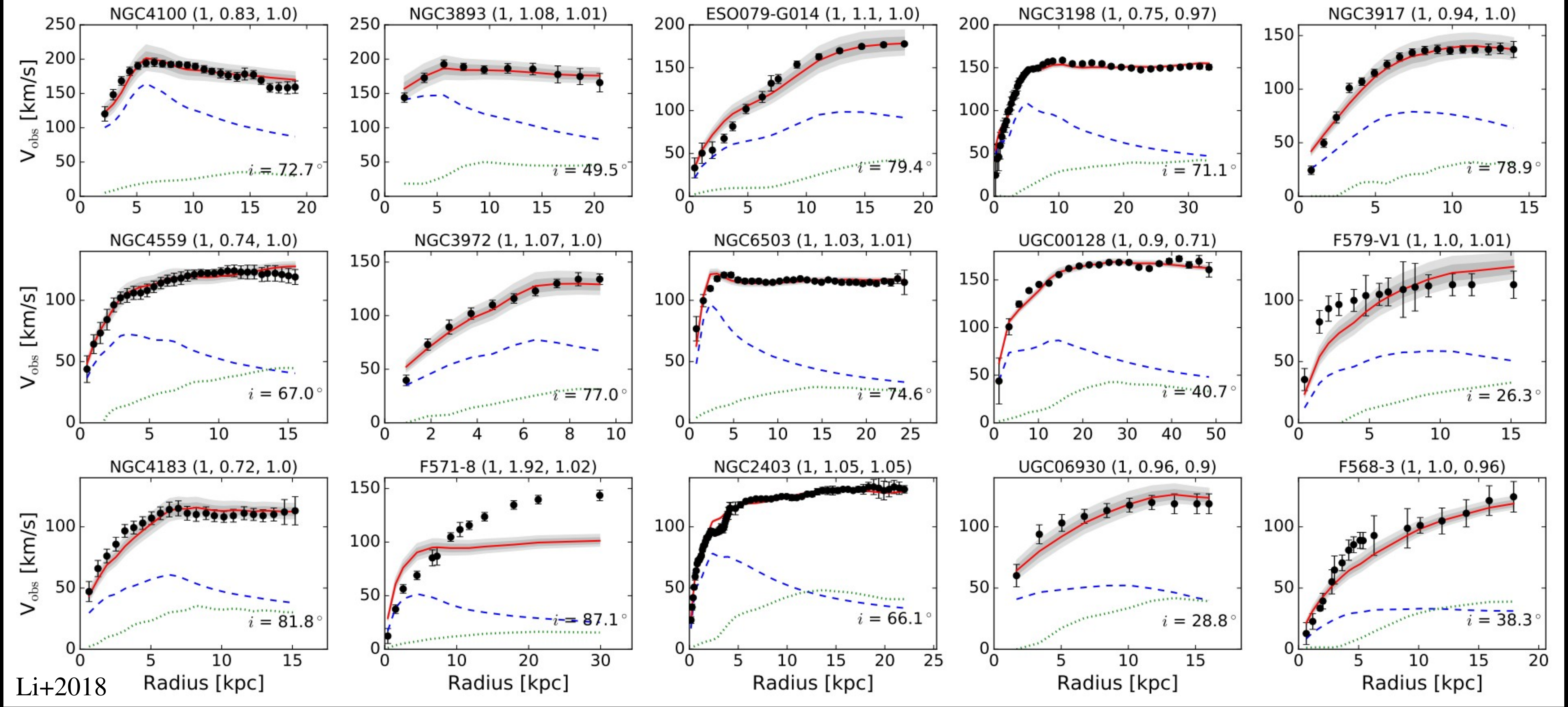
Li+2018

(3) Rotation curves can be predicted from the baryon distribution

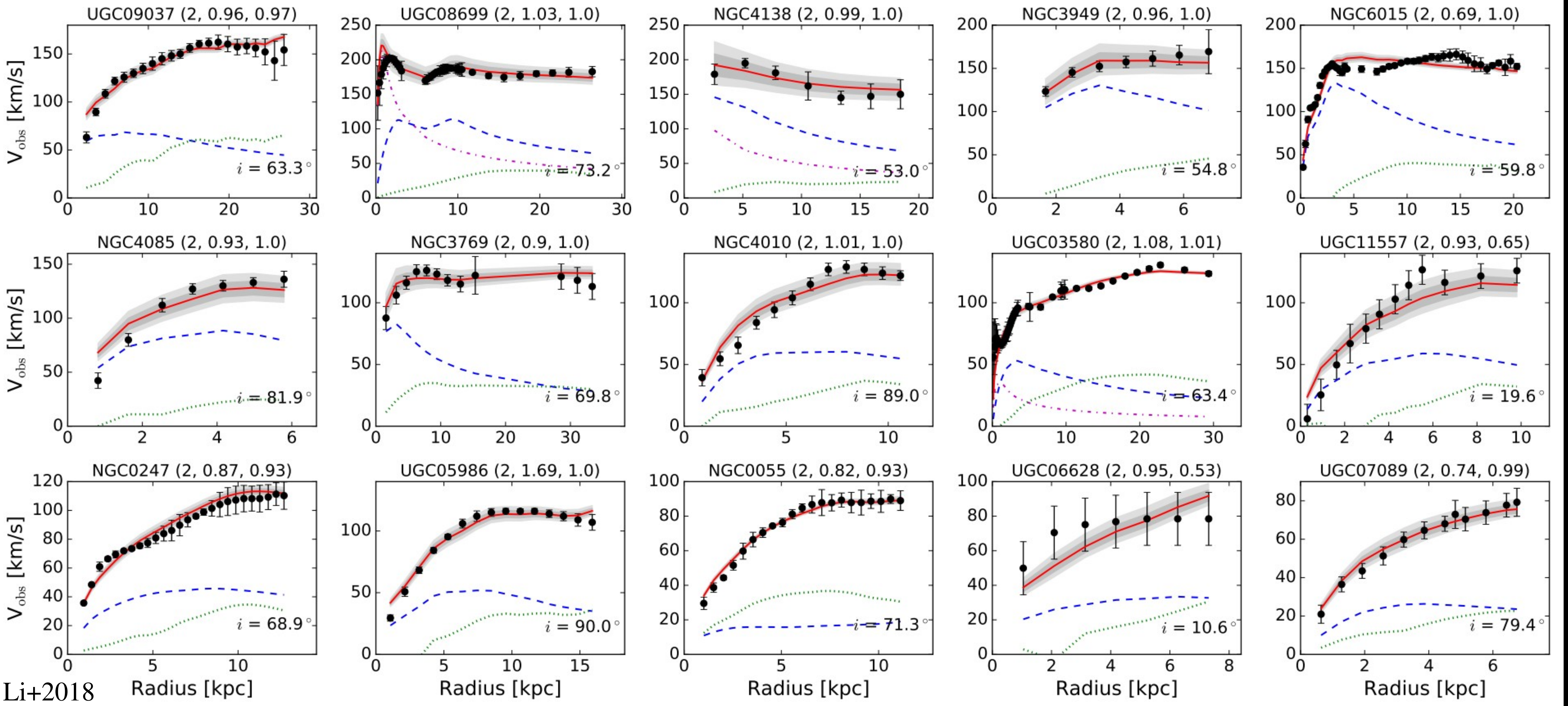


Li+2018

(3) Rotation curves can be predicted from the baryon distribution

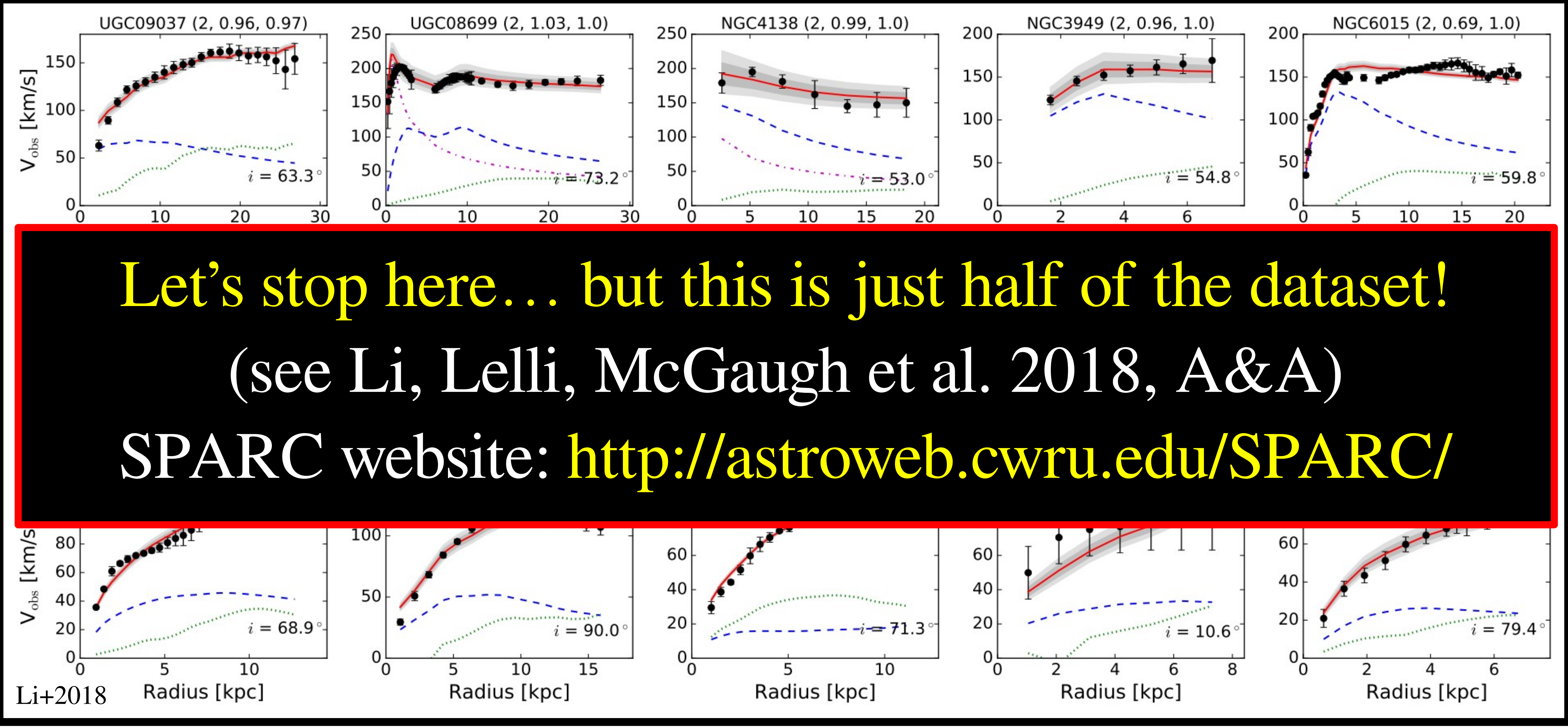


(3) Rotation curves can be predicted from the baryon distribution



Li+2018

(3) Rotation curves can be predicted from the baryon distribution

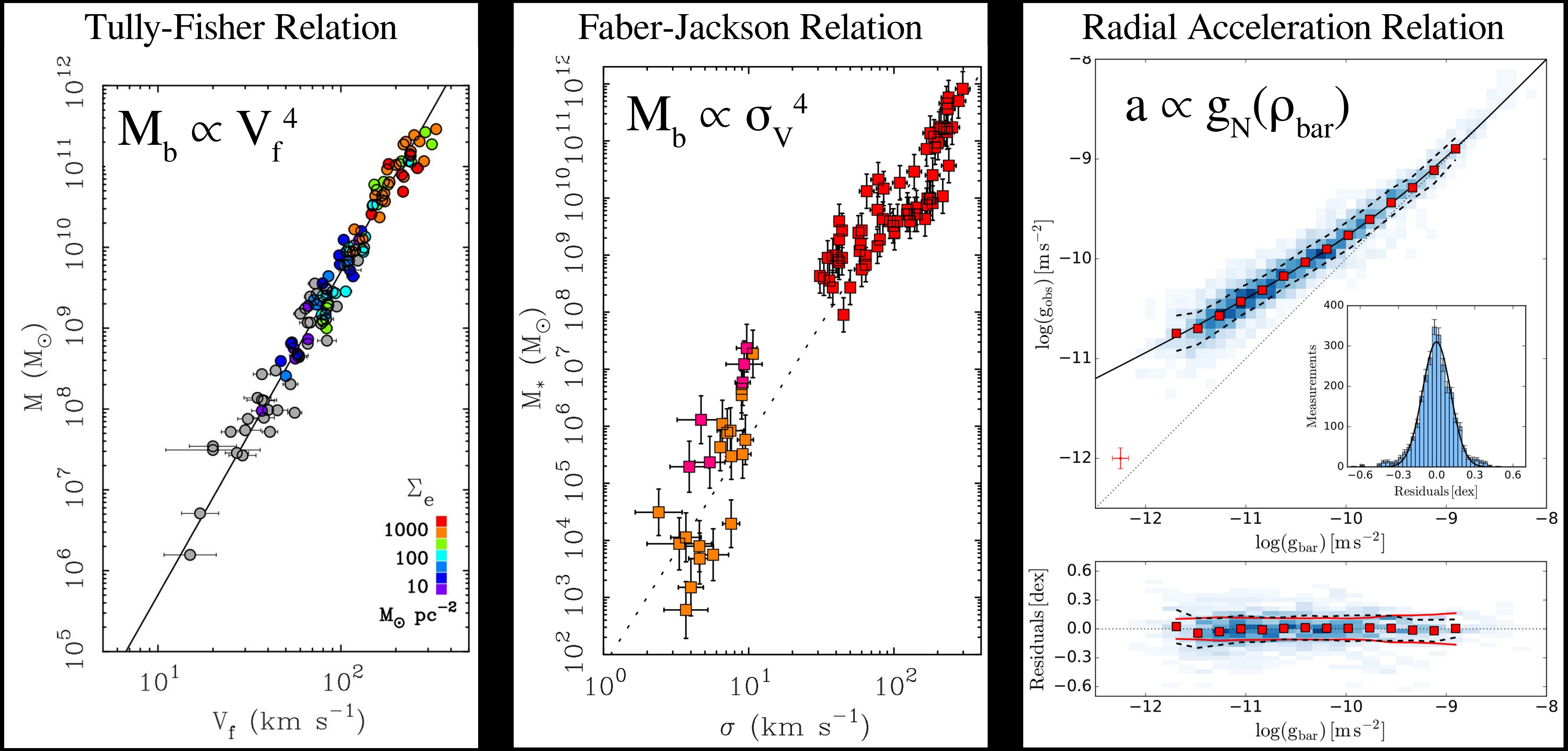


Let's stop here… but this is just half of the dataset!

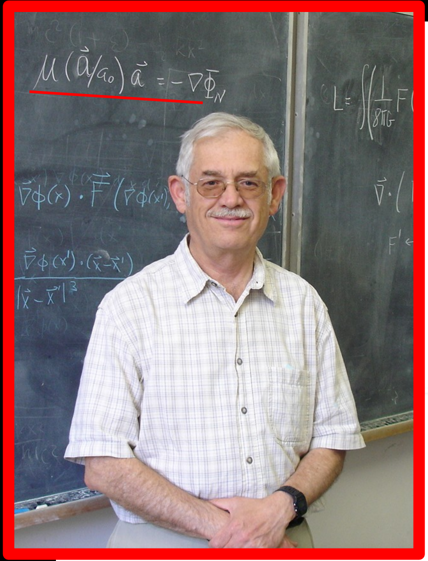
(see Li, Lelli, McGaugh et al. 2018, A&A)

SPARC website: http://astroweb.cwru.edu/SPARC/

Empirical Kepler-like laws of galaxies: emergence of a_0 + Baryon ↔ DM



II. Non-relativistic MOND theories



DOES THE MISSING MASS PROBLEM SIGNAL THE BREAKDOWN OF NEWTONIAN GRAVITY?

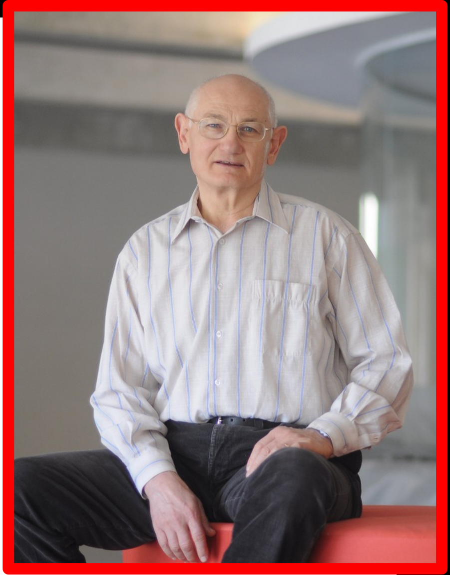
JACOB BEKENSTEIN
Department of Physics, Ben Gurion University of the Negev, Beer-Sheva

AND

MORDEHAI MILGROM[1]
Department of Physics, Weizmann Institute of Science, Rehovot

Received 1984 March 28; accepted 1984 May 17

1 year after the 1st trilogy



ABSTRACT

We consider a nonrelativistic potential theory for gravity which differs from the Newtonian theory. The theory is built on the basic assumptions of the modified dynamics, which were shown earlier to reproduce dynamical properties of galaxies and galaxy aggregates without having to assume the existence of hidden mass. The theory involves a modification of the Poisson equation and can be derived from a Lagrangian. The total momentum, angular momentum, and (properly defined) energy of an isolated system are conserved. The center-of-mass acceleration of an arbitrary bound system in a constant external gravitational field is independent of any property of the system. In other words, all isolated objects fall in exactly the same way in a constant external gravitational field (the weak equivalence principle is satisfied). However, the internal dynamics of a system in a constant external field is different from that of the same system in the absence of the external field, in violation of the strong principle of equivalence. These two results are consistent with the requirements of the modified dynamics. We sketch a toy relativistic theory which has a nonrelativistic limit satisfying the requirements of the modified dynamics.

Subject headings: cosmology — galaxies: internal motions — gravitation

Let's start from the classical Newtonian Action:

$$S = \int dt\, L = \int dt \left(L_{matter} + L_{gravity} + L_{coupling} \right) = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Principle of Least Action:

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla^2 \Phi = 4\pi G \rho$$

$$\frac{\delta S}{\delta \vec{x}} = 0 \rightarrow \vec{a} = -\vec{\nabla}\Phi$$

Let's start from the classical Newtonian Action:

$$S = \int dt\, L = \int dt \left(L_{matter} + L_{gravity} + L_{coupling} \right) = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Change this for modified inertia

Change this for modified gravity

Change this modify both

Principle of Least Action:

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla^2 \Phi = 4\pi G \rho$$

$$\frac{\delta S}{\delta \vec{x}} = 0 \rightarrow \vec{a} = -\vec{\nabla}\Phi$$

Let's start from the classical Newtonian Action:

$$S = \int dt\, L = \int dt \left(L_{matter} + L_{gravity} + L_{coupling} \right) = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Change this for modified inertia

Change this for modified gravity

Change this modify both

AQUAL

(Bekenstein & Milgrom 1984, Astrophysical Journal)

QUMOND

(Milgrom 2010, MNRAS)

Principle of Least Action:

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla^2 \Phi = 4\pi G \rho$$

$$\frac{\delta S}{\delta \vec{x}} = 0 \rightarrow \vec{a} = -\vec{\nabla}\Phi$$

AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984, ApJ)

$$S = \int dt\, L = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Lagrangian is quadratic in $\nabla\Phi$

$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla}\Phi|^2}{a_0^2} \right)$$

$F(z) \rightarrow z$ for $z = |\nabla\Phi|^2/a_0^2 \gg 1$

$F(z) \rightarrow z^{3/2}$ for $z = |\nabla\Phi|^2/a_0^2 \ll 1$

AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984, ApJ)

$$S = \int dt\, L = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Lagrangian is quadratic in $\nabla\Phi$

$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla}\Phi|^2}{a_0^2} \right)$$

$F(z) \rightarrow z$ for $z = |\nabla\Phi|^2/a_0^{\,2} \gg 1$

$F(z) \rightarrow z^{3/2}$ for $z = |\nabla\Phi|^2/a_0^{\,2} \ll 1$

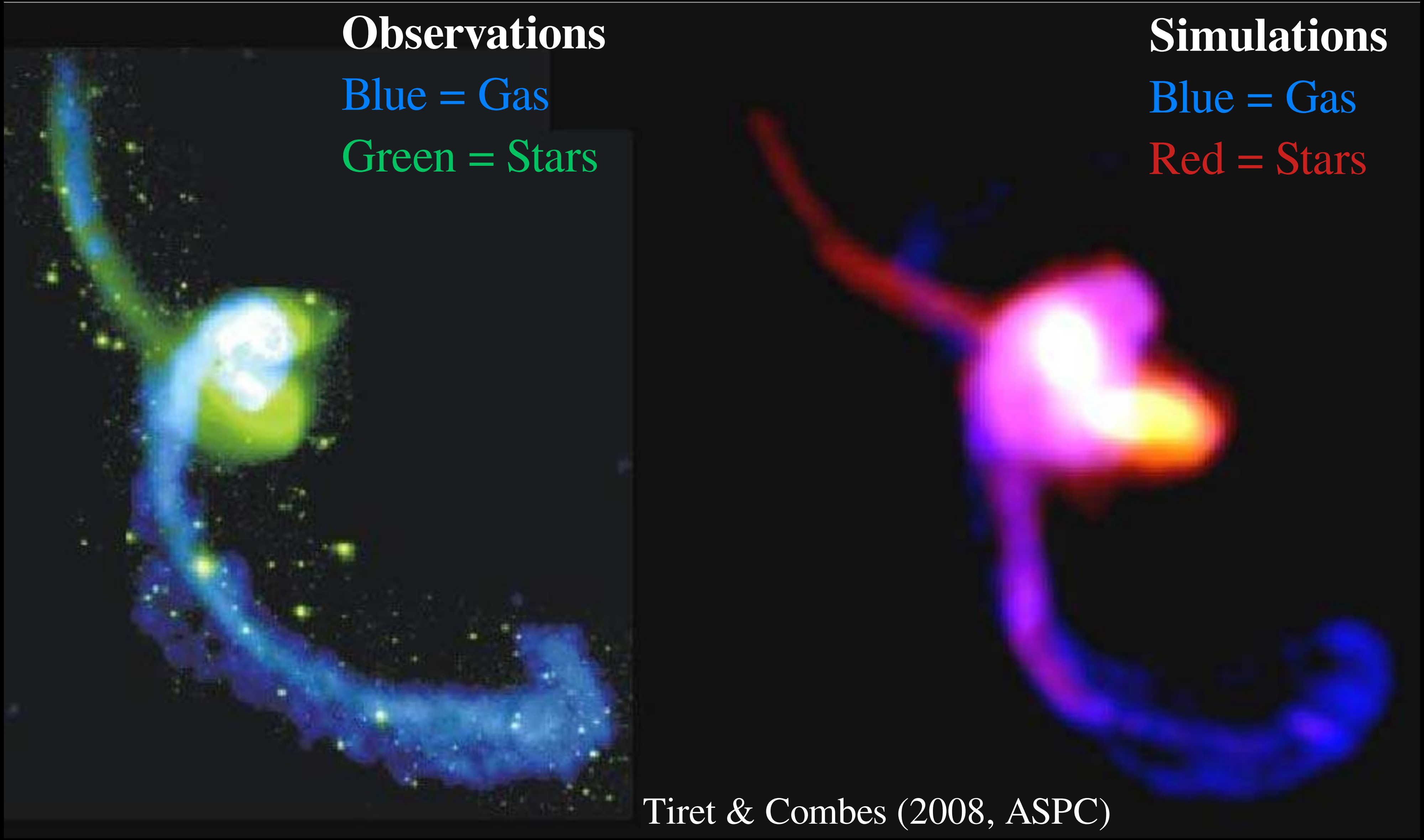
$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla \cdot \left[\mu\left(\frac{|\vec{\nabla}\Phi|}{a_0} \right) \vec{\nabla}\Phi \right] = 4\pi G \rho$$

Modified Poisson's Equation

$$\mu(x) = \frac{dF(z)}{dz} \qquad z = x^2$$

$F(z)$ provides the interpolation function $\mu = \upsilon^{-1}$

Application of AQUAL: The Antennae Merging Galaxies



QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S = \int dt\, L = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Single gravitational potential Φ

↓

$$\frac{-1}{8\pi G}\left[2\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi_N - a_0^2 Q\left(\frac{|\vec{\nabla}\Phi_N|^2}{a_0^2} \right) \right]$$

Two potentials: Φ and Φ_N!

QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S=\int dt\, L=\int dt\, d^3x \left(\rho \frac{V^2}{2}-\frac{|\vec{\nabla}\Phi|^2}{8\pi G}-\rho\Phi\right)$$

Single gravitational potential Φ

$$\downarrow$$

$$\frac{-1}{8\pi G}\left[2\vec{\nabla}\Phi\cdot\vec{\nabla}\Phi_N - a_0^2 Q\left(\frac{|\vec{\nabla}\Phi_N|^2}{a_0^2}\right)\right]$$

Two potentials: Φ and Φ_N!

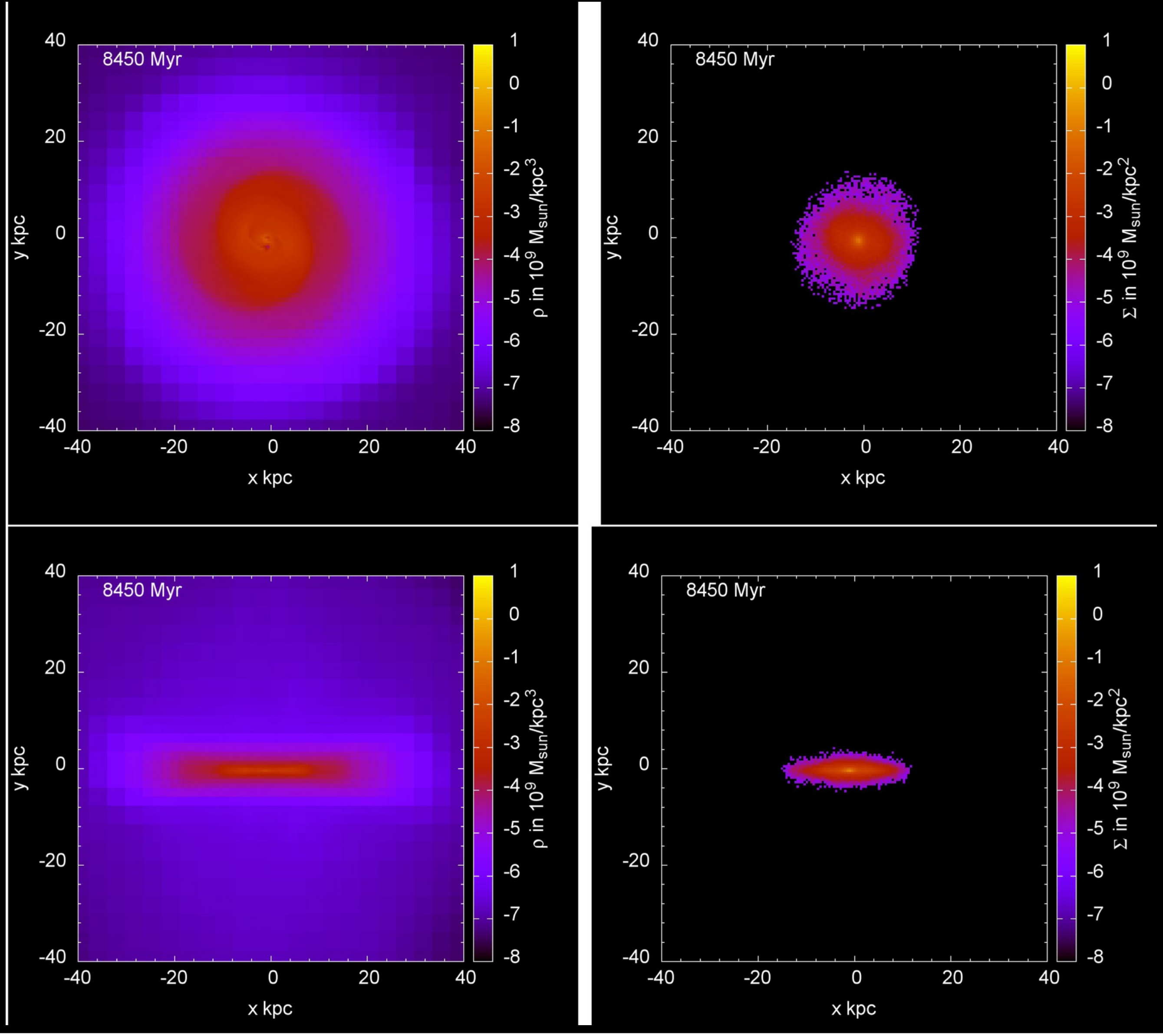
Principle of least Action varying Φ, Φ_N and $\bar{x}$ → set of 3 equations

$\nabla^2\Phi_N=4\pi G\rho$ ⟶ Standard, linear Poisson's equation for Φ_N

$\nabla^2\Phi=\vec{\nabla}\cdot\left[\nu\left(|\vec{\nabla}\Phi_N|/a_0\right)\vec{\nabla}\Phi_N\right]$ ⟶ Non-linear step: get Φ from Φ_N $\quad \nu(\sqrt{x})=\frac{dQ(x)}{x}$

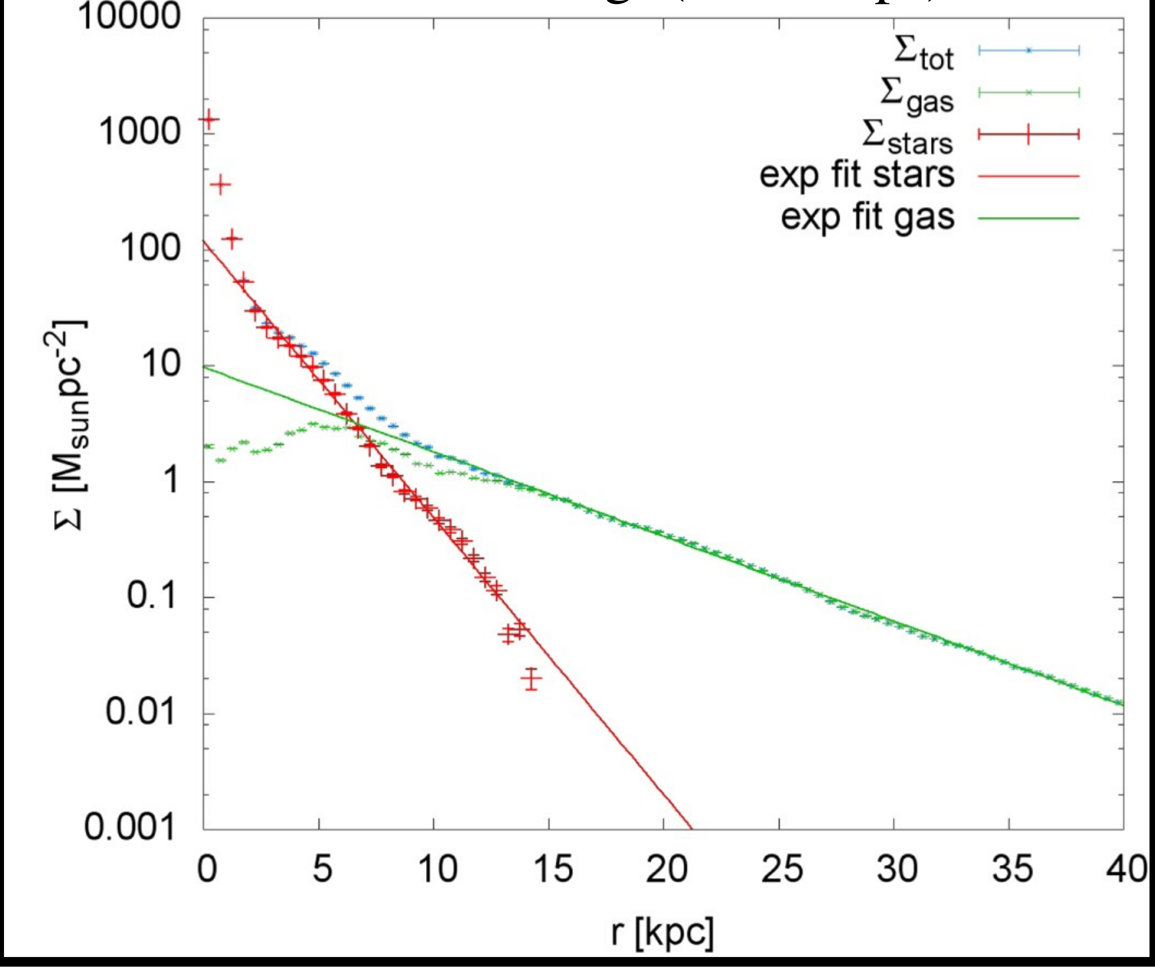
$\vec{a}=-\vec{\nabla}\Phi$ ⟶ Acceleration/force set by second potential Φ

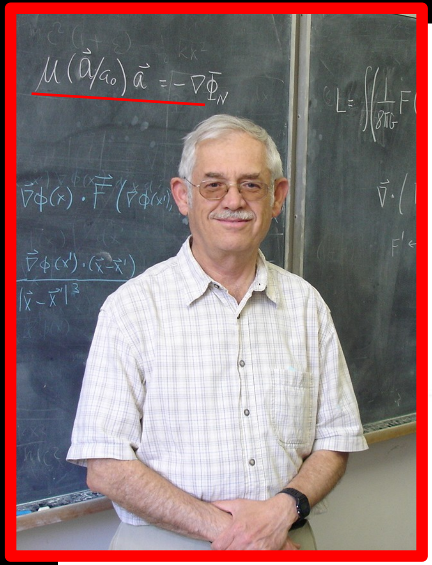
Application of QUMOND: Formation of Galaxy Disks



Gas collapse → Exponential disk

Wittenburg+(2020, ApJ)





DOES THE MISSING MASS PROBLEM SIGNAL THE BREAKDOWN OF NEWTONIAN GRAVITY?

JACOB BEKENSTEIN
Department of Physics, Ben Gurion University of the Negev, Beer-Sheva

AND

MORDEHAI MILGROM[1]
Department of Physics, Weizmann Institute of Science, Rehovot

Received 1984 March 28; accepted 1984 May 17

ABSTRACT

We consider a nonrelativistic potential theory for gravity which differs from the Newtonian theory. The theory is built on the basic assumptions of the modified dynamics, which were shown earlier to reproduce dynamical properties of galaxies and galaxy aggregates without having to assume the existence of hidden mass. The theory involves a modification of the Poisson equation and can be derived from a Lagrangian. The total momentum, angular momentum, and (properly defined) energy of an isolated system are conserved. The center-of-mass acceleration of an arbitrary bound system in a constant external gravitational field is independent of any property of the system. In other words, all isolated objects fall in exactly the same way in a constant external gravitational field (the weak equivalence principle is satisfied). However, the internal dynamics of a system in a constant external field is different from that of the same system in the absence of the external field, in violation of the strong principle of equivalence. These two results are consistent with the requirements of the modified dynamics. We sketch a toy relativistic theory which has a nonrelativistic limit satisfying the requirements of the modified dynamics.

Subject headings: cosmology — galaxies: internal motions — gravitation

Weak Equivalence Principle (WEP):

→ Universality of free fall (center-of-mass motion)

Weak Equivalence Principle (WEP):

→ Universality of free fall (center-of-mass motion)

Einstein Equivalence Principle (EEP):

→ WEP + Lorentz invariance (spacetime rotations)

+ Local Position Invariance (LPI) for non-gravitational experiments: the results of experiments do not depend on where/when they are done

Weak Equivalence Principle (WEP):

→ Universality of free fall (center-of-mass motion)

Einstein Equivalence Principle (EEP):

→ WEP + Lorentz invariance (spacetime rotations)

+ Local Position Invariance (LPI) for non-gravitational experiments: the results of experiments do not depend on where/when they are done

Strong Equivalence Principle (SEP):

→ EEP + LPI for gravitational experiments too

Weak Equivalence Principle (WEP):

→ Universality of free fall (center-of-mass motion)

Einstein Equivalence Principle (EEP):

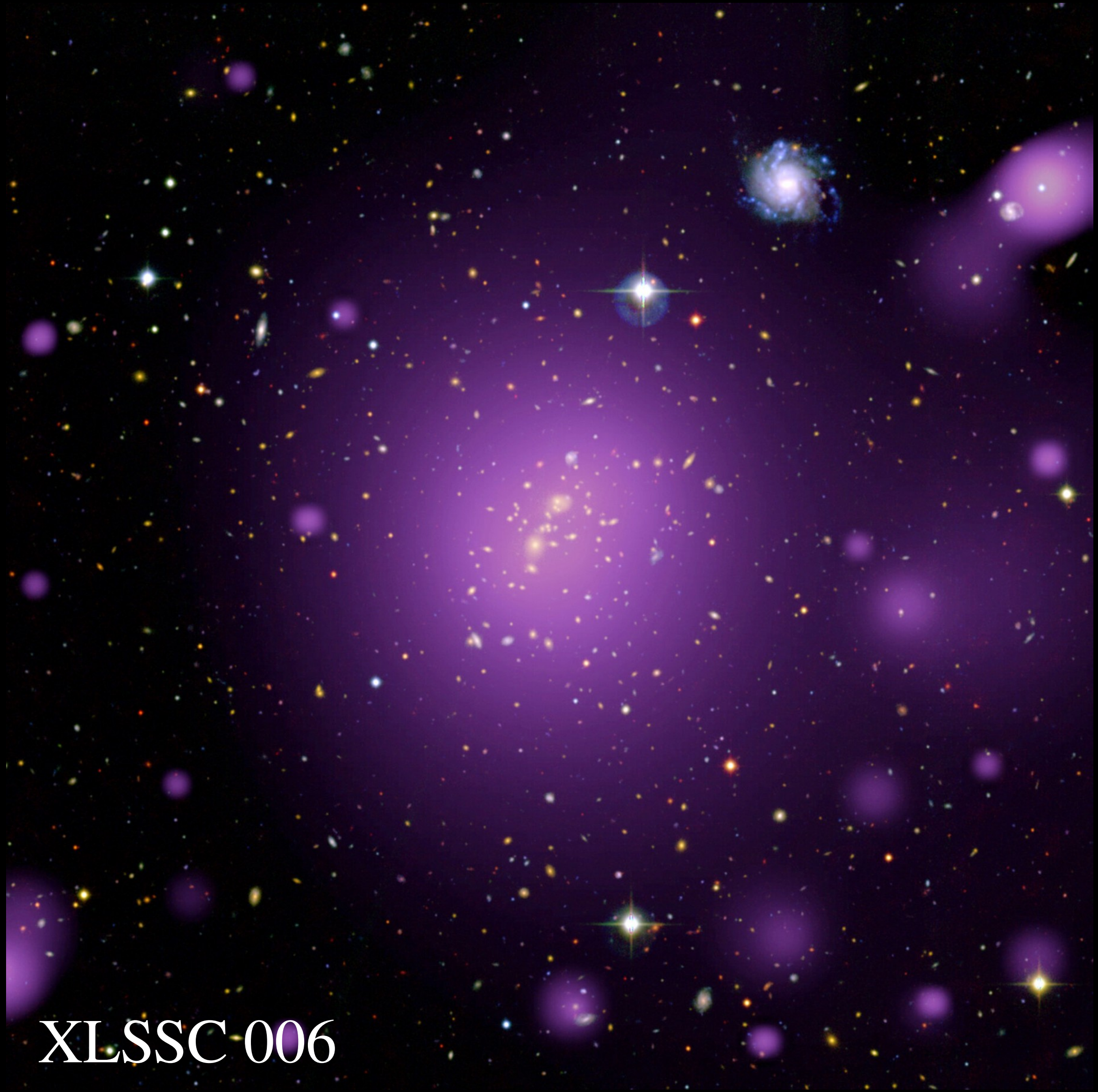
→ WEP + Lorentz invariance (spacetime rotations)
+ Local Position Invariance (LPI) for non-gravitational experiments:
the results of experiments do not depend on where/when they are done

Strong Equivalence Principle (SEP):

→ EEP ~~+ LPI for gravitational experiments too~~

Broken by MOND:
External Field Effect
(Chae+2020, 2021, ApJ)

Galaxy Clusters: systems with ~100-1000 galaxies

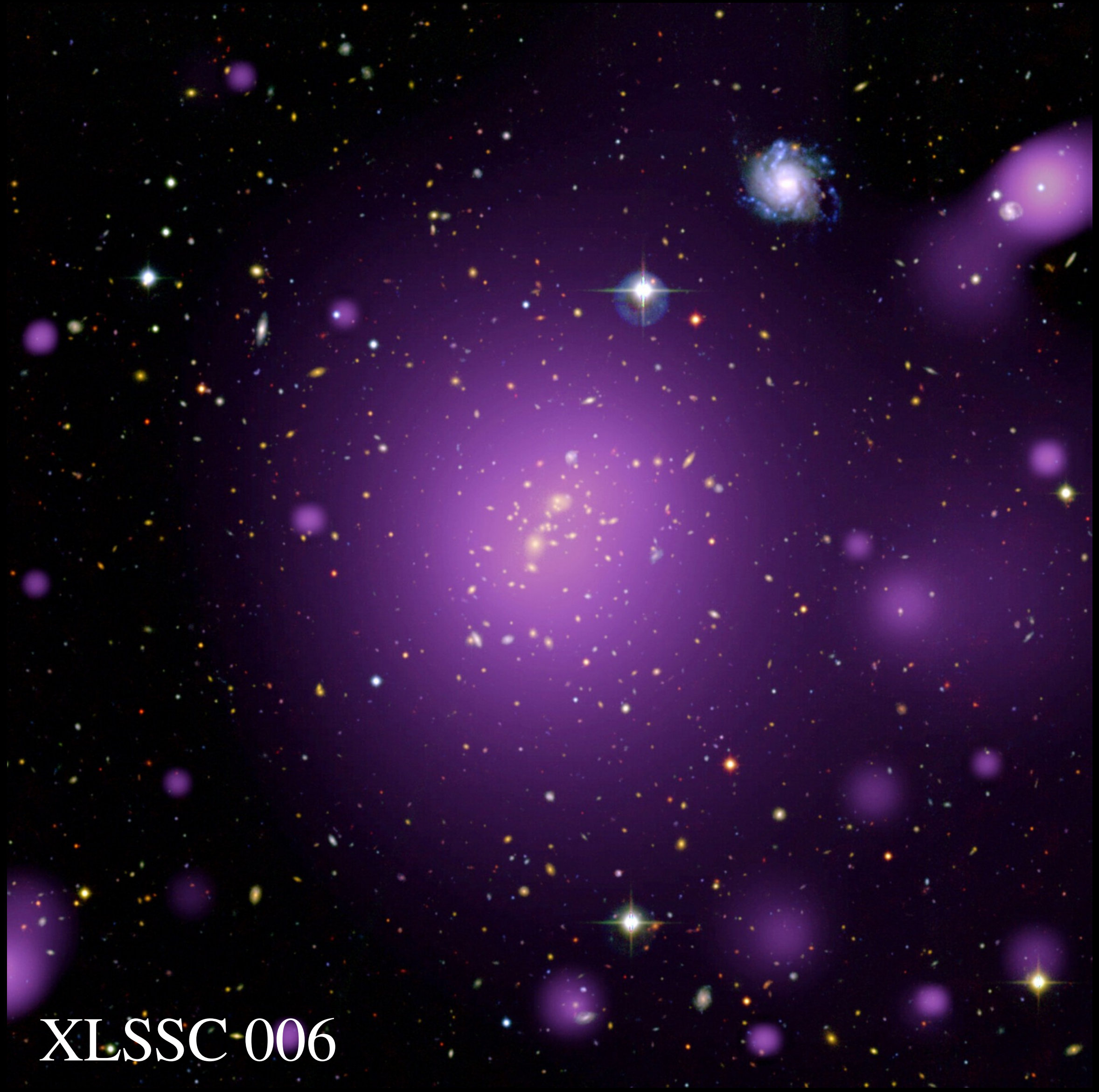


Observed baryon budget:

10% galaxies (optical & NIR)

90% hot ionized gas (X rays)

Galaxy Clusters: systems with ~100-1000 galaxies



Observed baryon budget:

10% galaxies (optical & NIR)

90% hot ionized gas (X rays)

Sphere in Hydrostatic Equilibrium

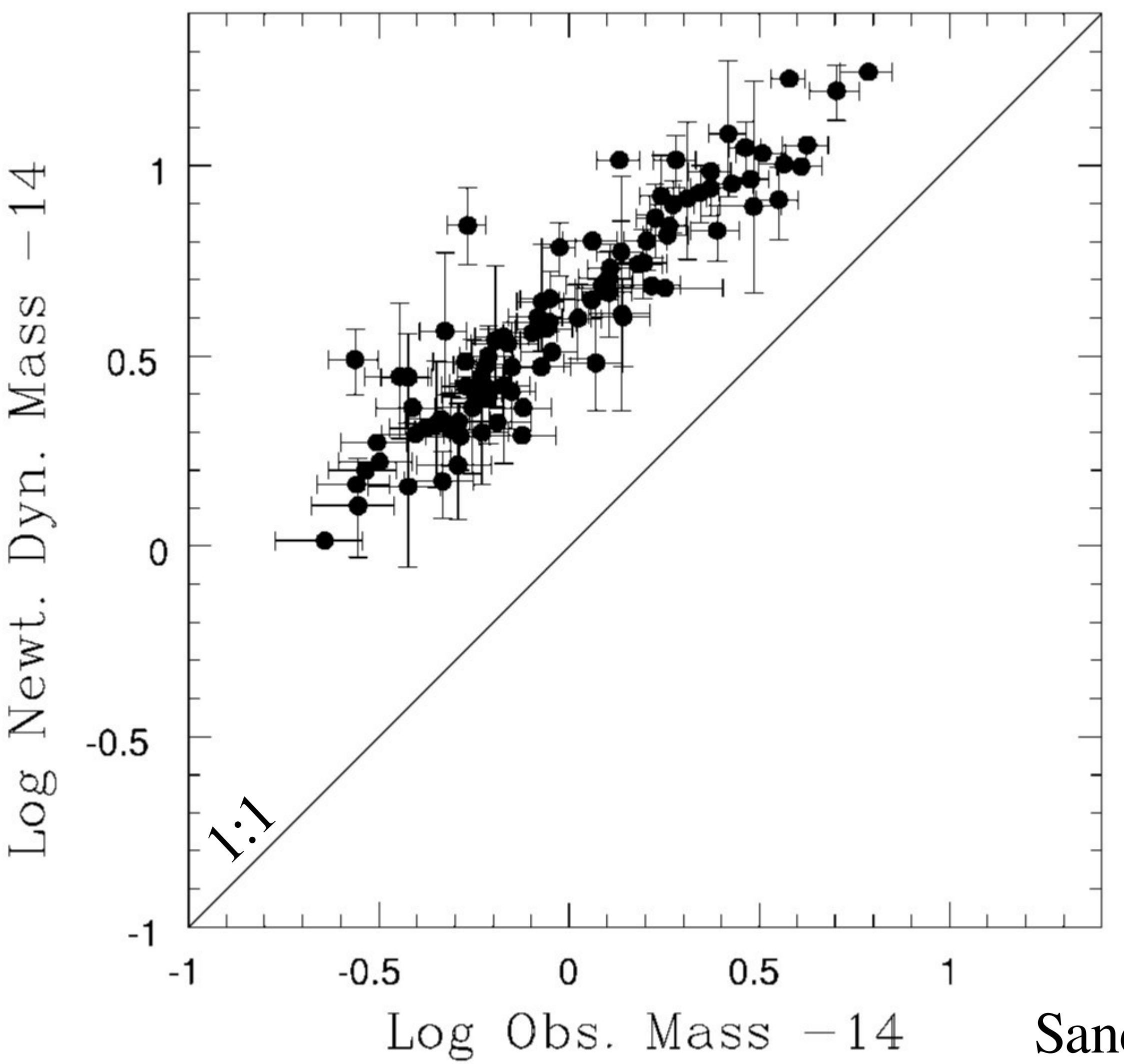
$$\frac{dP_{gas}}{dr} = \rho_{gas} \frac{d\Phi}{dr}$$

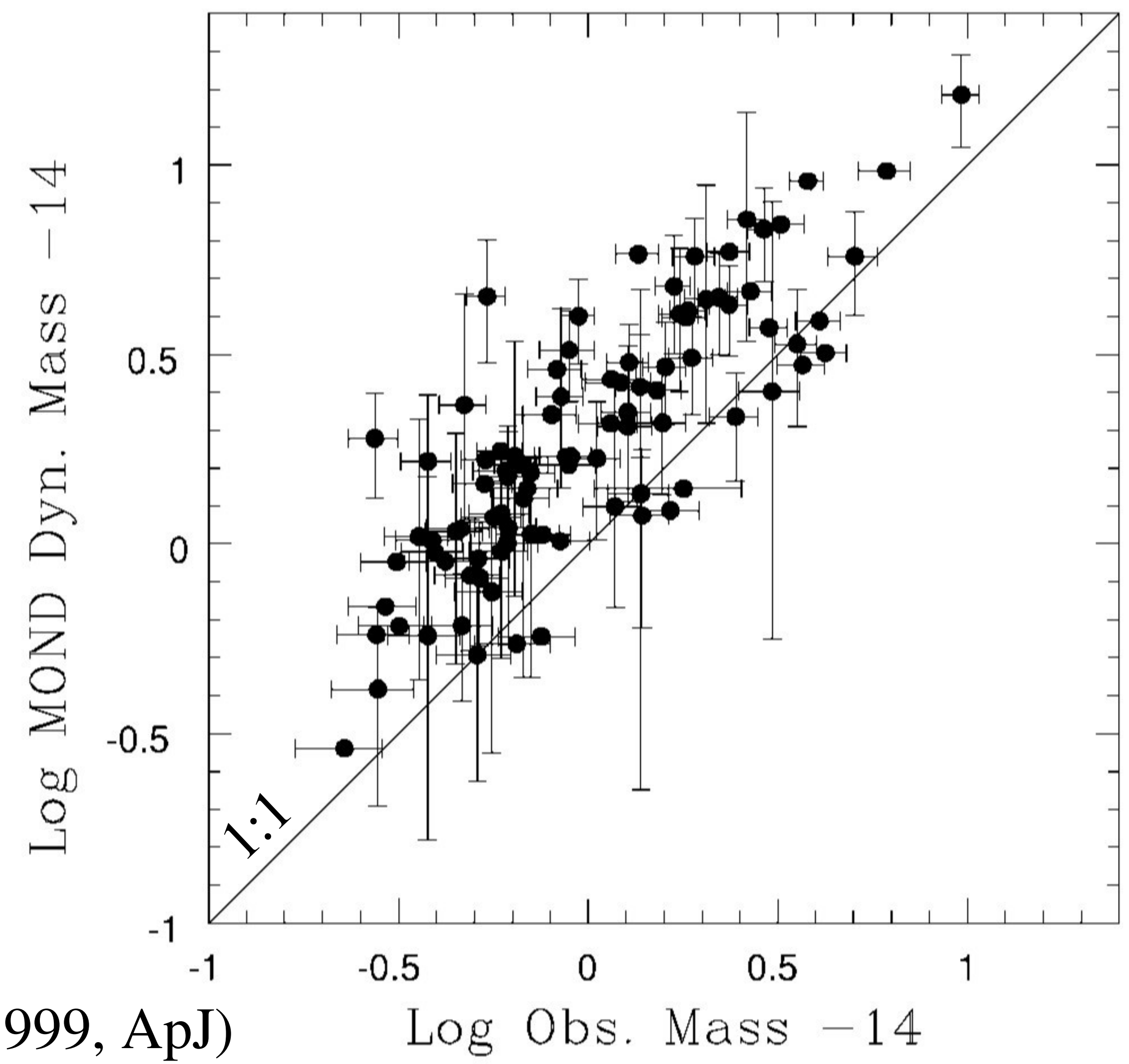
$$\frac{d}{dr}\left(\frac{\rho_{gas} k T_{gas}}{w\, m_p}\right) = \rho_{gas} \frac{d\Phi}{dr}$$

Galaxy Clusters: Long-standing problem for MOND

Newtonian analysis: $M_{dyn}/M_{bar} \sim 4\text{-}5$

MOND analysis: $M_{dyn}/M_{bar} \sim 2$





Sanders (1999, ApJ)

Galaxy Clusters: Long-standing problem for MOND

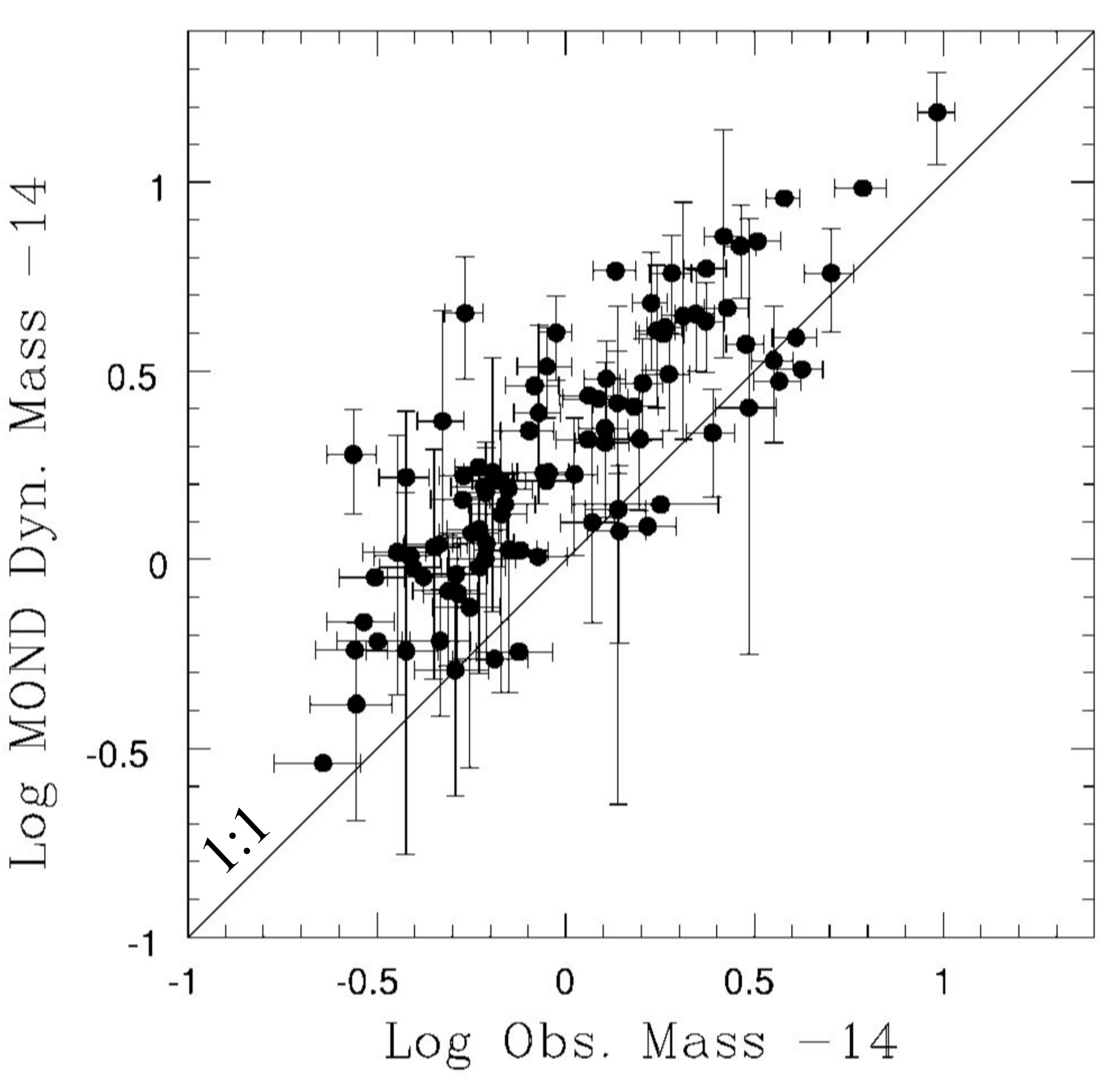
Proposed Solutions:

- Undetected (missing) baryons

  → Compact clouds of cold gas?

  (Milgrom 2008, NAR)

MOND analysis: $M_{dyn}/M_{bar} \sim 2$

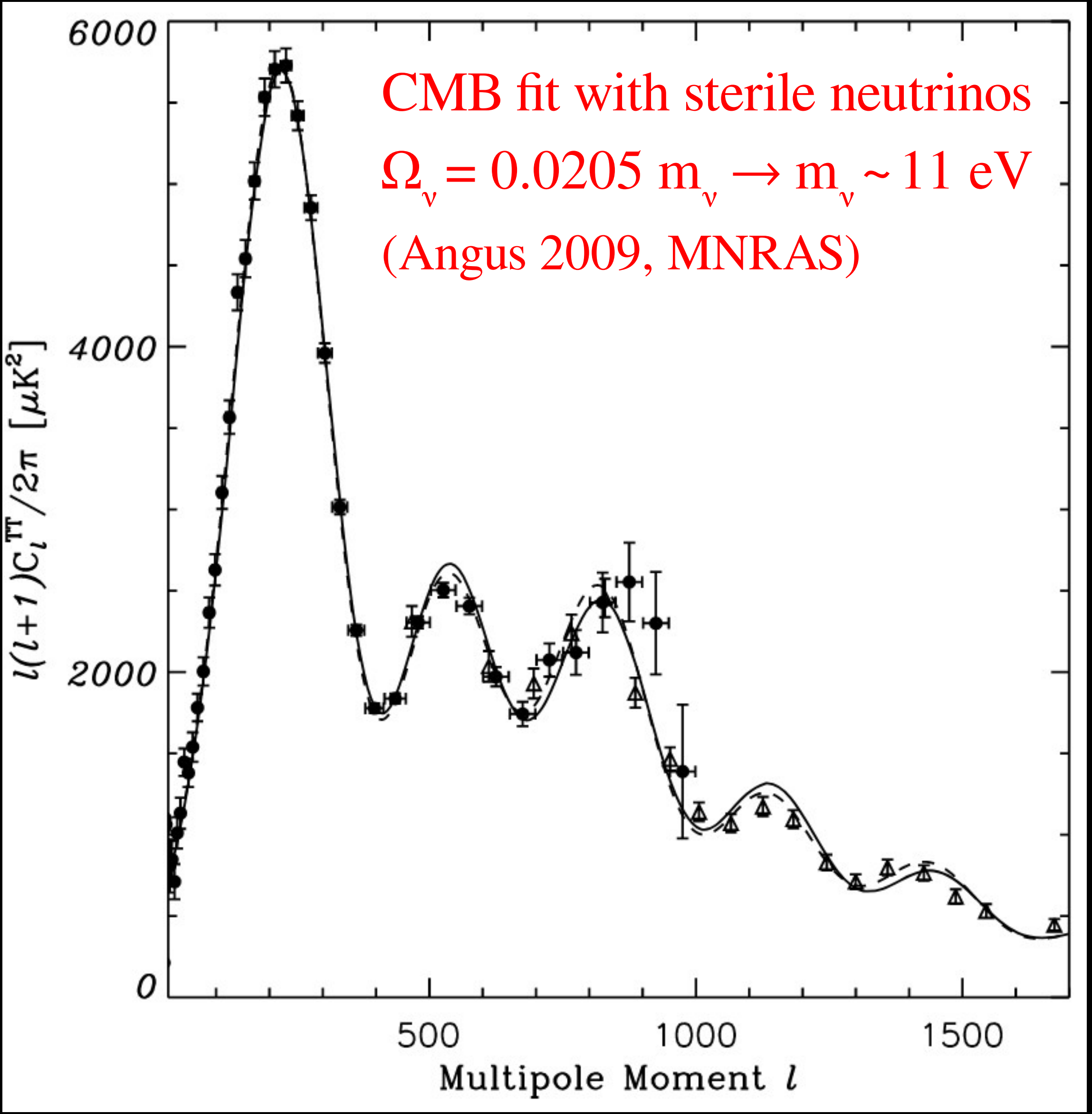


Galaxy Clusters: Long-standing problem for MOND

Proposed Solutions:

- Undetected (missing) baryons
  → Compact clouds of cold gas?
  (Milgrom 2008, NAR)

- Sterile neutrinos of ~11 eV
  → Neutrino oscillations & CMB fit!
  (Angus+2010, MNRAS)



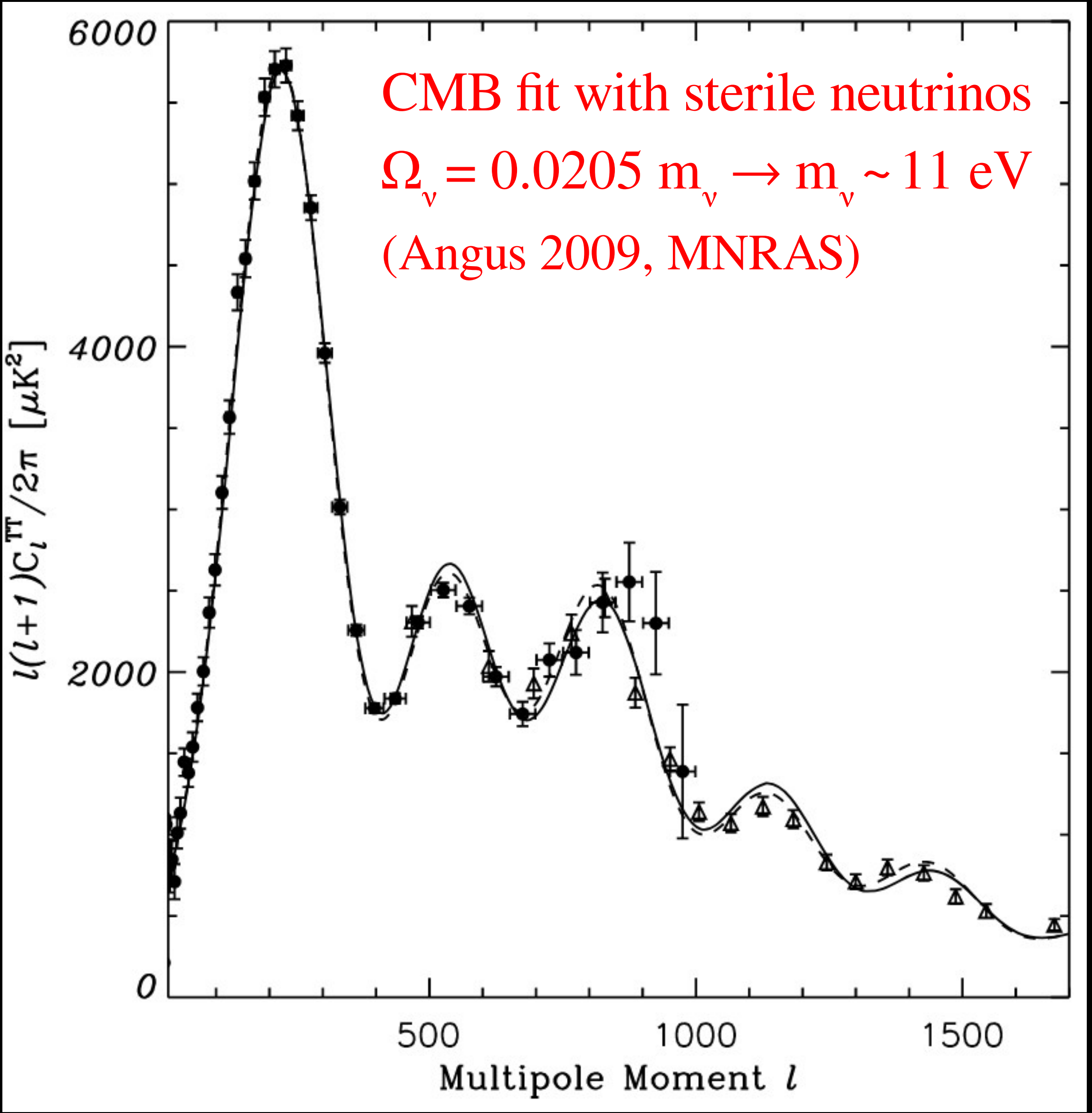
Galaxy Clusters: Long-standing problem for MOND

Proposed Solutions:

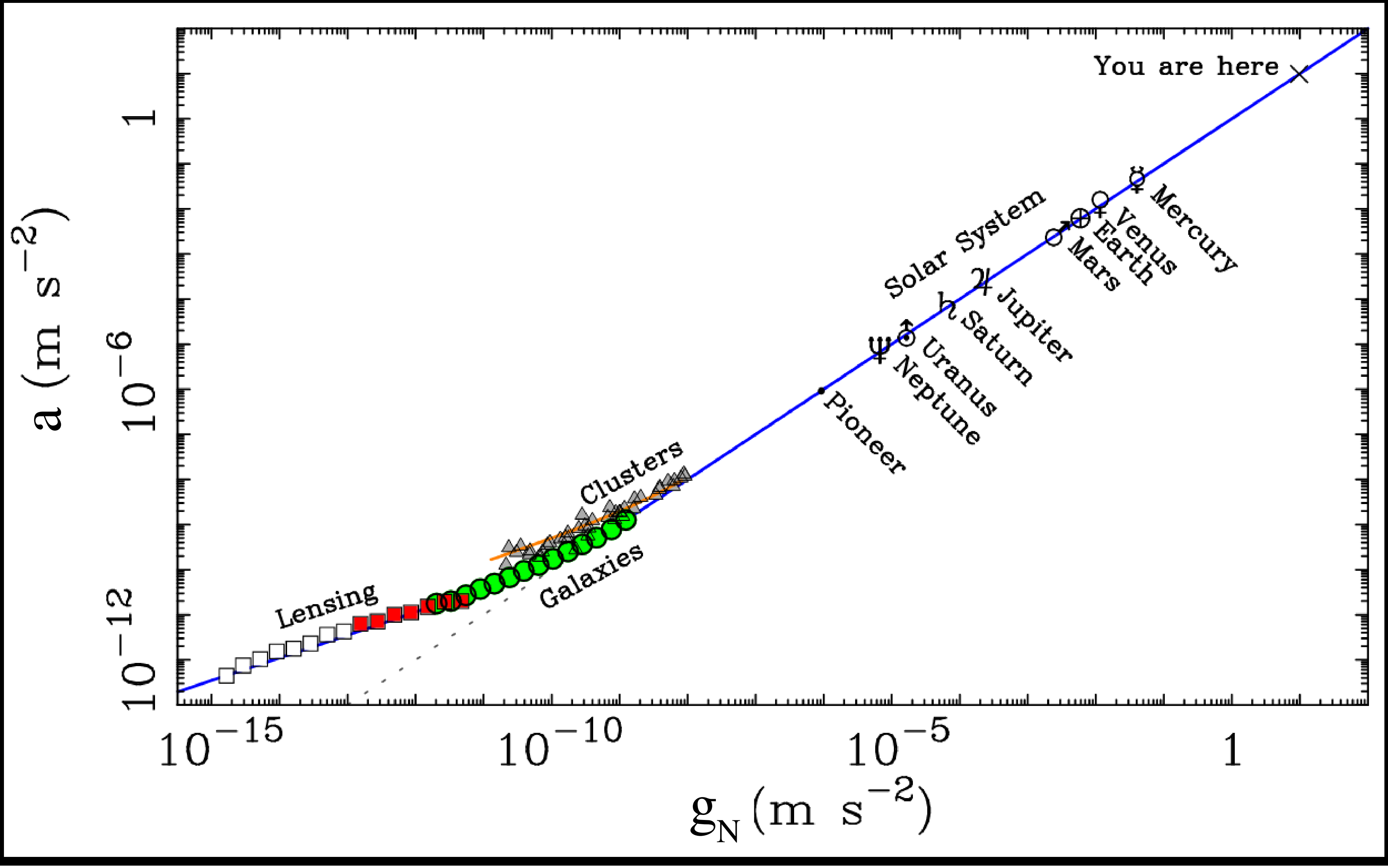
- Undetected (missing) baryons
  → Compact clouds of cold gas?
  (Milgrom 2008, NAR)

- Sterile neutrinos of ~11 eV
  → Neutrino oscillations & CMB fit!
  (Angus+2010, MNRAS)

- Extended MOND: $a_0 \propto \Phi$
  → Deeper theory? But more freedom…
  (Zhao & Famaey 2012, PRD)



Putting Galaxy Clusters in context on the RAR



III. Relativistic MOND theories

TeVeS (Tensor-Vector-Scalar) - Bekenstein (2004, PRD)

- Tensor $g_{\mu\nu}$ → Einstein's metric
- Vector A^{μ} → to get the "right" gravitational lensing (Sanders 1997, ApJ)
- Scalar Φ → to get the DM effect for matter (Bekenstein & Milgrom 1984, ApJ)
- Free Function → interpolation function (similar to AQUAL & QMOND)

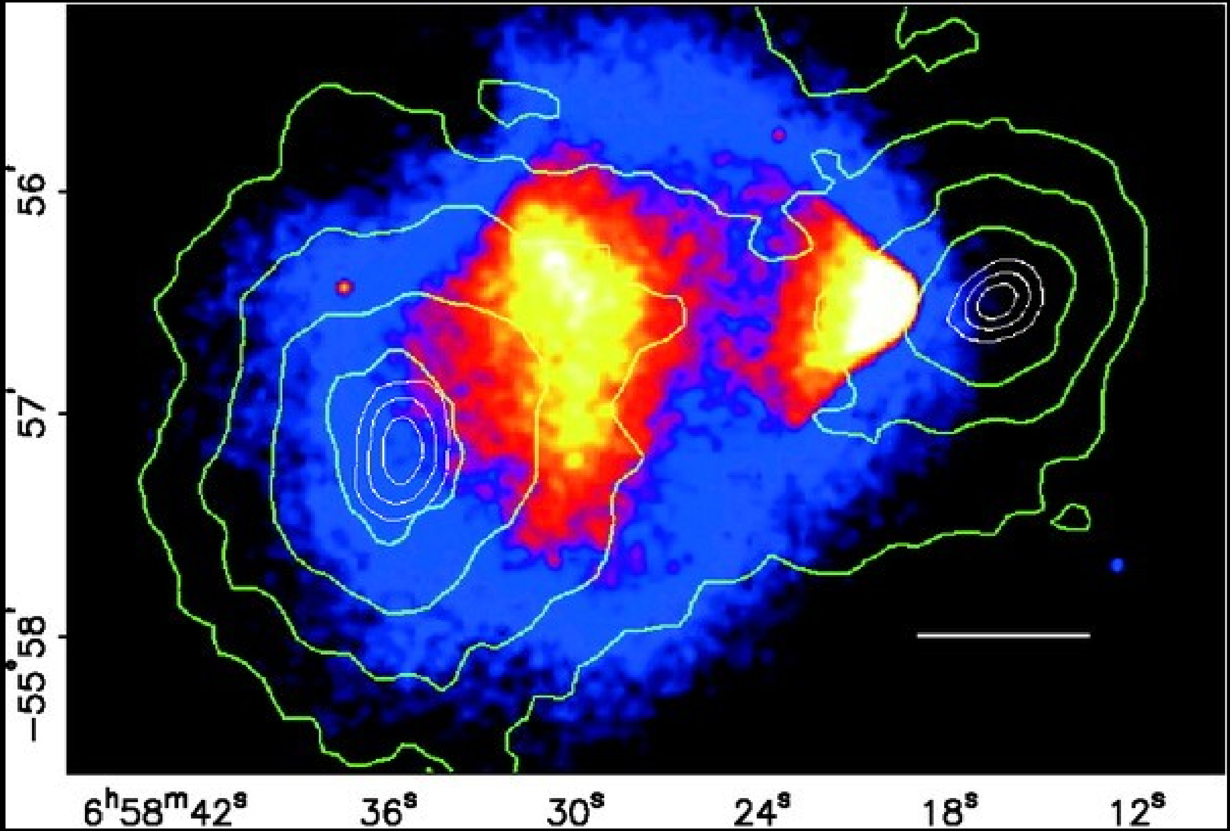
TeVeS (Tensor-Vector-Scalar) - Bekenstein (2004, PRD)

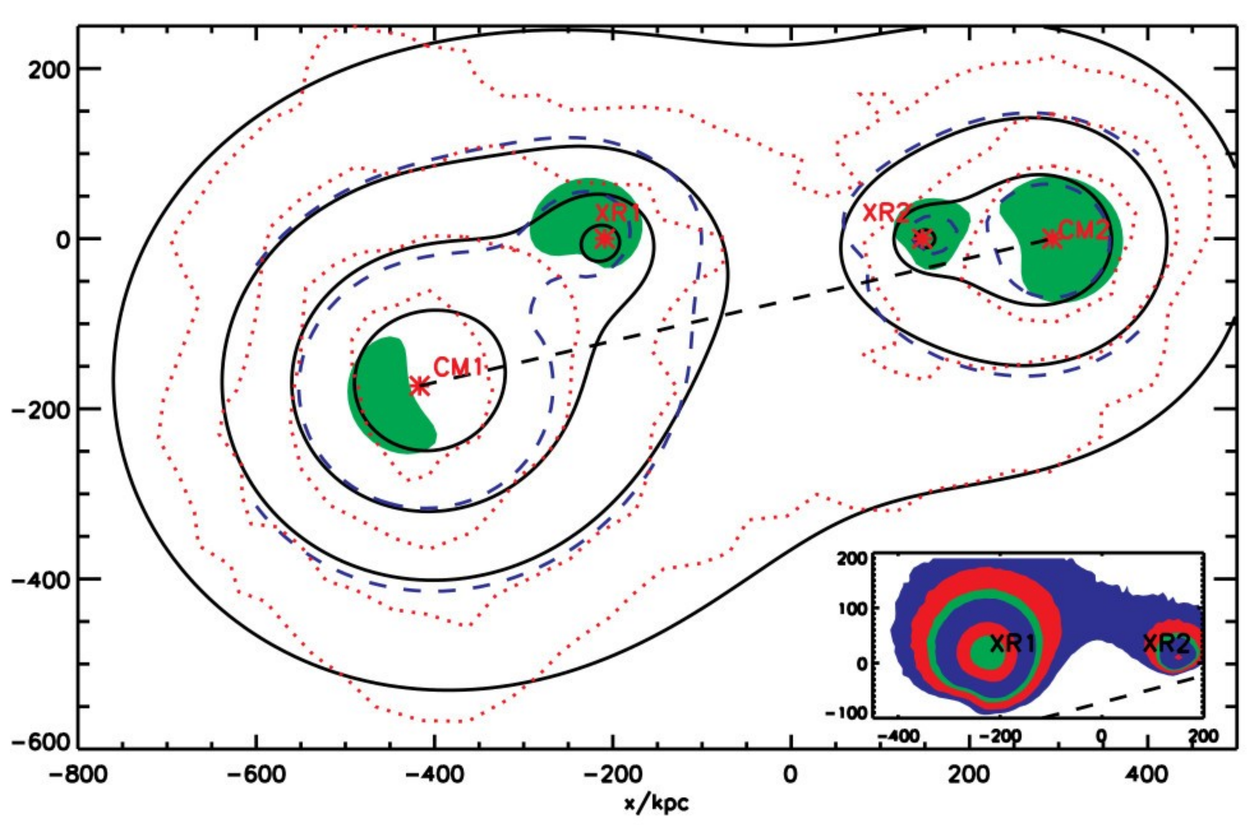
- Tensor $g_{\mu\nu}$ → Einstein's metric
- Vector A^{μ} → to get the "right" gravitational lensing (Sanders 1997, ApJ)
- Scalar Φ → to get the DM effect for matter (Bekenstein & Milgrom 1984, ApJ)
- Free Function → interpolation function (similar to AQUAL & QMOND)

Matter follows a "physical metric" given by a disformal transformation:

$$\tilde{g}_{\mu,\nu} = g_{\mu,\nu} e^{-2\phi} + A_{\mu} A_{\nu} e^{-2\phi} - A_{\mu} A_{\nu} e^{2\phi} = e^{-2\phi} g_{\mu,\nu} - 2 A_{\mu} A_{\nu} \sinh(2\phi)$$

Bullet Cluster: Not a problem in MOND/TeVeS

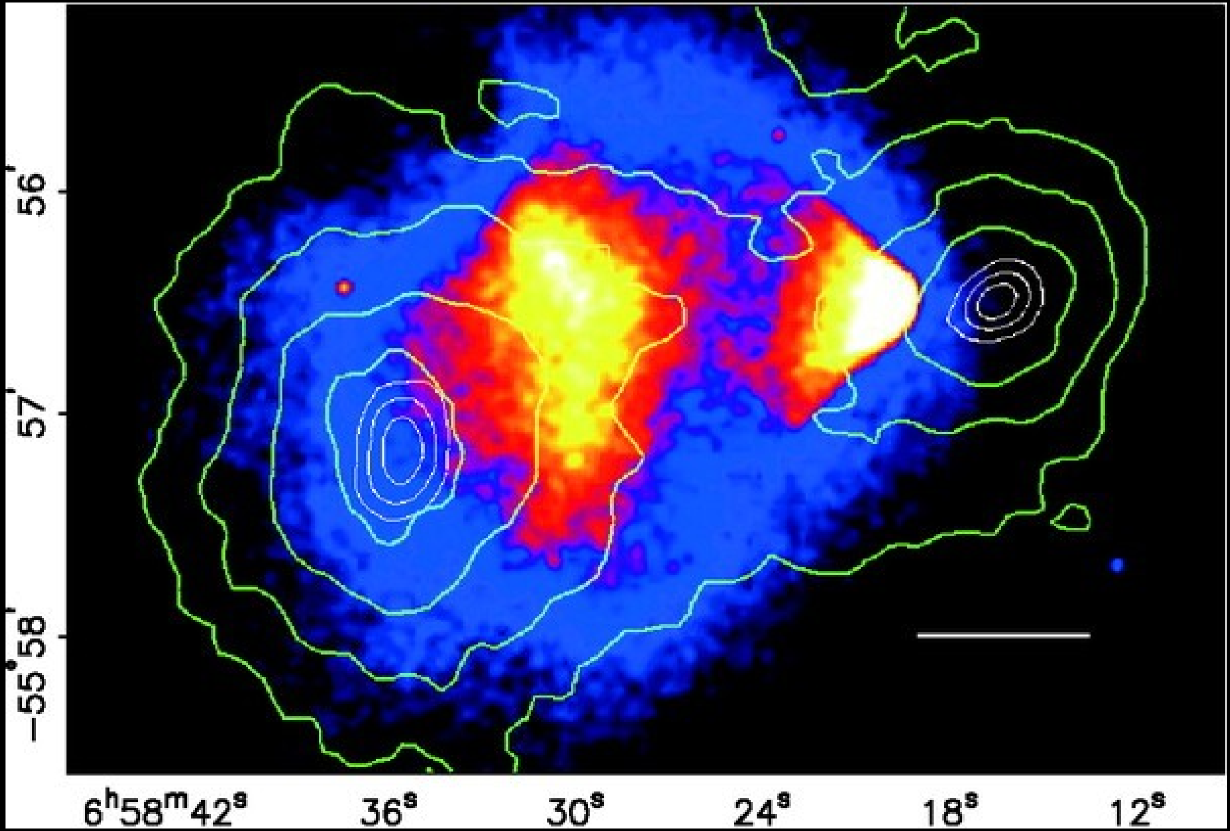


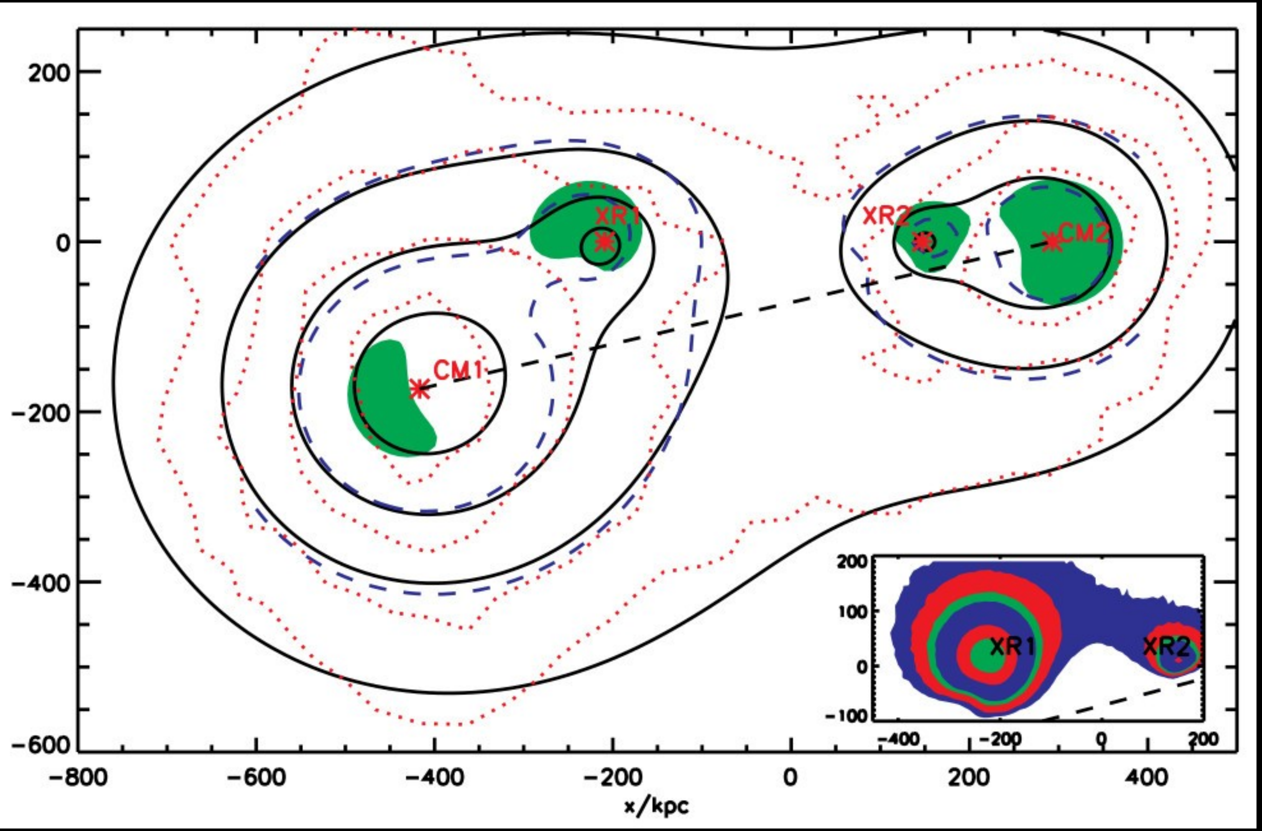


OBSERVATIONS (Clowe+2004, ApJ)
Green: Observed lensing map (total mass)
Blue/Red/Yellow: X-ray emission (hot gas)

MOND (Angus+2006, MNRAS; 2007, ApJ)
Red: Observed lensing convergence map
Black: MOND model with 2eV neutrinos
Blue: total surface densities (baryons+υ)

Bullet Cluster: Not a problem in MOND/TeVeS



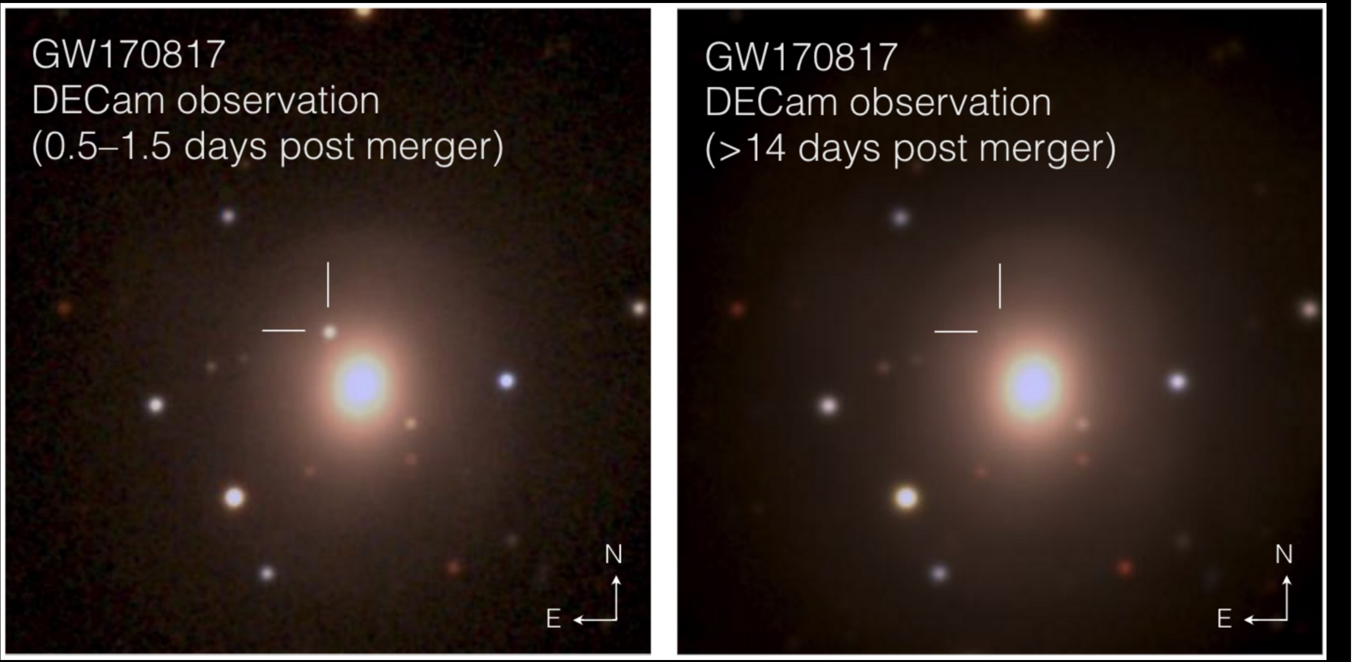


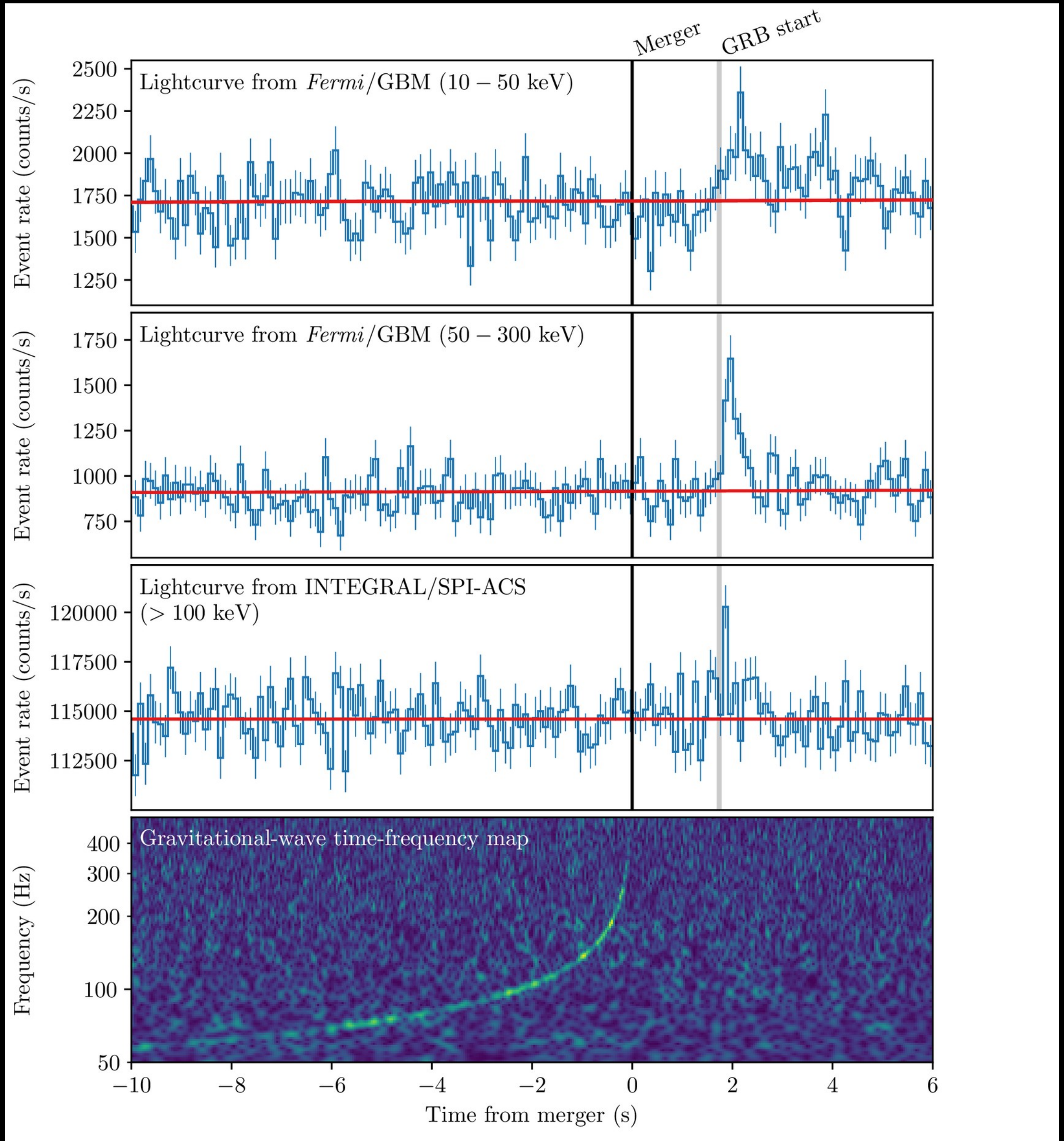
X-ray bow shock → High collision speed of ~4500 km/s

Very rare ($p \sim 10^{-7}$) in ΛCDM simulations (Farrar & Rosen, 2006, PRL)

But natural in MOND simulations (Angus & McGaugh 2008, MNRAS)

TeVeS is ruled out by kilonova discovery (GW170817)





Gravitational wave signal immediately followed by gamma-ray signal:

$|c_{GW} - c_{EM}| < 10^{-15}\, c_{EM}$

But TeVeS predicted $c_{GW} \neq c_{EM}$!

New Class of TeVeS-like theories (Skordis & Zlosnik)

Combine scalar & vector in a time-like vector (Skordis & Zlosnik 2019, PRD):

$B^{\mu} = e^{-2\phi} A^{\mu}$ such that $B^2 = g^{\mu\nu} B_{\mu} B_{\nu} = -e^{-2\phi}$ ⇨ $c_{GW} = c_{EM}$

New Class of TeVeS-like theories (Skordis & Zlosnik)

Combine scalar & vector in a time-like vector (Skordis & Zlosnik 2019, PRD):

$B^{\mu} = e^{-2\phi} A^{\mu}$ such that $B^2 = g^{\mu\nu} B_{\mu} B_{\nu} = -e^{-2\phi}$ ⇨ $c_{GW} = c_{EM}$

Fix free terms in the Action by requiring (Skordis & Zlosnik 2021, PRL):

(1) Newton in non-rel. limit when $|\nabla\Phi| \gg a_0$ in *quasi-static situations*

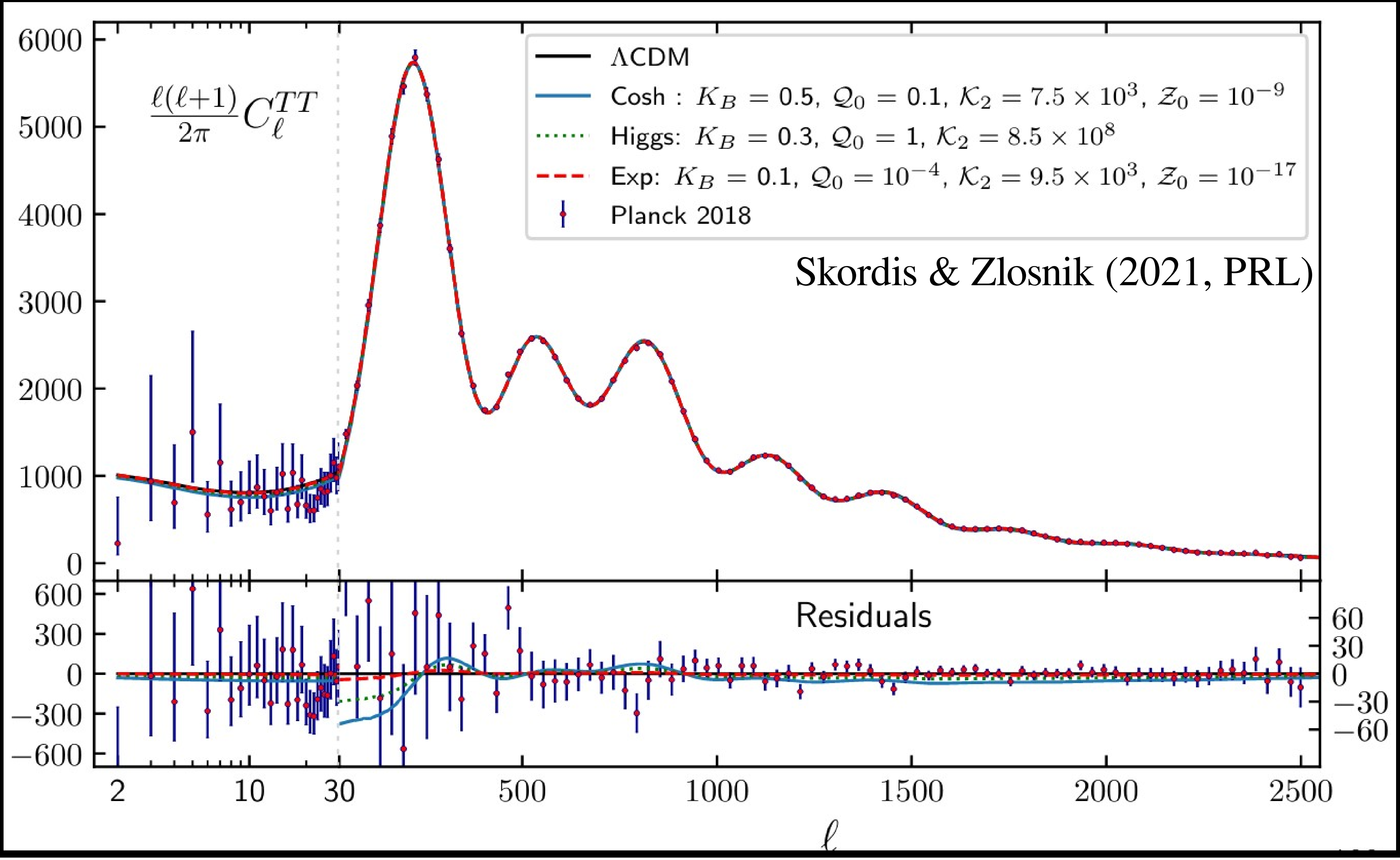
(2) AQUAL in non-rel. limit when $|\nabla\Phi| \ll a_0$ in *quasi-static situations*

(3) Gravitational lensing without dark matter

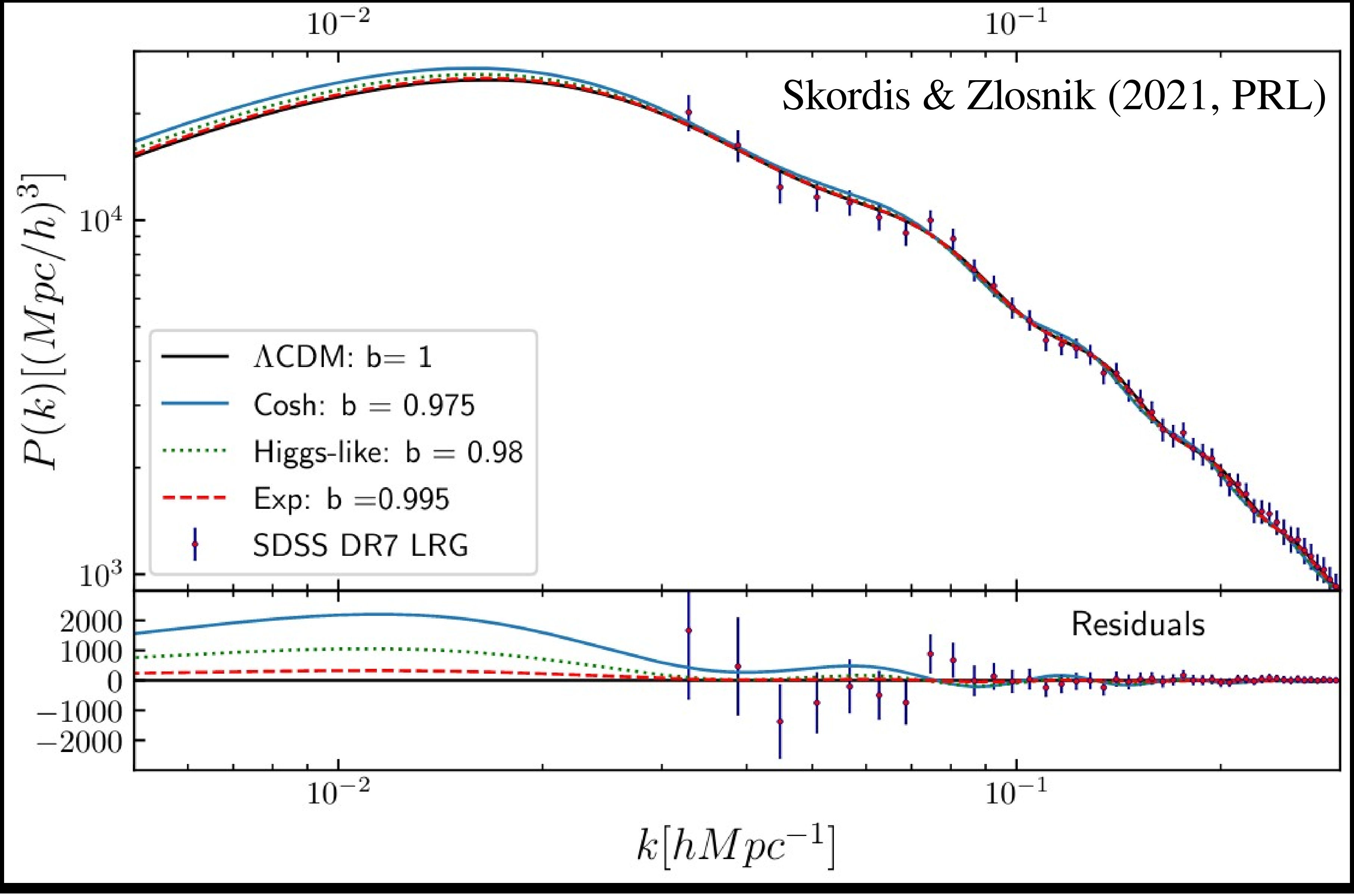
(4) Tensor mode of GW propagates at the speed of light

(5) FLRW background with the same expansion history as LCDM

CMB Power Spectrum well fitted by Relativistic MOND

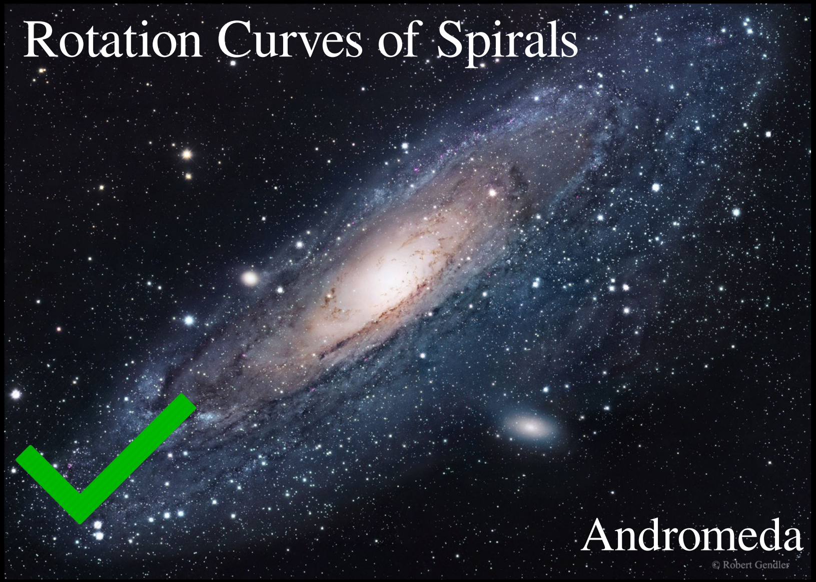


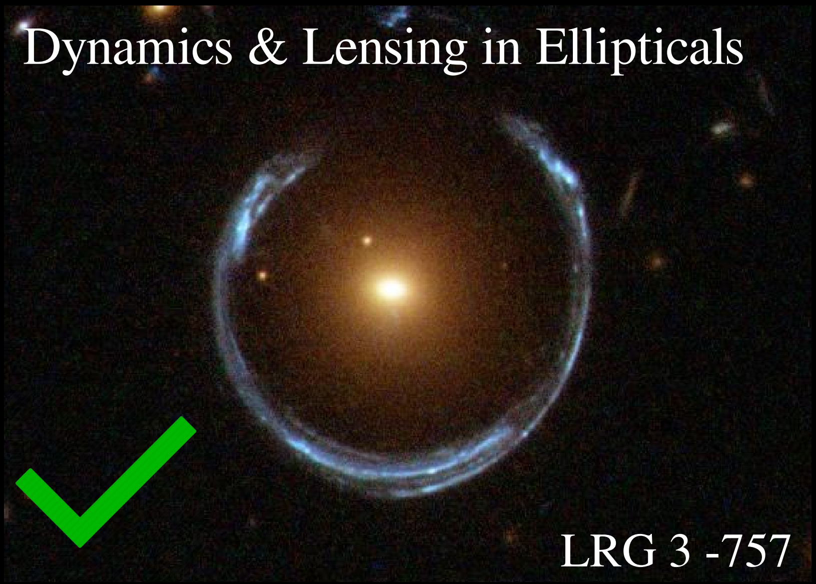
Matter Power Spectrum well fitted by Relativistic MOND



Status of MOND at Various Scales

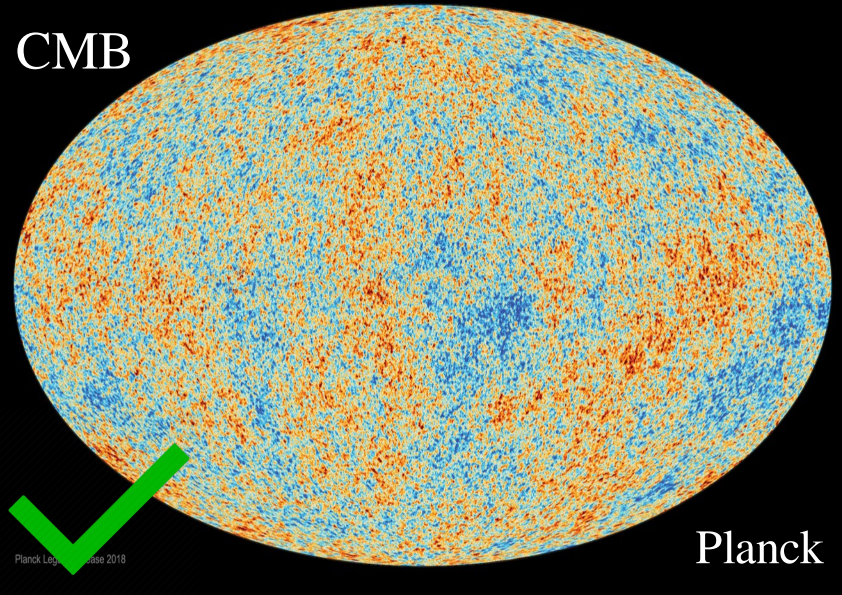
Galaxy Scales (~1-100 kpc)

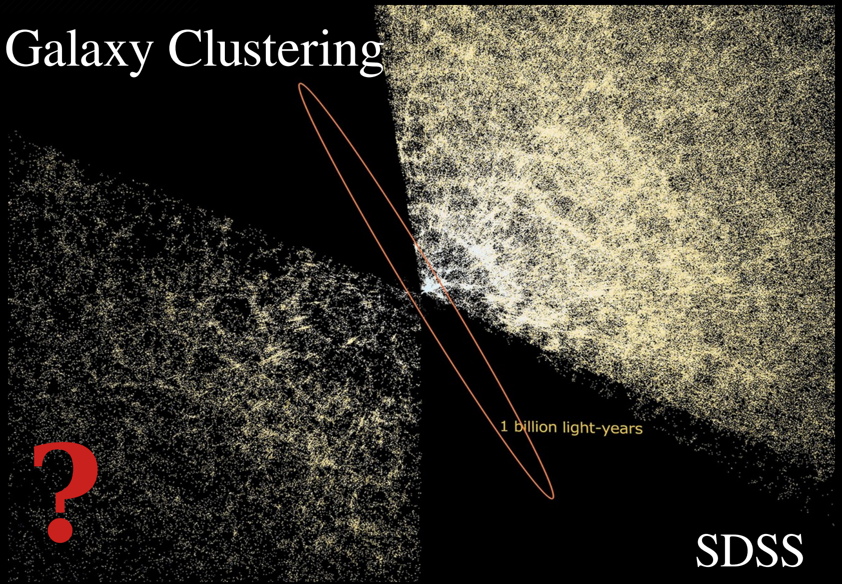




Groups/Clusters Scales (~1-5 Mpc)

Cosmological Scales (>100 Mpc)





Further readings on MOND

Springer Link

Review Article | Open Access | Published: 07 September 2012

Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions

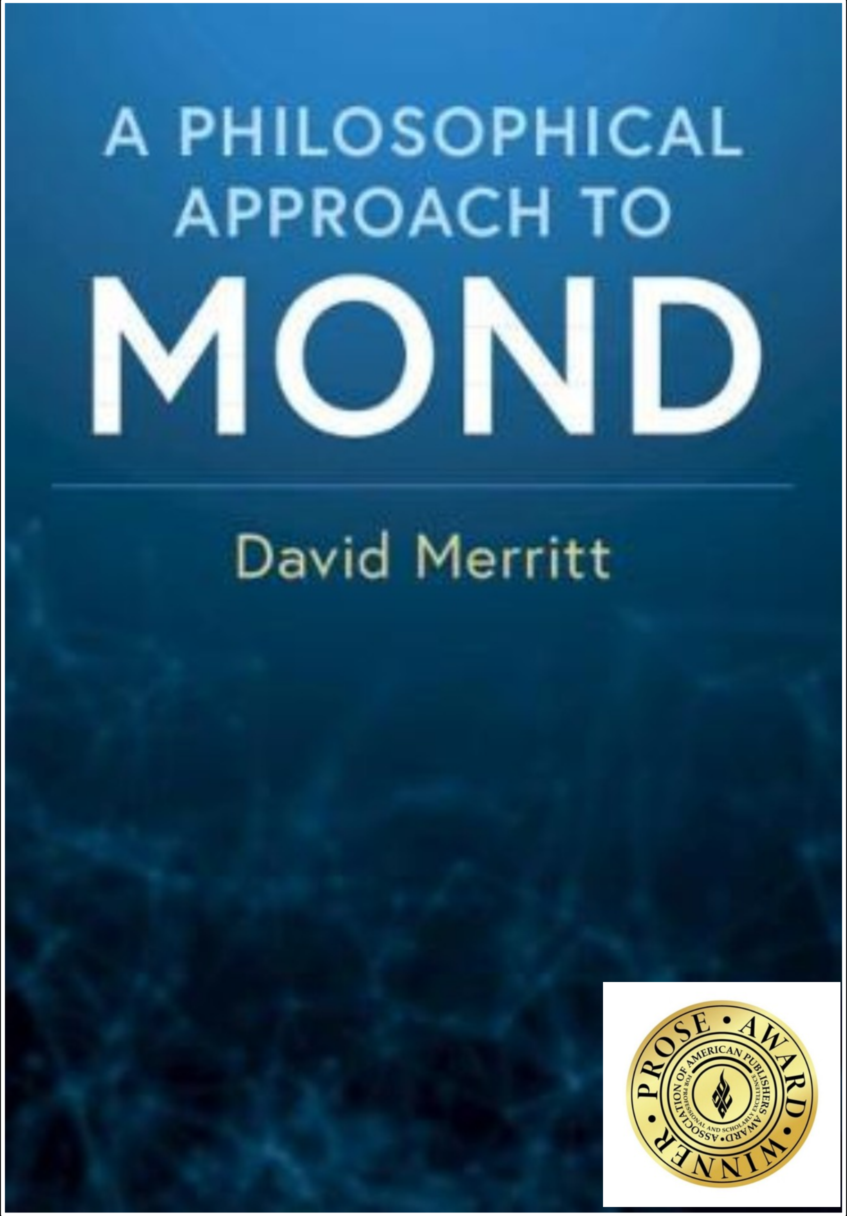
Famaey & McGaugh, 2012, Living Reviews in Relativity, 15, 10

symmetry MDPI

Review

From Galactic Bars to the Hubble Tension: Weighing Up the Astrophysical Evidence for Milgromian Gravity

Banik & Zhao, 2022, Symmetry, 14, 7, 1331



More Slides

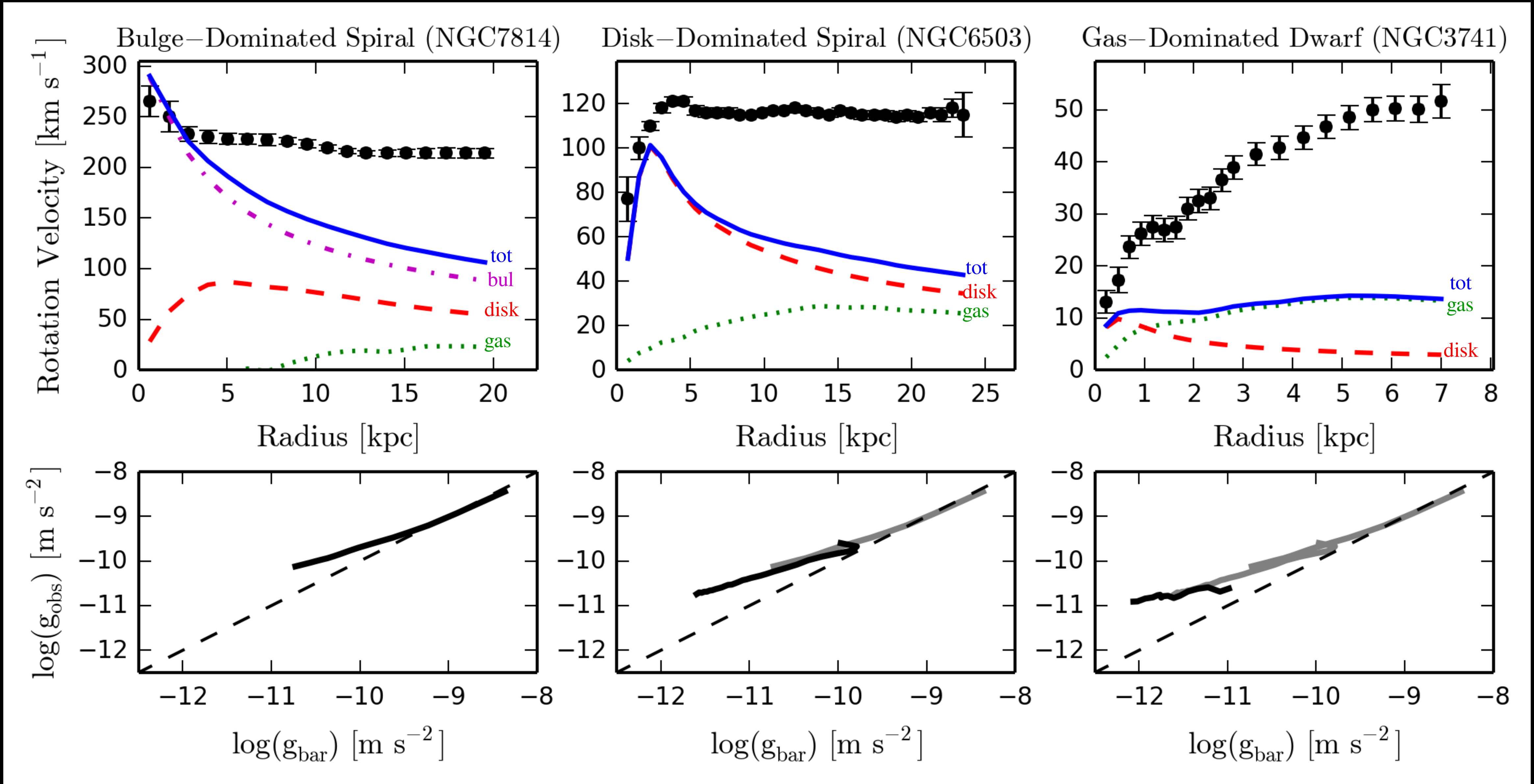
MOND is predictive → can be falsified (Popper is happy)

ΛCDM is reactive → can reproduce almost everything a-posteriori (after the observations)

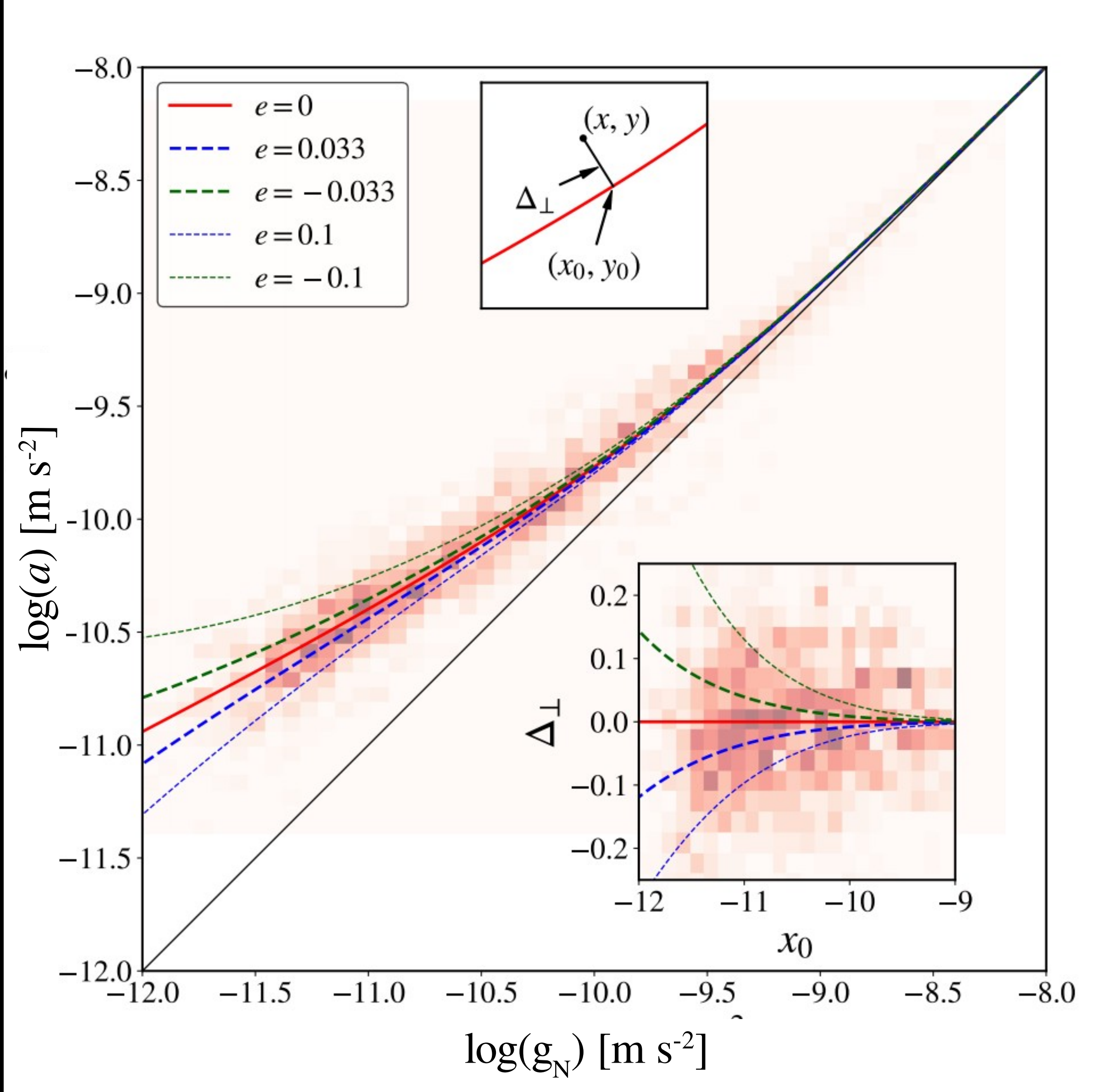
Why did MOND get any prediction right?

If MOND turns out to be wrong, galaxy formation in ΛCDM (baryonic physics) must work in a precise way to mimic the MOND phenomenology on galaxy scales.

Galaxies lie on the same RAR *despite* their diversity



External field effect (EFE): implications for the RAR



- For truly *isolated* galaxies:

  $a = \nu_0(g_{N,int}/a_0)g_{N,int}$

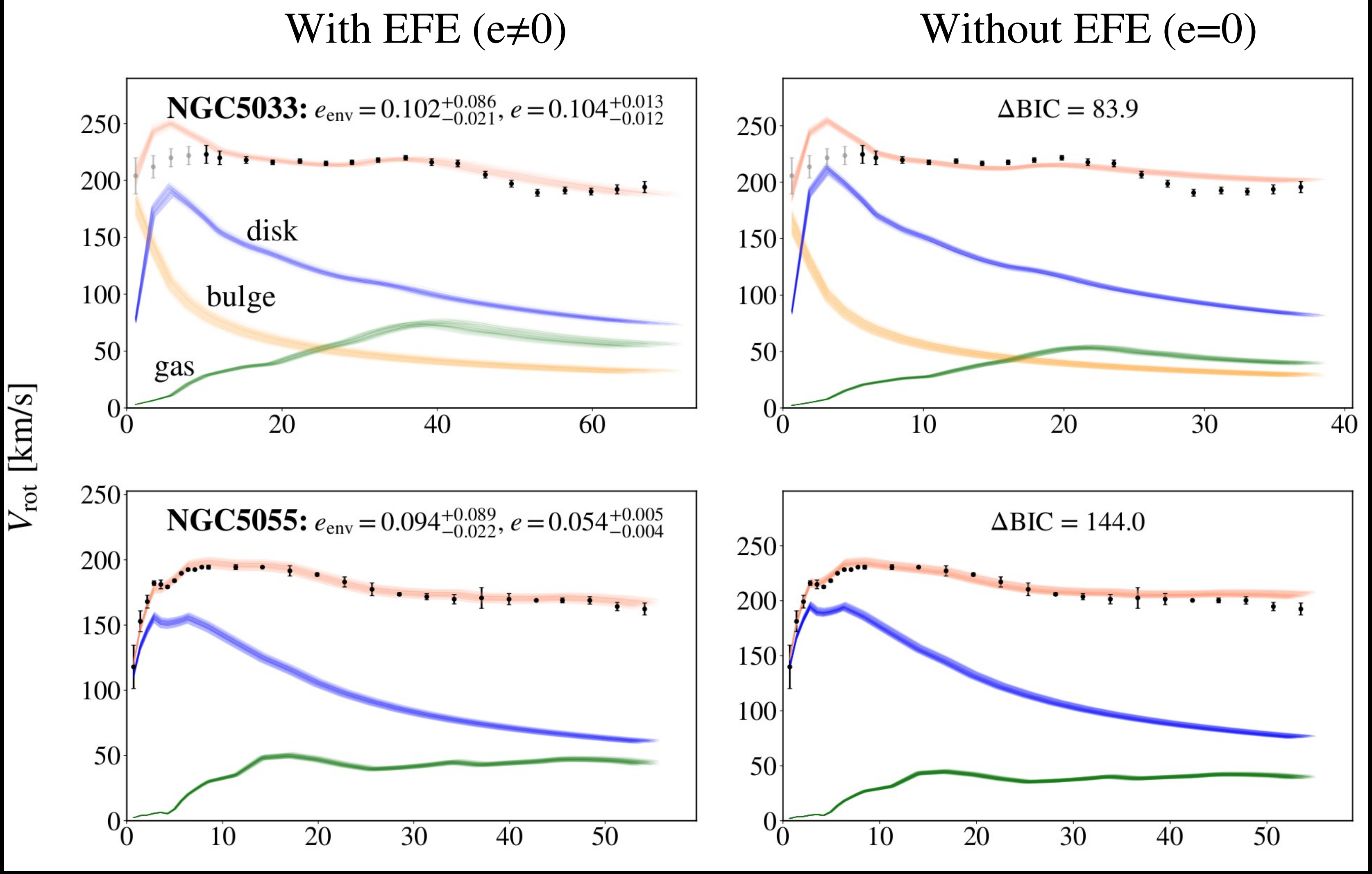
- For galaxies subjected to $e = g_{N,ext}/a_0$:

  $a = \nu_e(g_{N,int}/a_0; e)g_{N,int}$

- RAR should be a *family of curves* depending on the galaxy environment
- We can fit RCs to infer the value of e and independently estimate e_{env} from the galaxy large-scale environment.

Chae, Lelli, Desmond et al. (2020)

EFE is weak: individual detections only in extreme cases

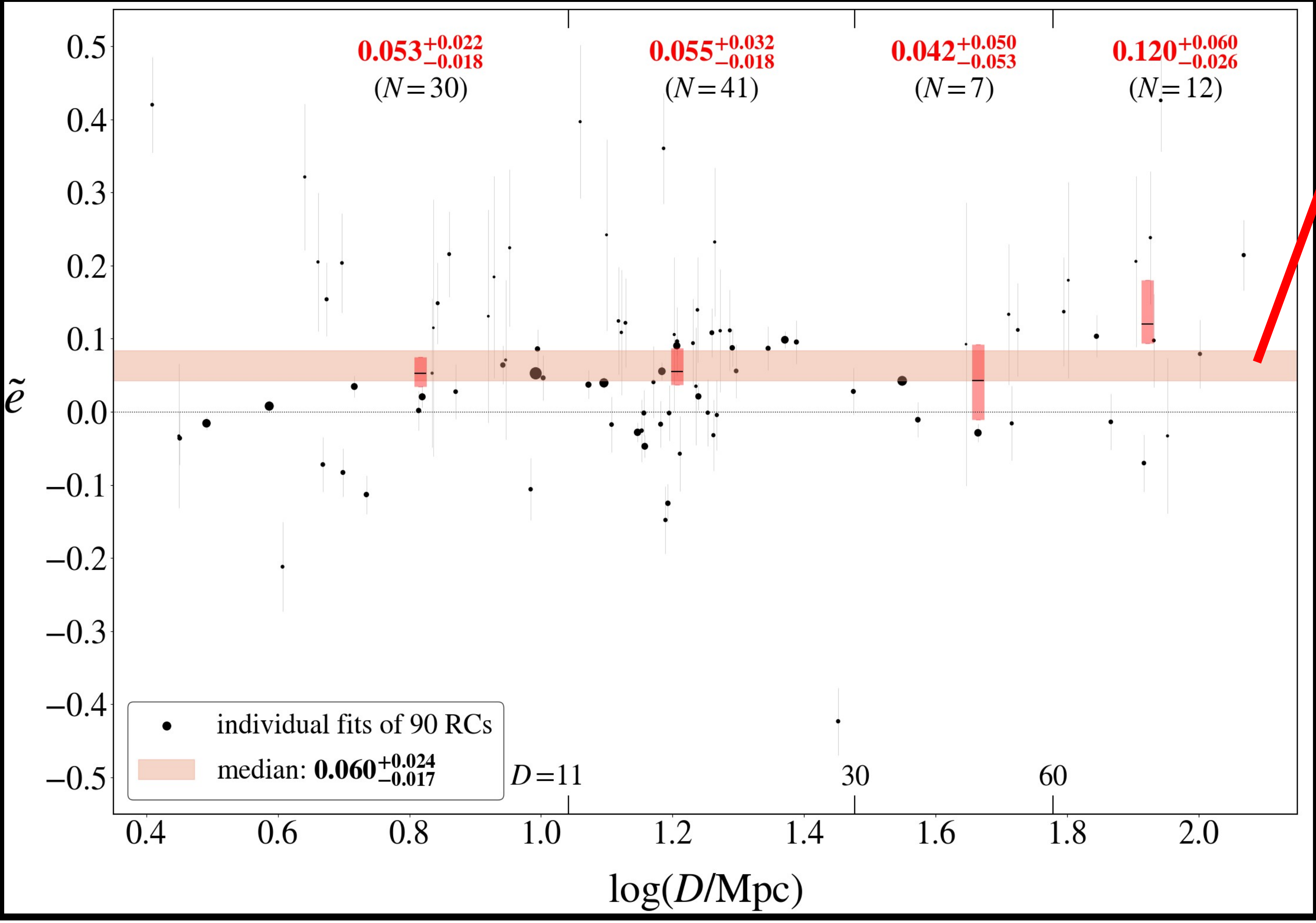


NGC 5033

NGC 5055

Chae+2020, 2021

Statistical approach: EFE>0 at >4σ and agrees with LSS



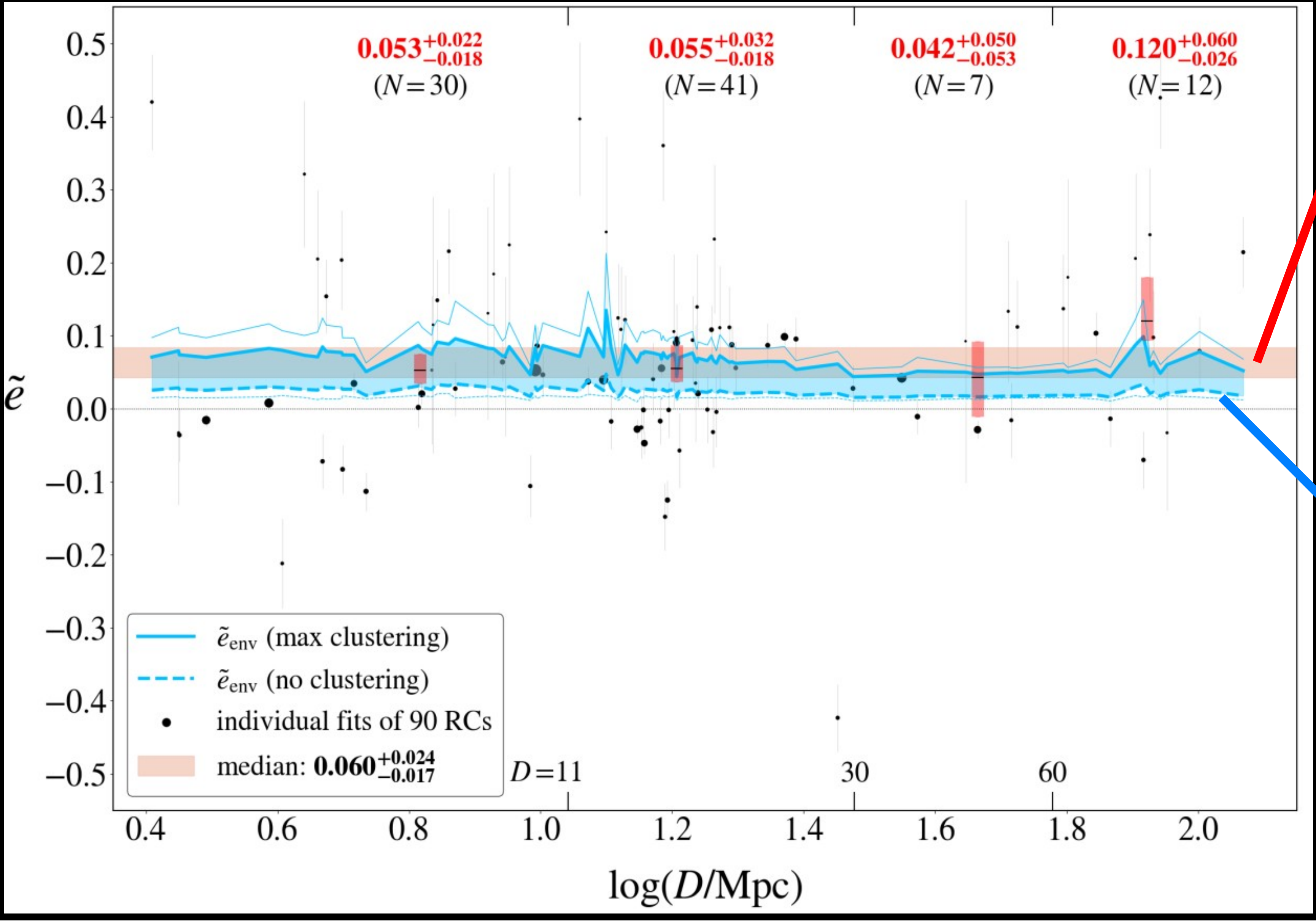
$\frac{g_{ext}}{a_0}$ ←

From Rotation Curve Fits

Chae+2020, 2021

Statistical approach: EFE>0 at >4σ and agrees with LSS

$$\frac{g_{ext}}{a_0}$$ ←



From Rotation Curve Fits

From Baryon Large Scale Structure

Chae+2020, 2021

MOND External Field effect (EFE)

MOND is non-linear → both internal ($g_{N,int}$) and external ($g_{N,ext}$) fields

For non-isolated systems, three possibilities:

(1) $g_{N,ext} \ll g_{N,int} \ll a_0$

→ MOND regime (e.g. nearly isolated galaxies)

(2) $g_{N,int} \ll a_0 \ll g_{N,ext}$

→ Newtonian regime (e.g. star clusters in the inner MW)

(3) $g_{N,int} \ll g_{N,ext} \ll a_0$

→ Newton with $G_{eff} \sim G\, a_0/g_{N,ext}$ (e.g. some satellites of MW)

EFE is a general MOND prediction but details depend on the specific theory

MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \sim \frac{H_0 \cdot c}{2\pi}$$

H_0 = Hubble constant → maybe $a_0(\mathrm{t}) \sim H(\mathrm{t})$?

$$a_0 \sim \frac{c^2 \sqrt{\Lambda/3}}{2\pi}$$

Λ = Cosmological constant → relation to Dark Energy?

IF this numerology has some deeper, fundamental meaning:

either the state of the Universe at large enters in local dynamics, or

the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

MOND as Modified Inertia (Milgrom 1994, Annals of Physics)

$$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla}\Phi_N$$

$\overline{A}$ is a functional of the full trajectory $\overline{x}(t)$ with dimension of m/s^2.
For $a \gg a_0$, $A \rightarrow a = d^2x/dt^2$ (Newton's 2nd Law).

In MOND no full theory yet setting A from varying S but two general results (Milgrom 1994):

(A) IF we impose the Newtonian and MOND limits at high and low accelerations +
Galilei Invariance → Eq. of motions are the same in all inertial frames: $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

Theory is time non-local: $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i \vec{x}}{dt^i}; i = 1, 2, \ldots N\right)$

Accelerations at ($\overline{x}$, t) depend on the full orbital history!

(B) For purely circular orbits: $\vec{a}\,\mu\left(\frac{a}{a_0}\right) = \vec{g}_N$ holds exactly (e.g. RAR for disk galaxies)

The interpolation function is a derived concept valid for circular orbits

Lovelock-Grigore Theorem:

GR (+Λ) is the only theory that satisfy these assumptions:

1- Geometry is Reimannian

2- The Action depends only on $g_{\mu\upsilon}$

3- It is diffeomorphism invariant

4- It is local

5- It leads to 2nd order field equations

Path towards a relativistic version of MOND