

# MOND: An Alternative to Particle Dark Matter

Federico Lelli

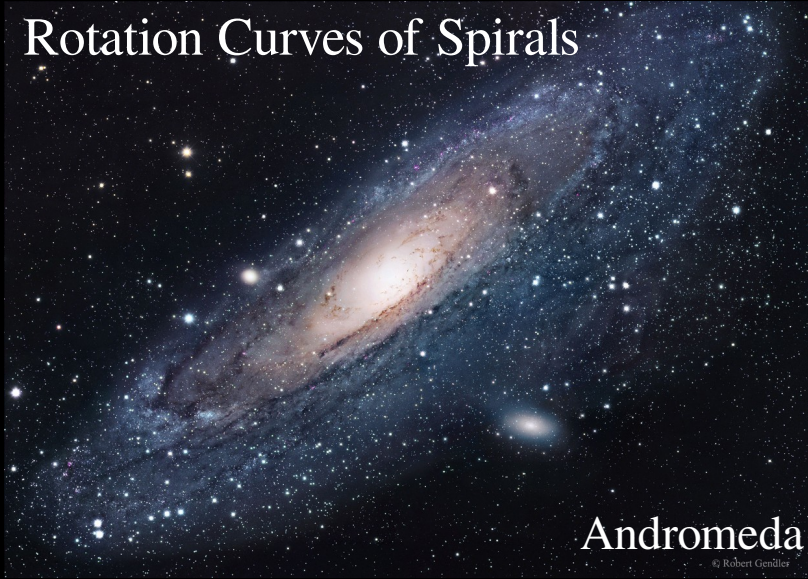
INAF – Arcetri Astrophysical Observatory



# The Dark Matter Effect at Various Scales

## Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



Andromeda

© Robert Gendler

Dynamics & Lensing in Ellipticals

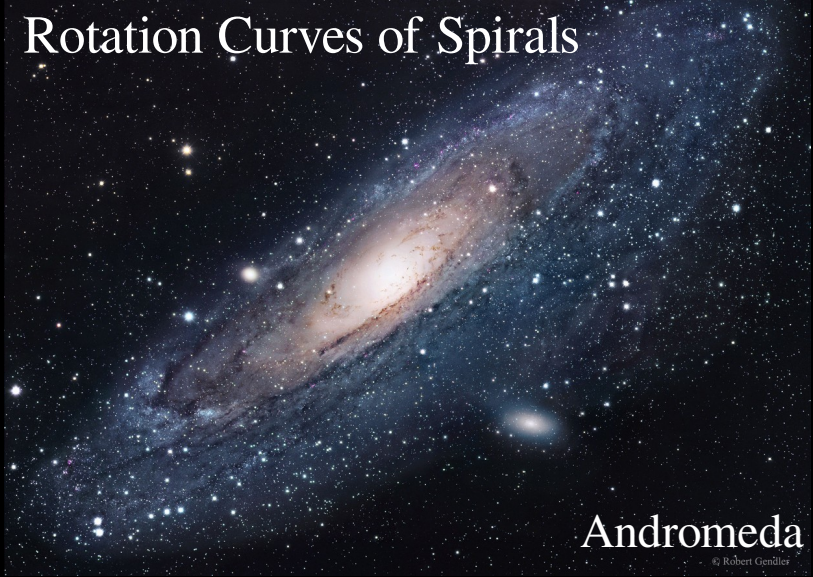


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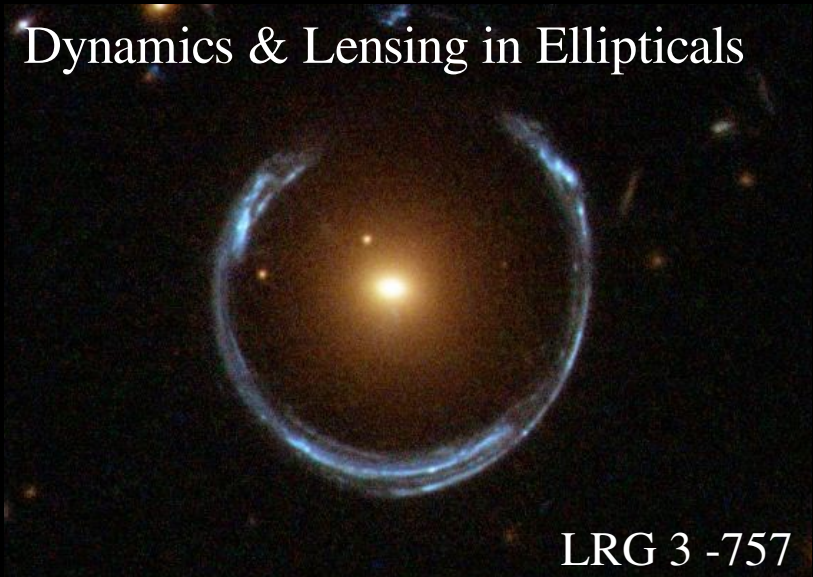


## Groups/Clusters Scales (~1-5 Mpc)

Interactions & Mergers in Groups



Dynamics & Lensing in Ellipticals



Dynamics & Lensing in Clusters



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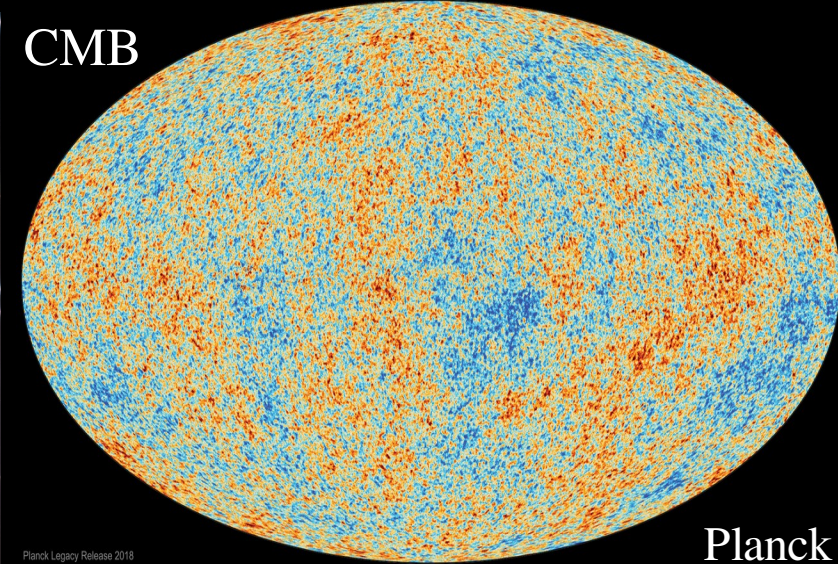
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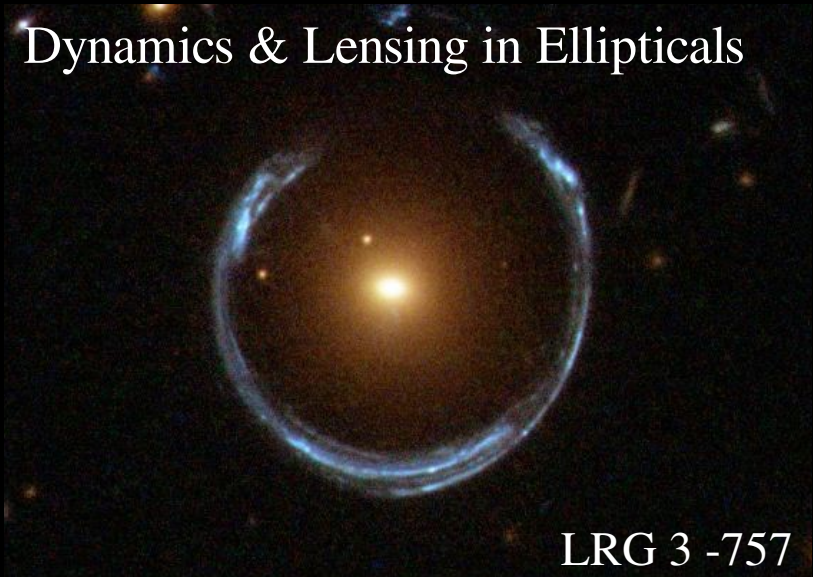


## Cosmological Scales (>100 Mpc)

CMB



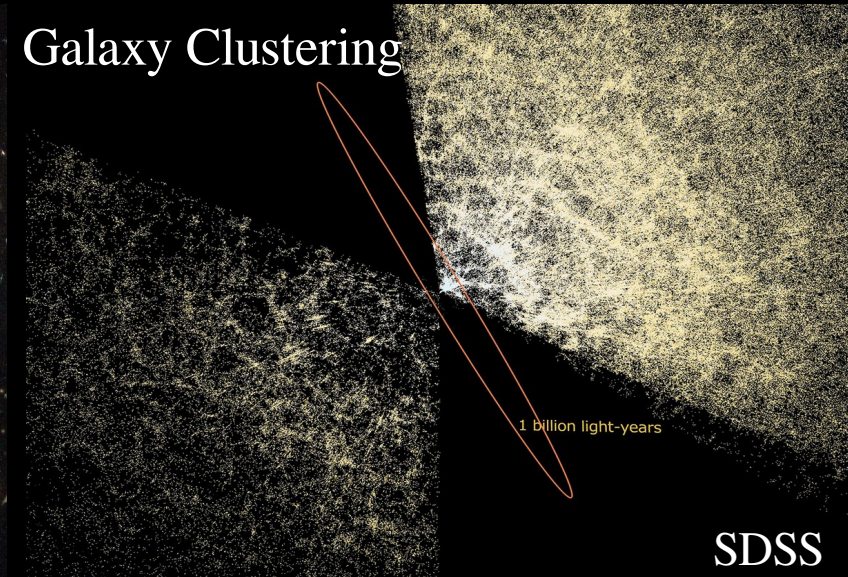
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Dynamics & Lensing in Clusters



Galaxy Clustering



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Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



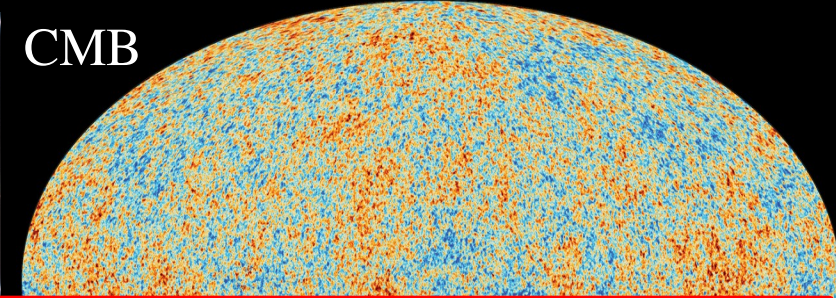
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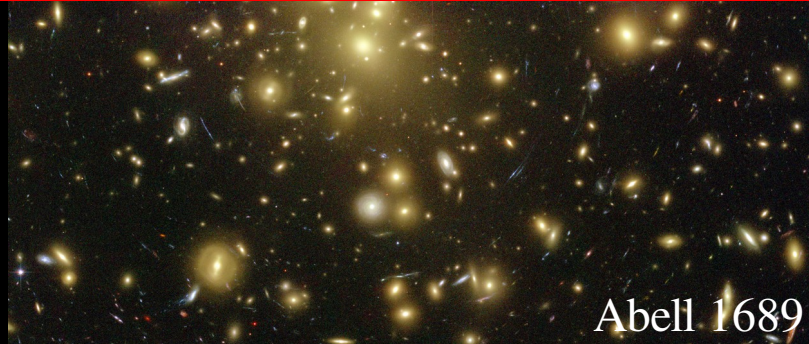


This is *not* direct evidence for *particle* dark matter:  
Standard Laws of Gravity (Einstein & Newton) +  
Standard Model of Particle Physics = Do NOT work

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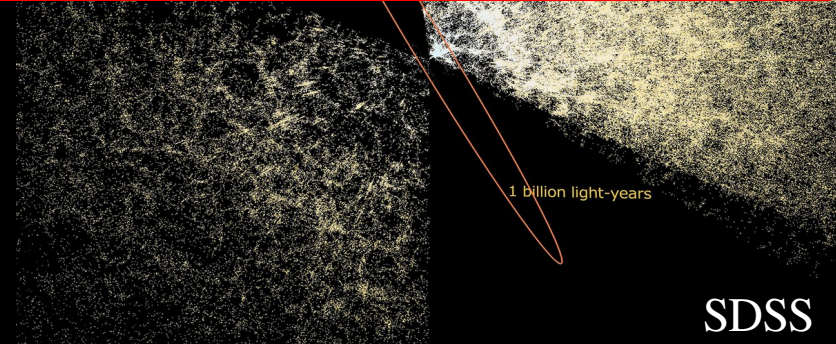


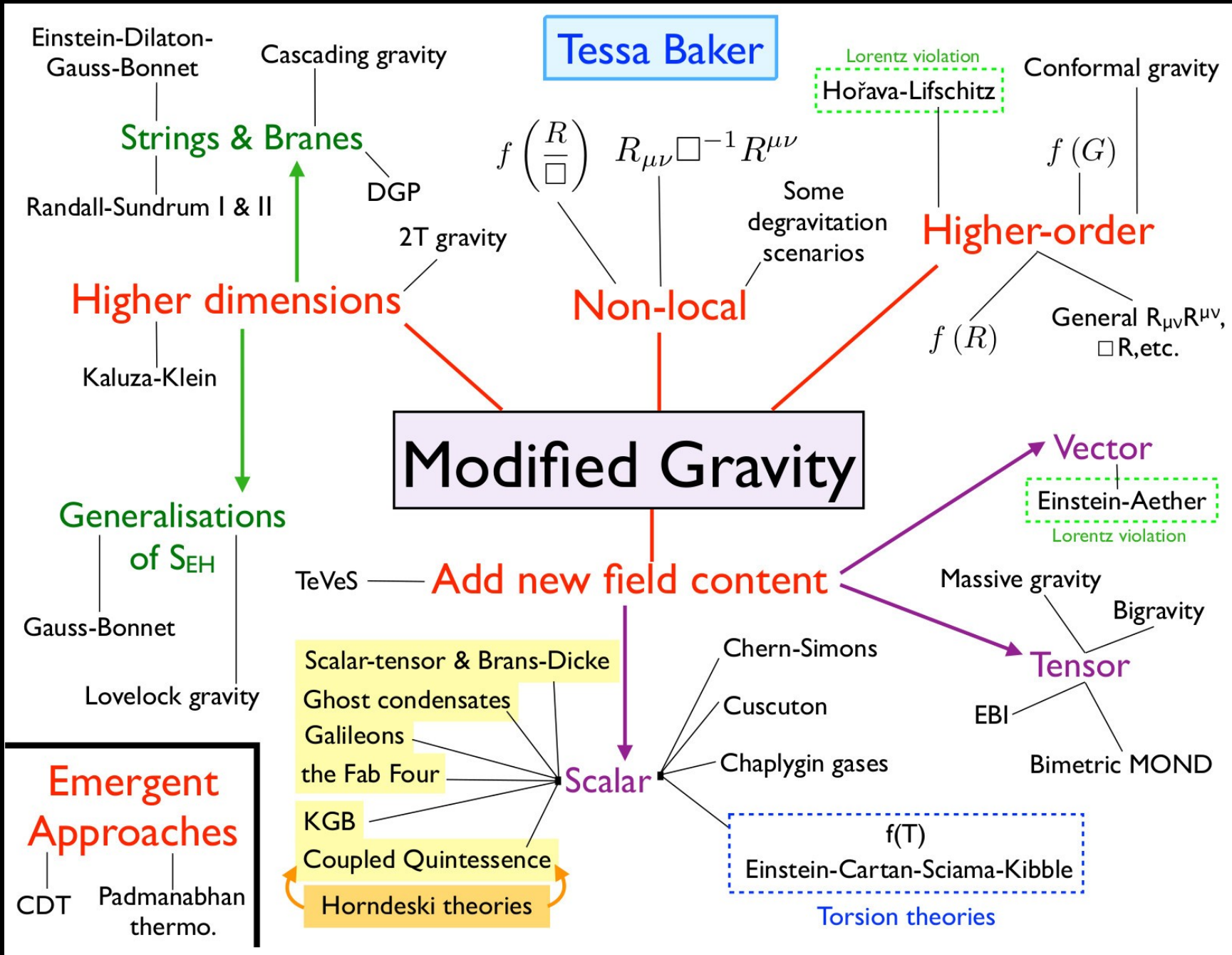
Abell 1689



1 billion light-years

SDSS





Many versions of Modified Gravity to explain DM or DE.

This talk will not cover all this.

I will focus on Milgromian Dynamics (aka MOND). Empirically motivated alternative to CDM.

# Talk Outline:

## I. General MOND paradigm

- Theory postulates & general predictions
- Tests on stationary systems: “isolated” galaxies & clusters

## II. Specific MOND theories

- Tests on non-stationary systems: interacting galaxies & clusters
- Relativistic theories: cosmology and the CMB

# I. The general MOND paradigm



# MOND = Modified Newtonian Dynamics or MilgrOmiaN Dynamics



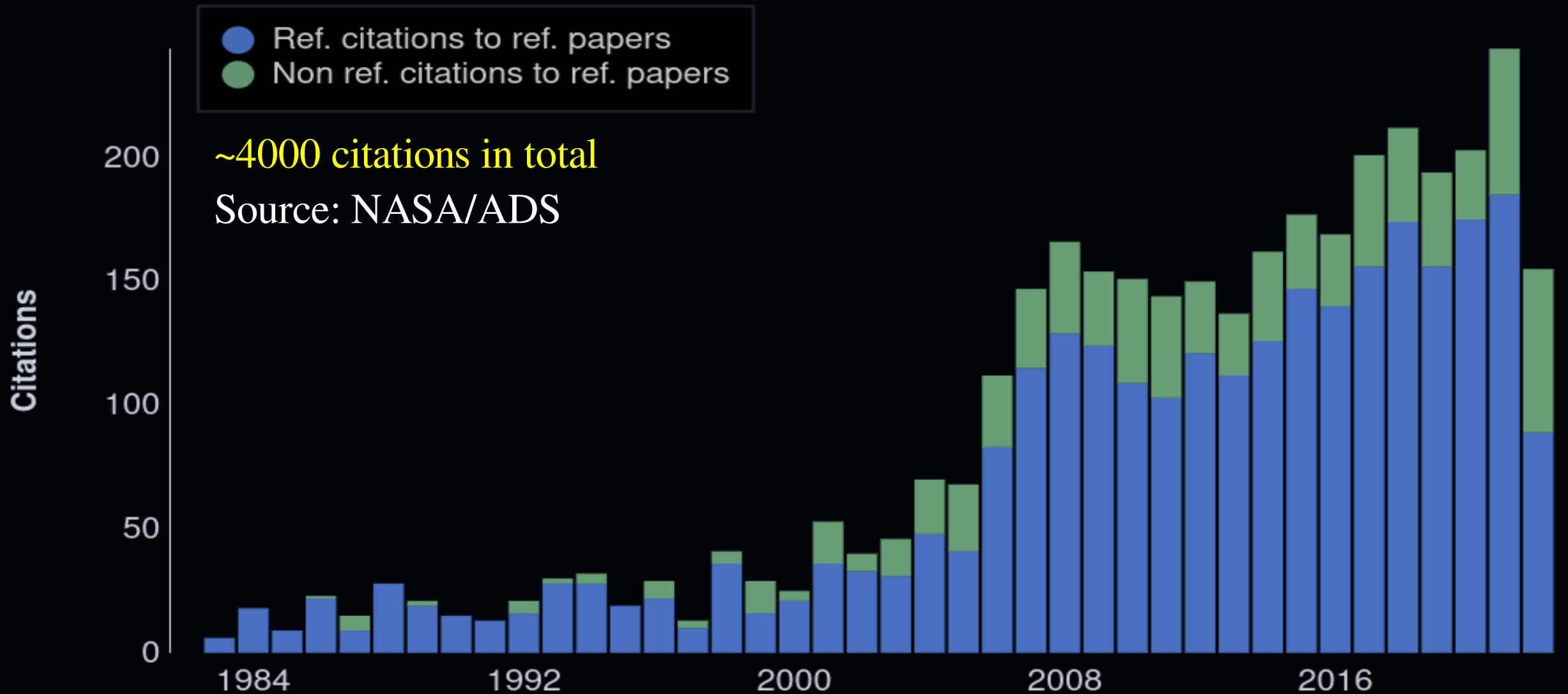
Proposed by Moderhai Milgrom (1983a, b, c; ApJ).

“40 years of MOND” conference

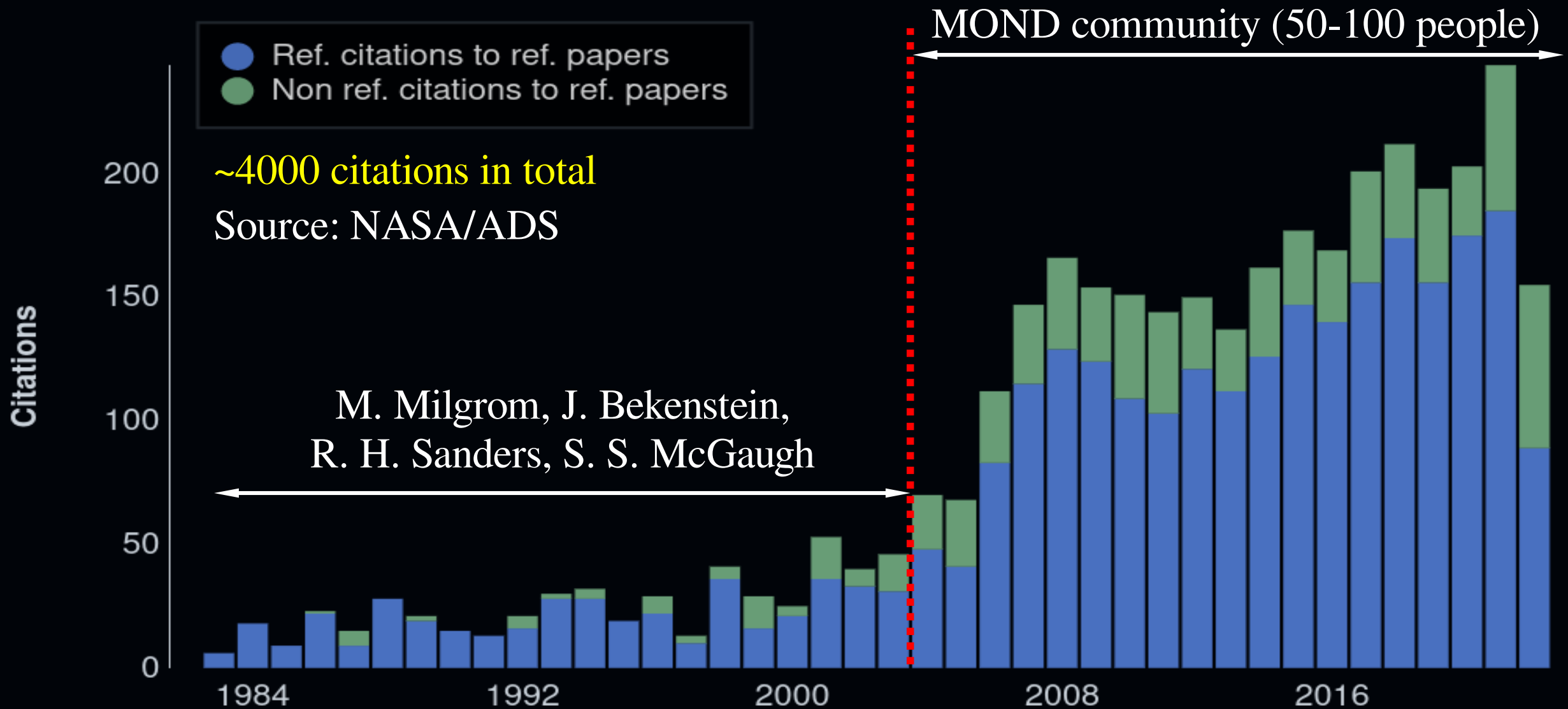
St. Andrews (Scotland), 5-9 June 2023

Registration deadline: 1<sup>st</sup> of May

# Citations to the original MOND trilogy (Milgrom 1983a, b, c)



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**Or impose scale invariance** (Milgrom 2009, ApJ):  $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$   $V$  is invariant!

# A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES<sup>1</sup>

M. MILGROM

Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study

Received 1982 February 4; accepted 1982 December 28

**40 years ago!**

## ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

1. The Keplerian, circular velocity around a finite galaxy becomes independent of  $r$  at large radii, thus resulting in asymptotically flat velocity curves.

2. The asymptotic circular velocity ( $V_\infty$ ) is determined only by the total mass of the galaxy ( $M$ ):  $V_\infty^4 = a_0 GM$ , where  $a_0$  is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.

3. The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.

4. The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density  $\Sigma$  falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of  $\Sigma$ , the velocity rises more slowly to the asymptotic value.

5. The value of the acceleration constant,  $a_0$ , determined in a few independent ways is approximately  $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$ , which is of the order of  $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$ .

The main predictions are:

①. Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.

②. The  $V_\infty^4 = a_0 GM$  relation should hold exactly.

③. An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing  $r$  in a predictable way.



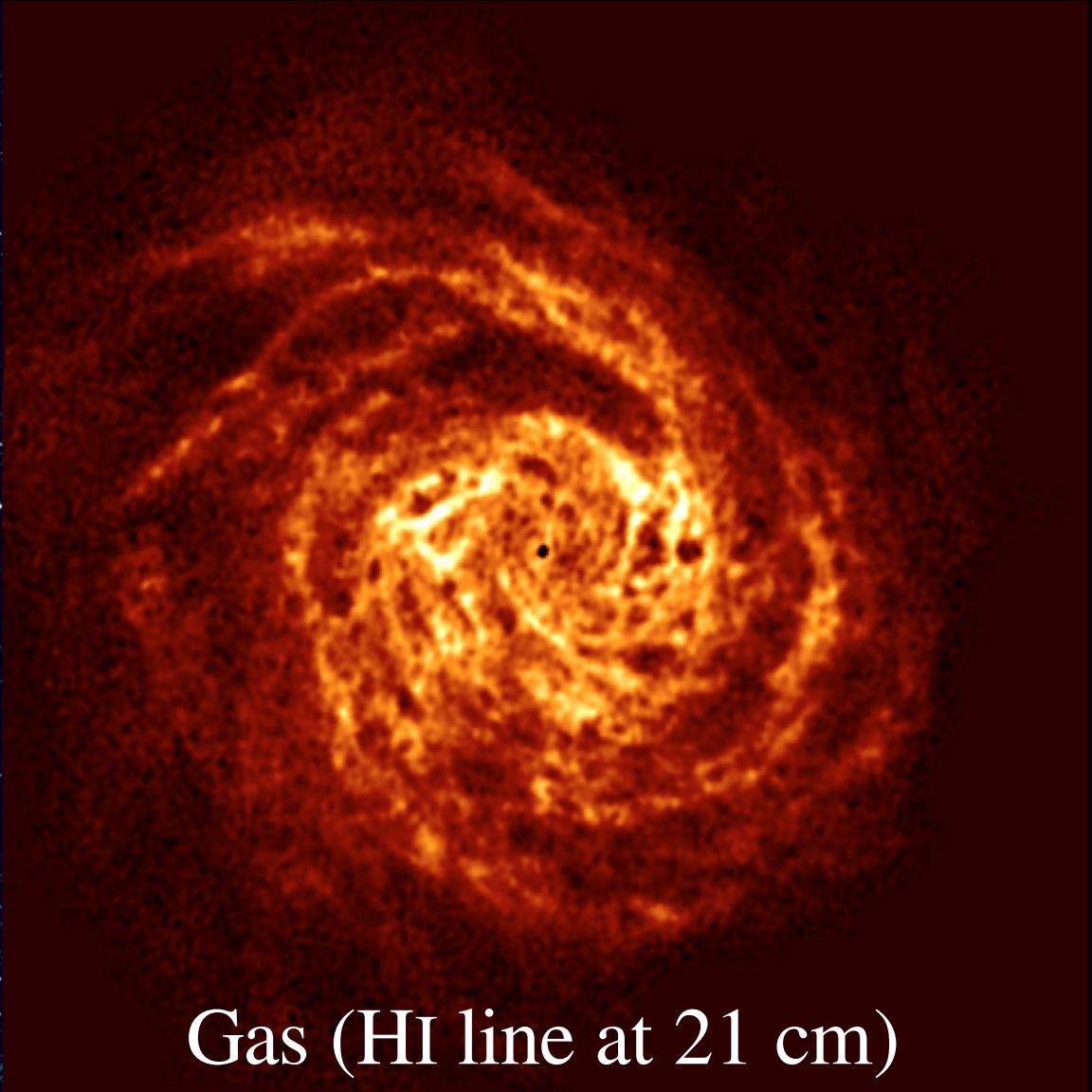
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NGC 6946 (Boomsma+2008, A&A)



Stars (optical image)



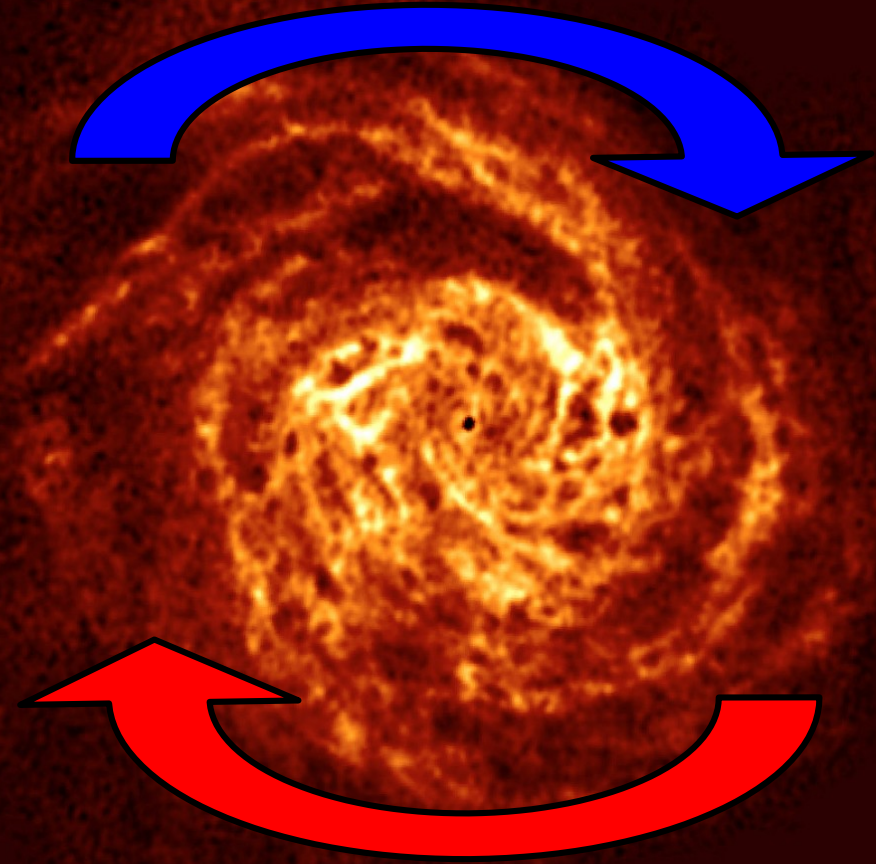
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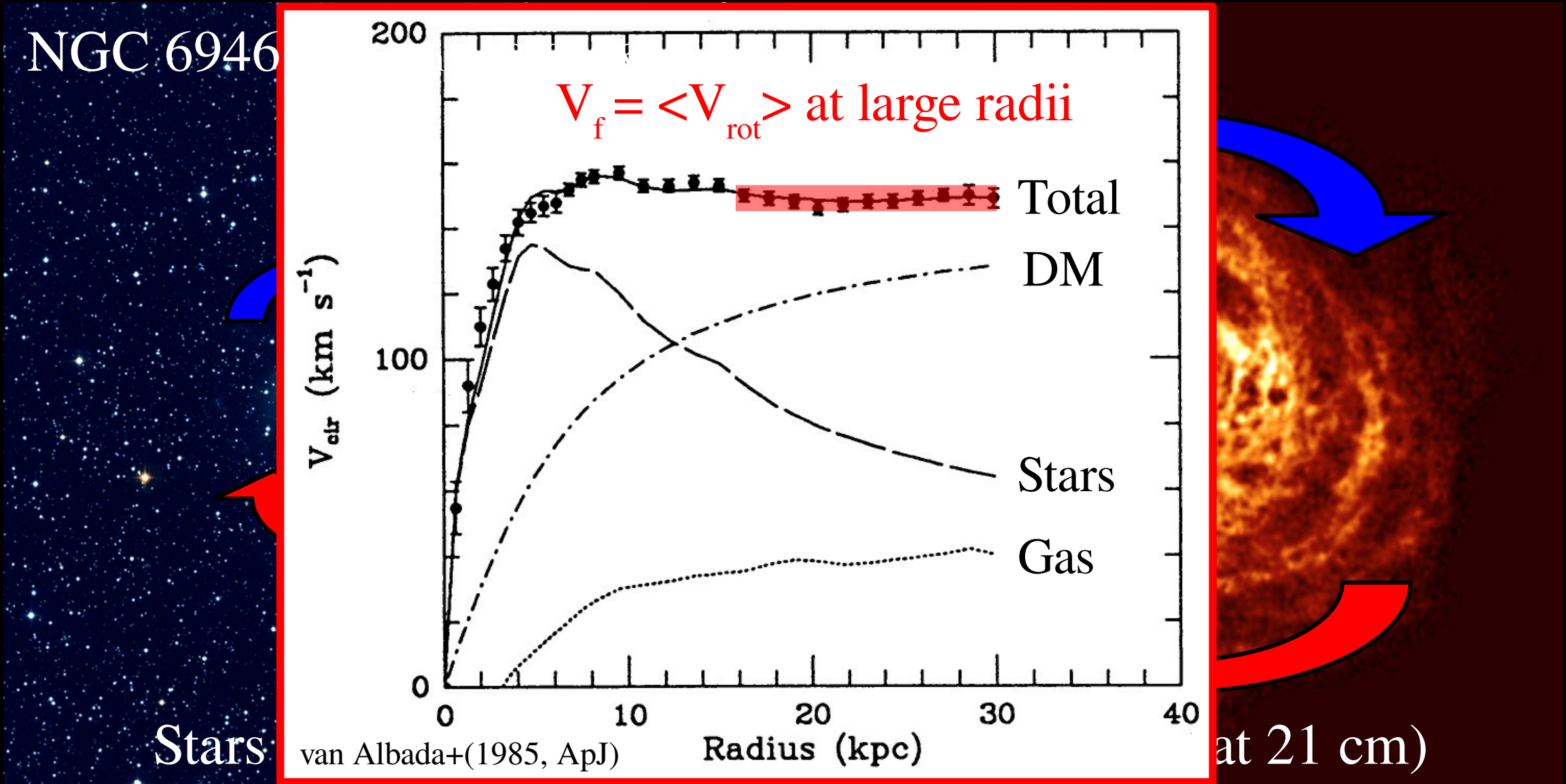


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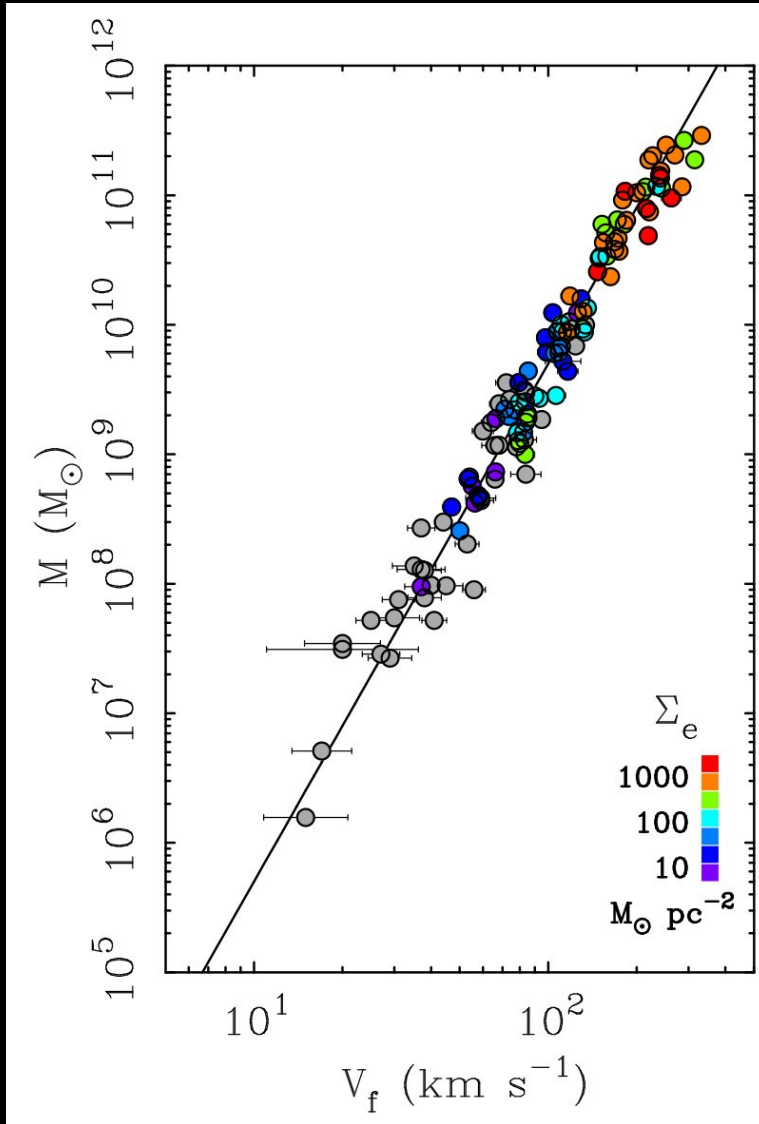


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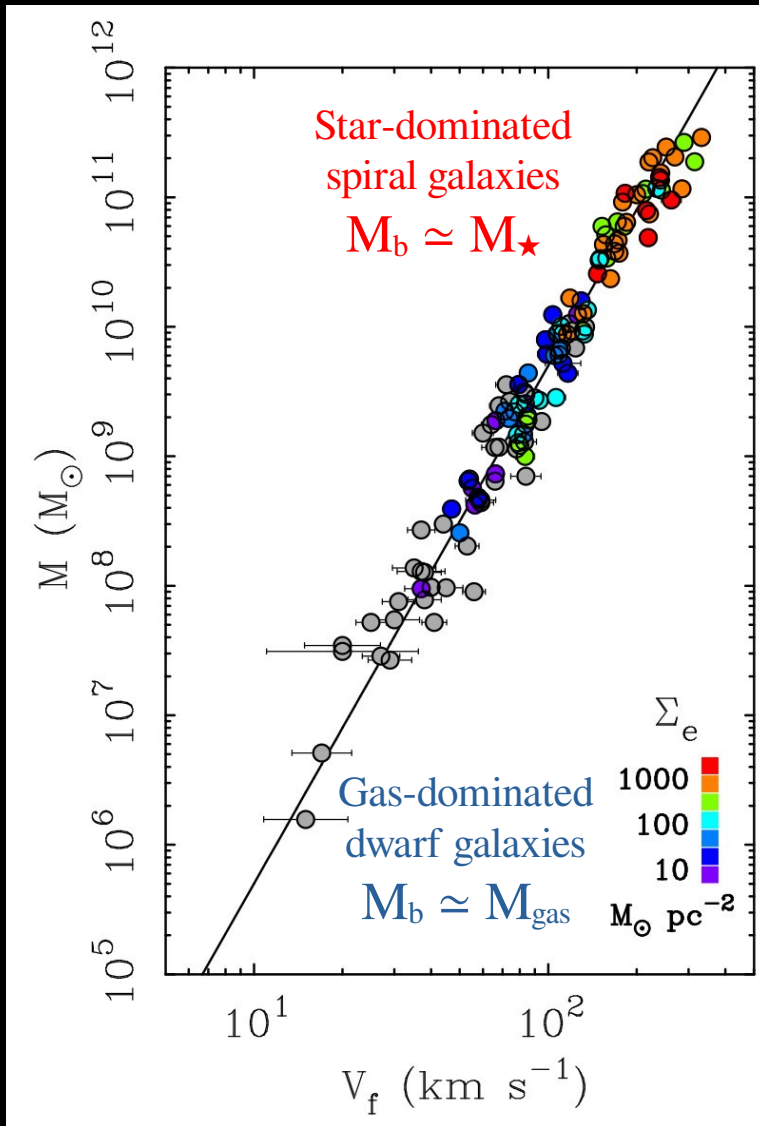
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Tully-Fisher relation (1977, A&A):  $L_B$  vs HI linewidth

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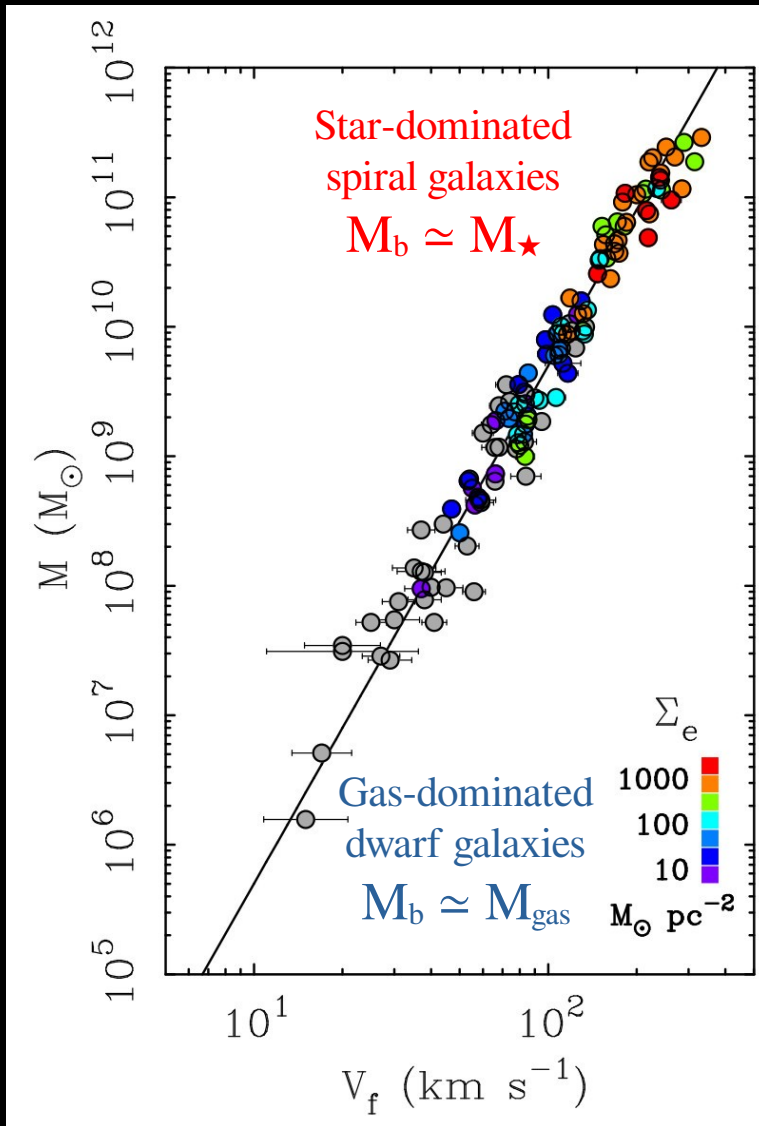
Four a-priori independent predictions in one equation:

(i) The relevant quantities are  $M_b$  (stars+gas) and  $V_f \rightarrow$  **OK**

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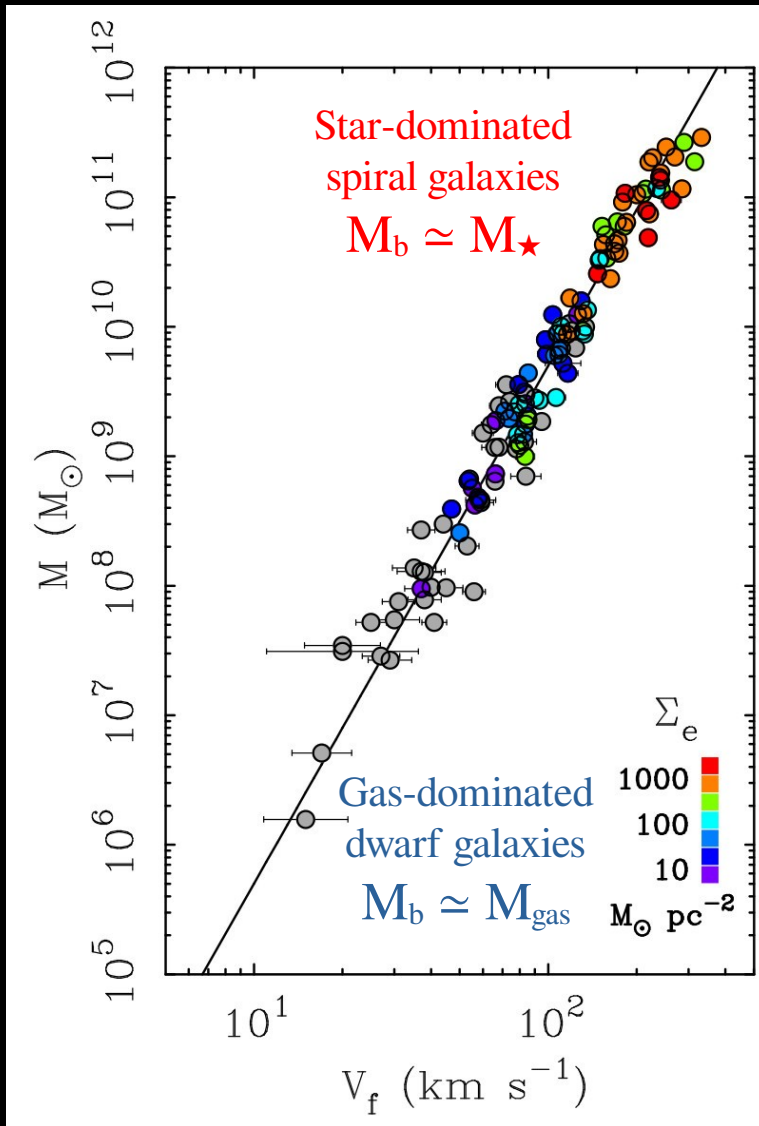
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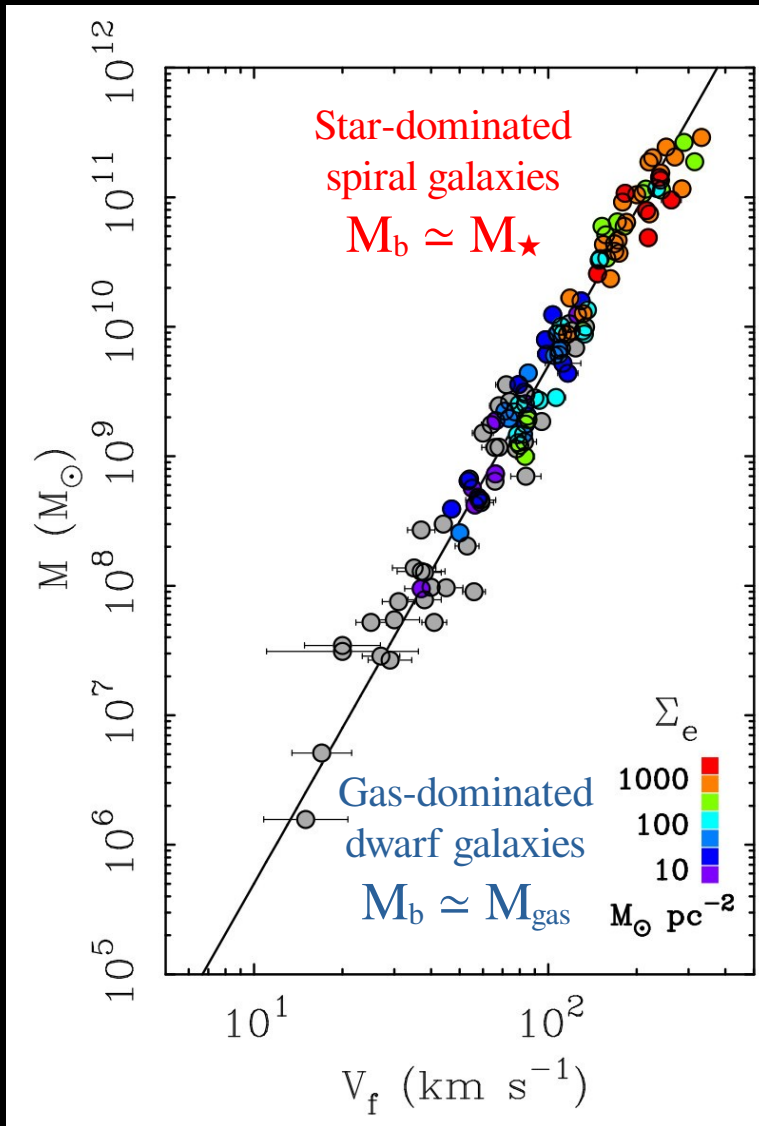
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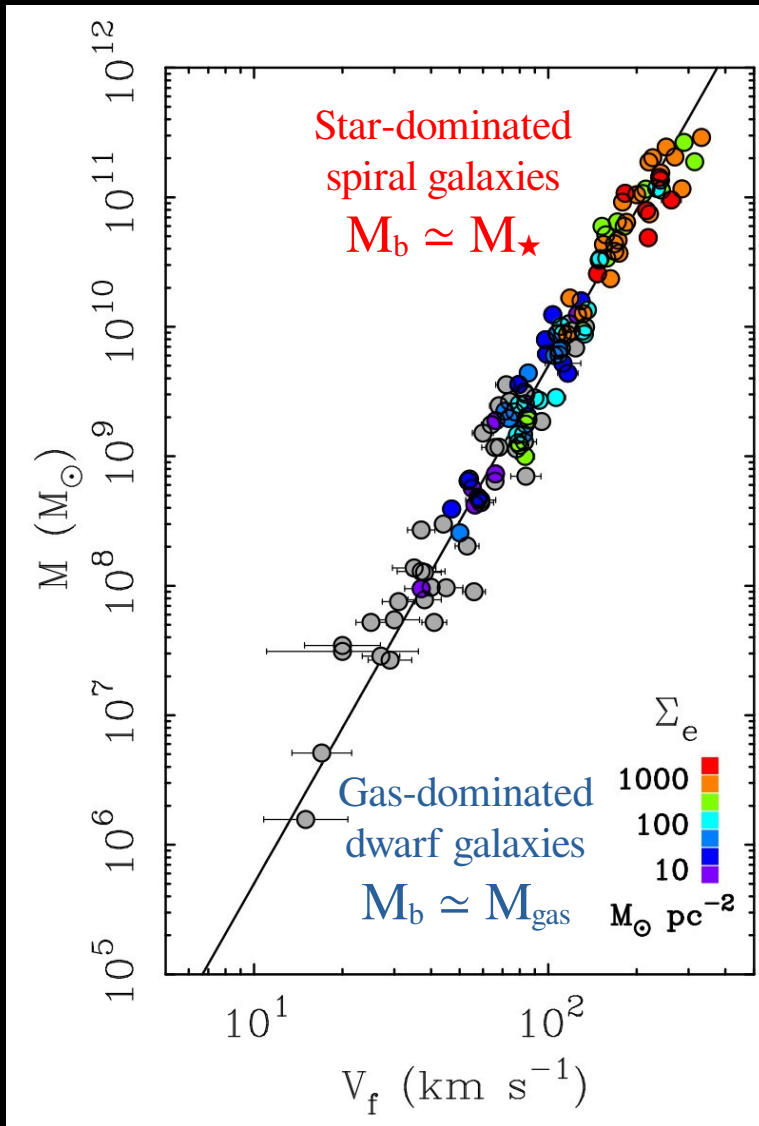
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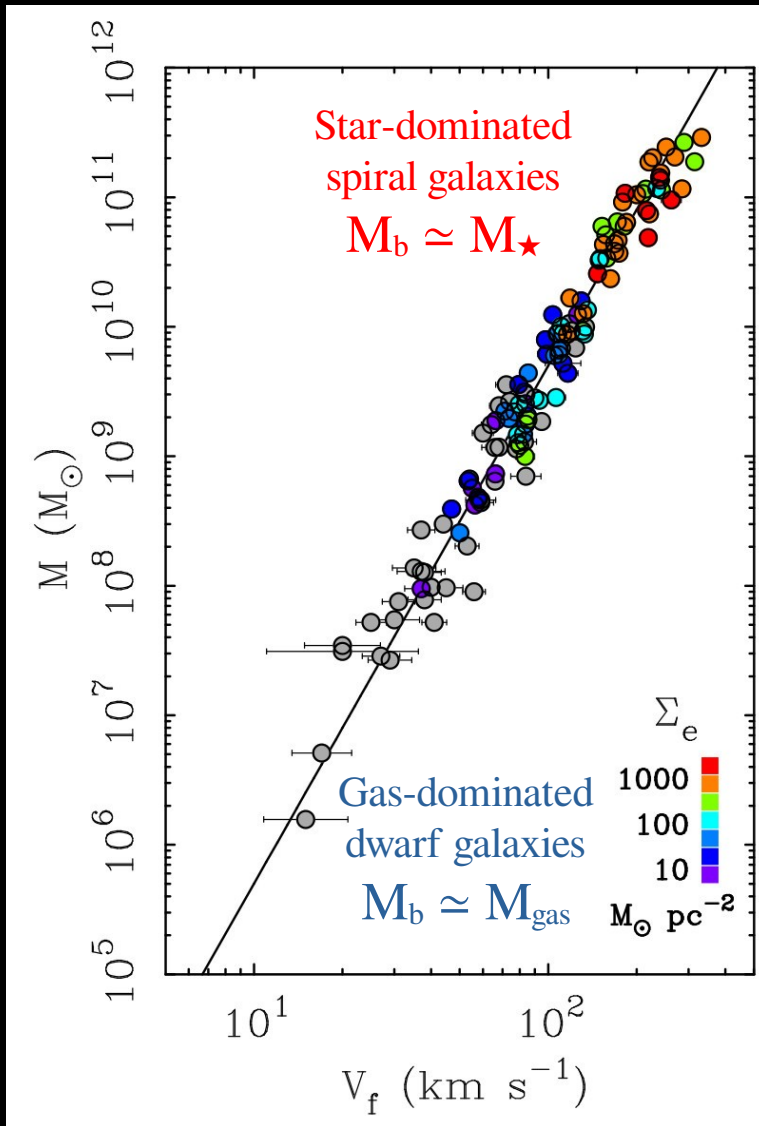
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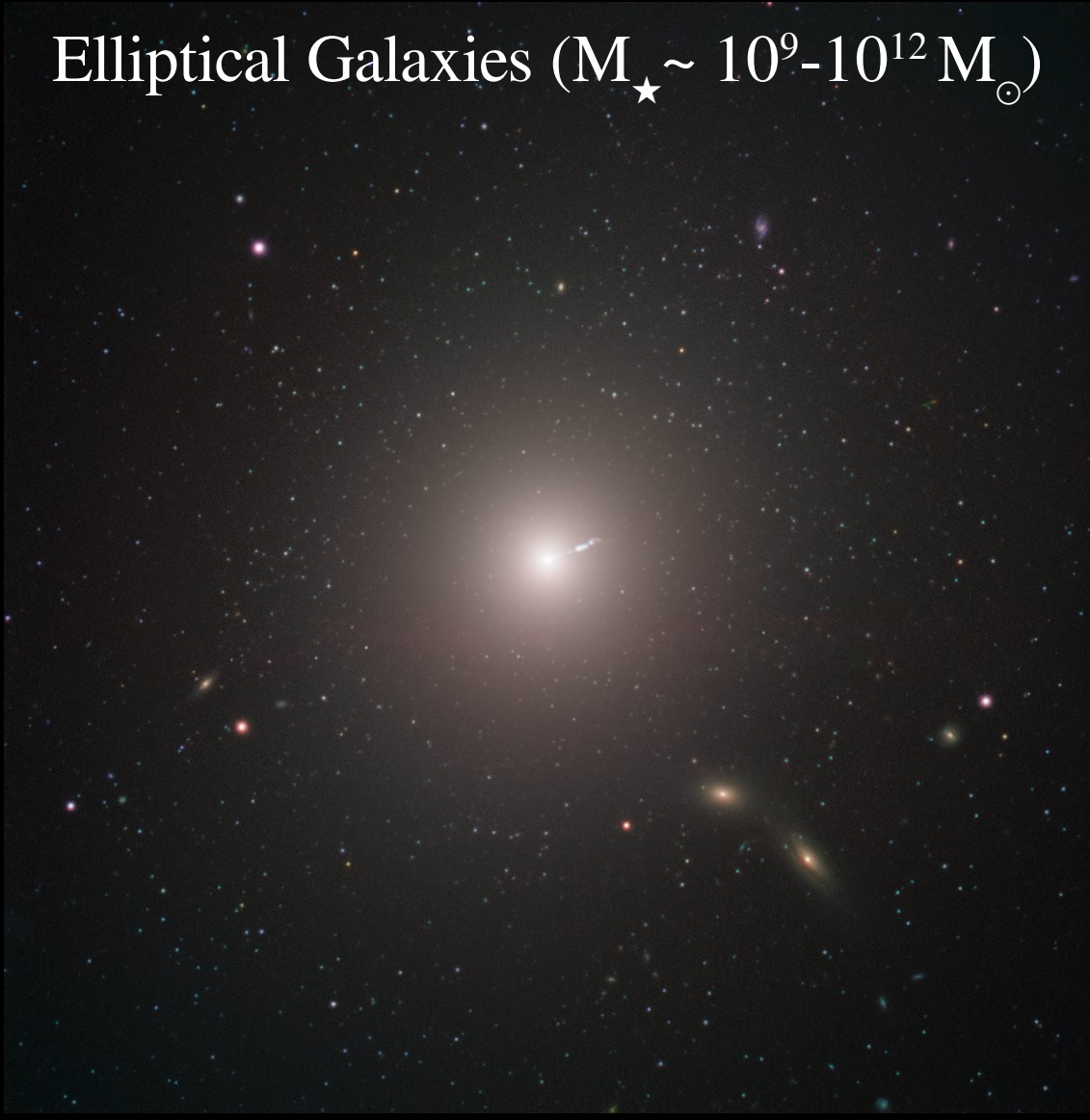
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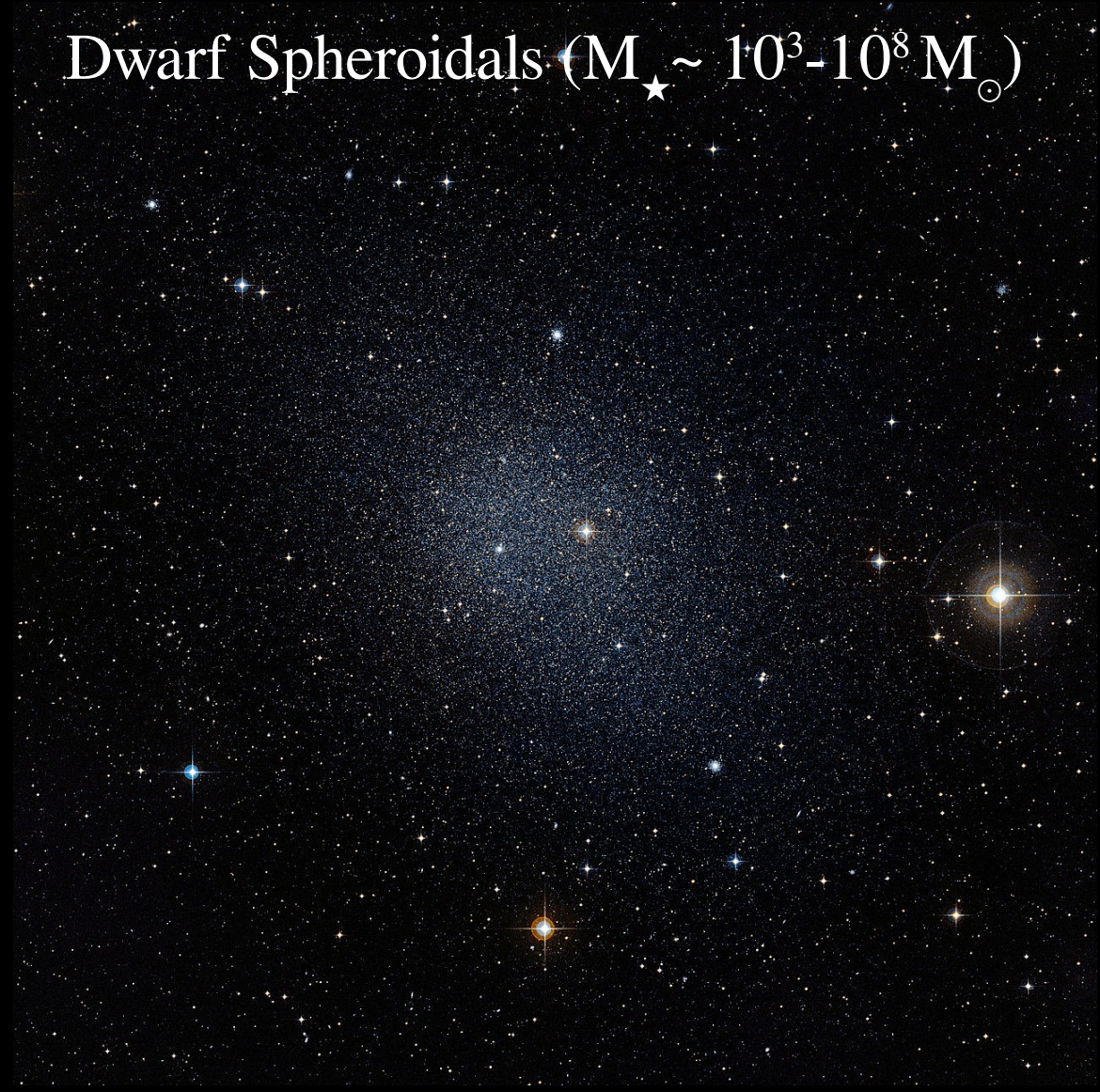
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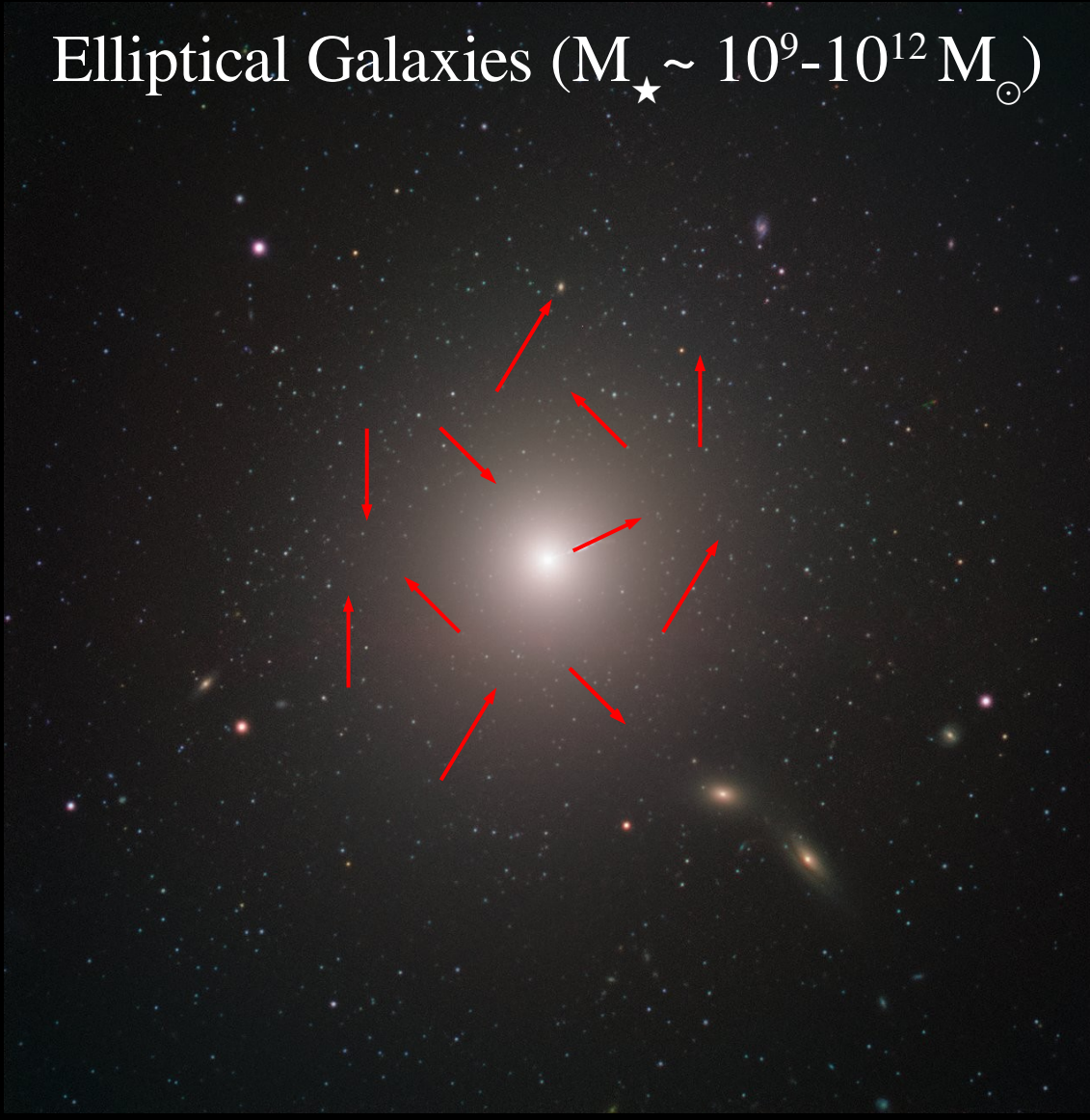


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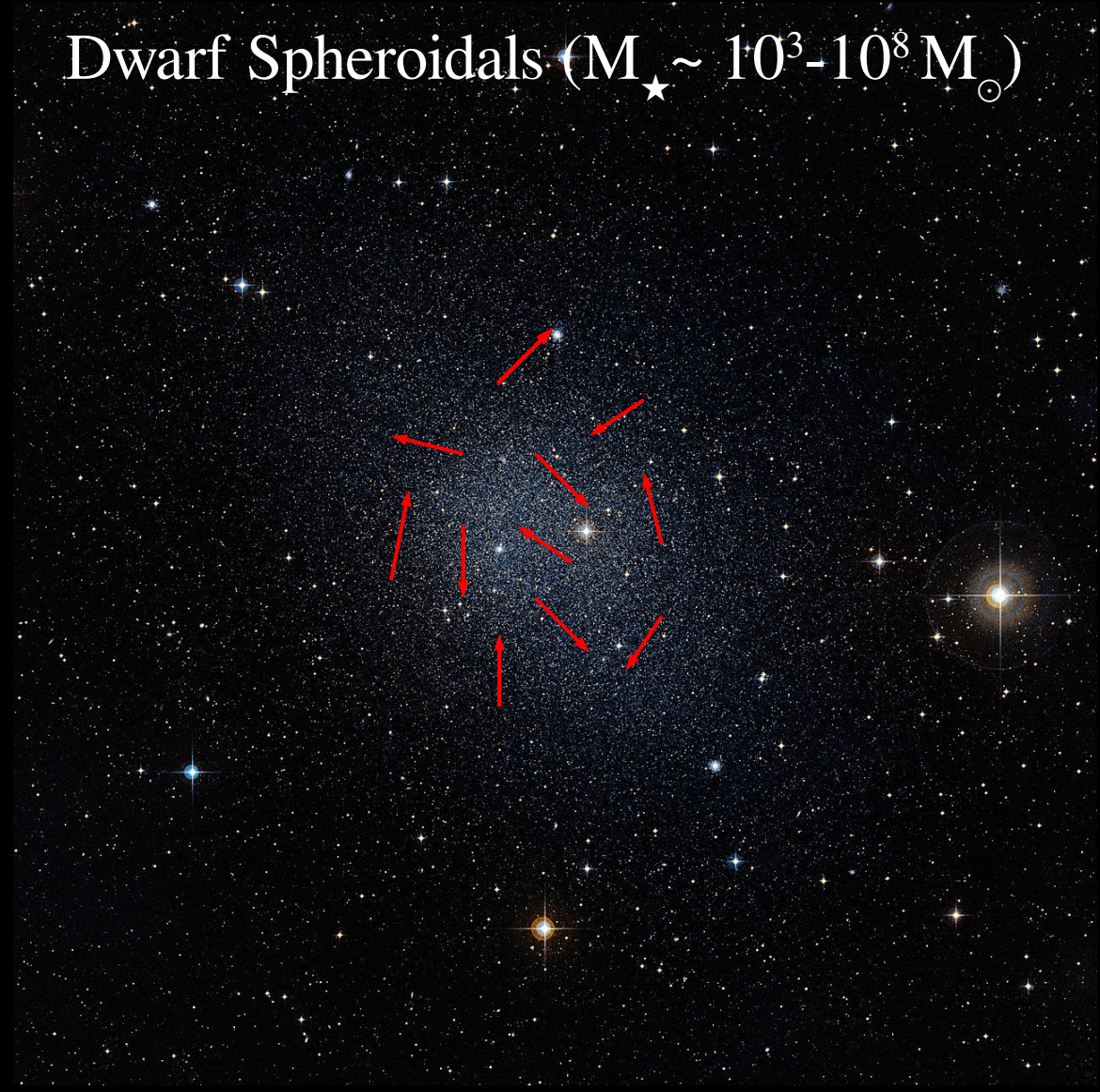


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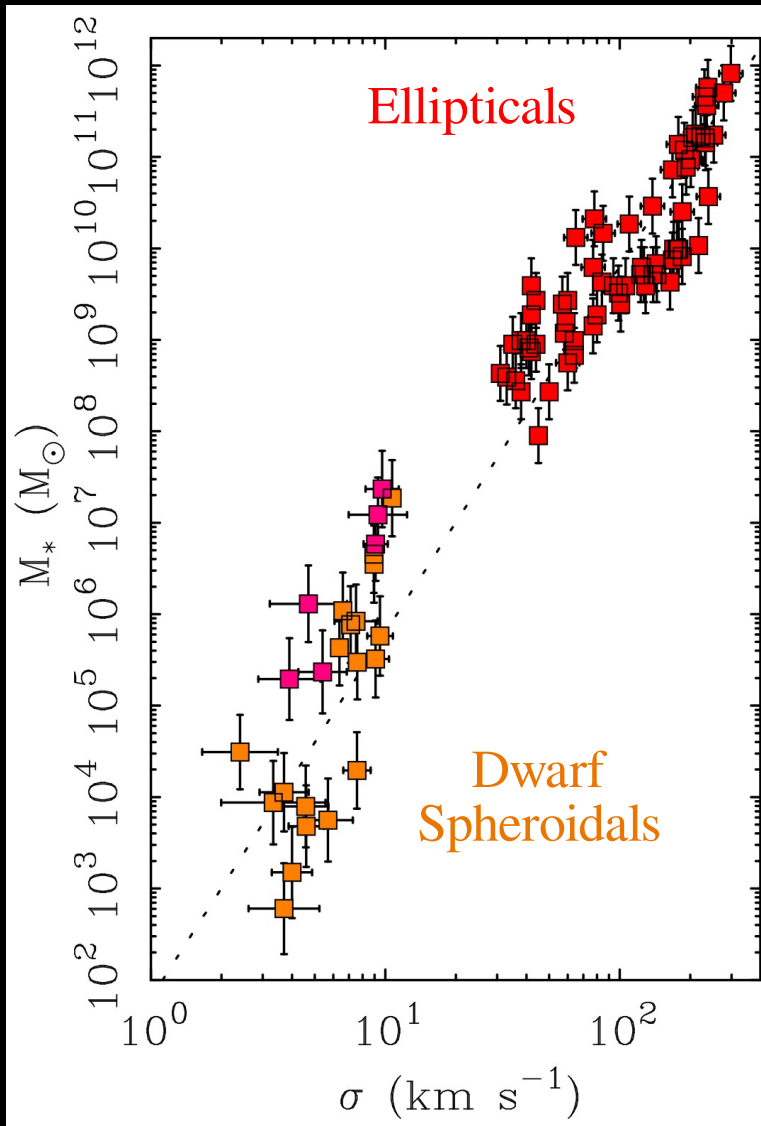


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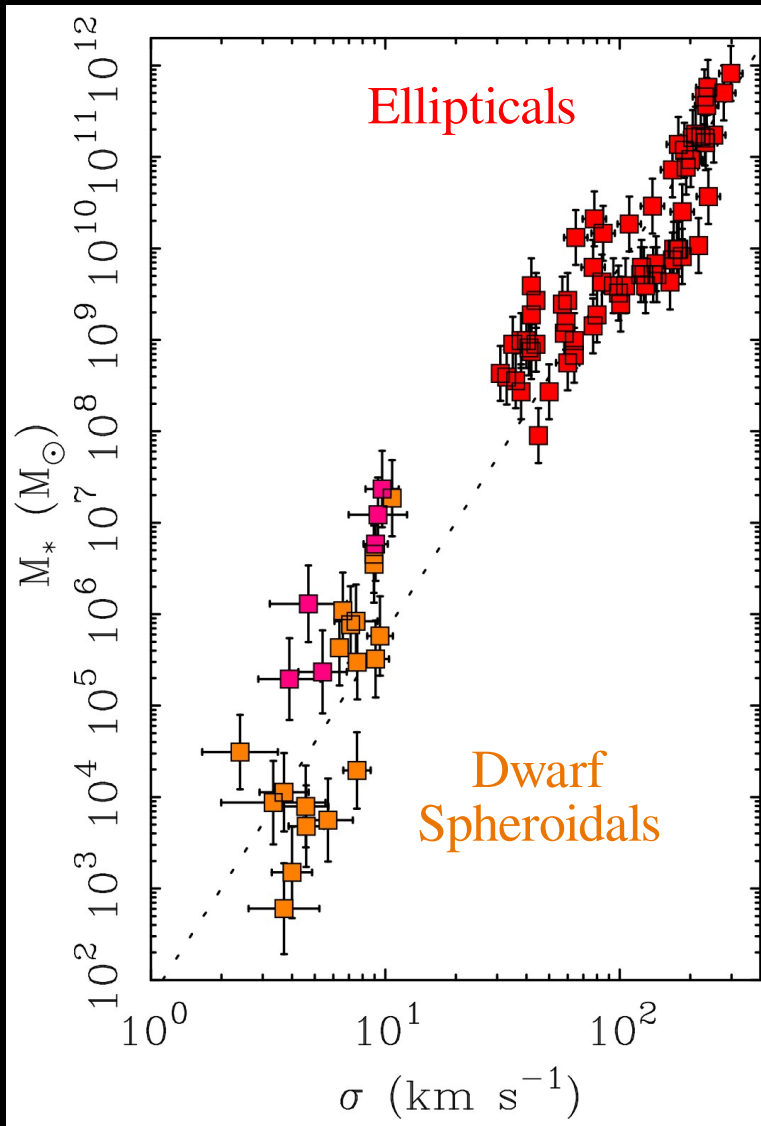
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Faber-Jackson relation (1976, ApJ) for ellipticals

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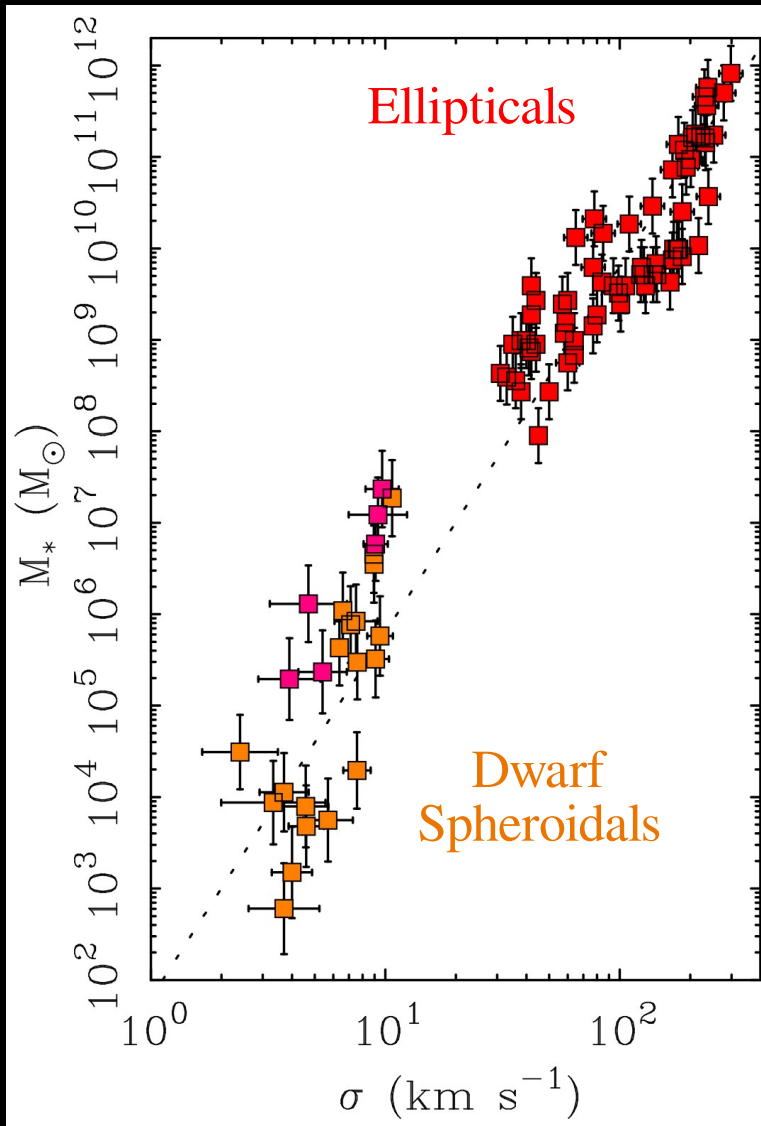


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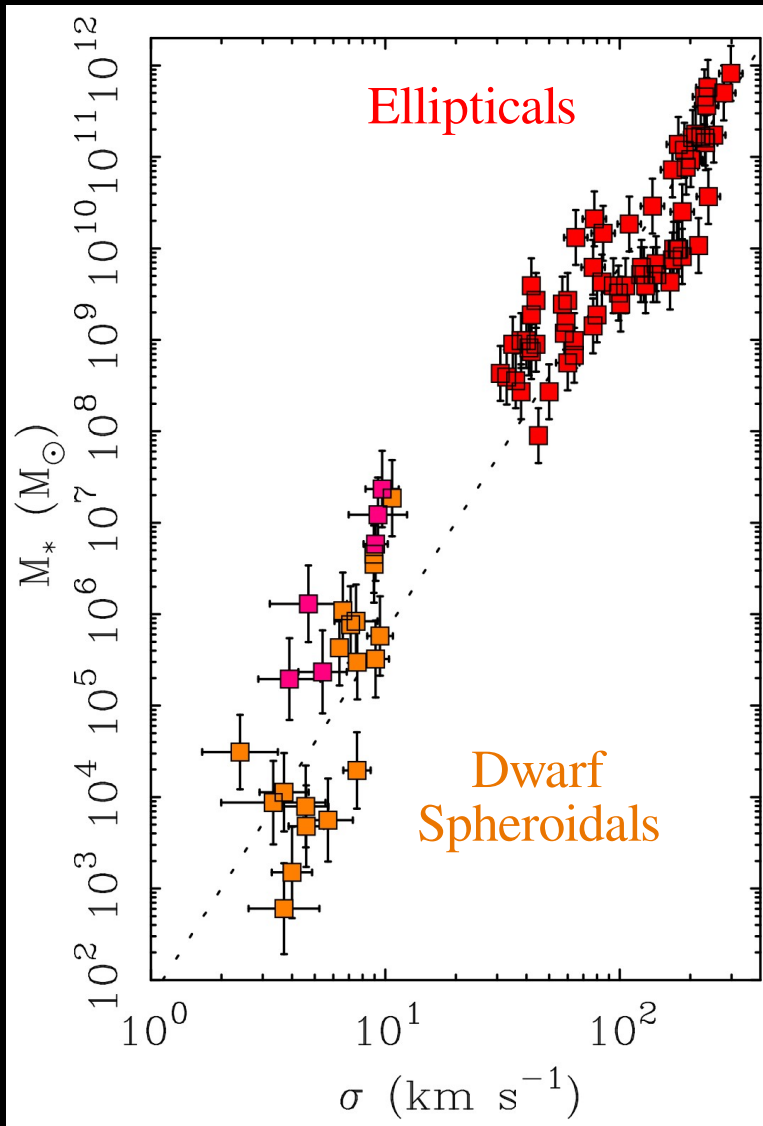
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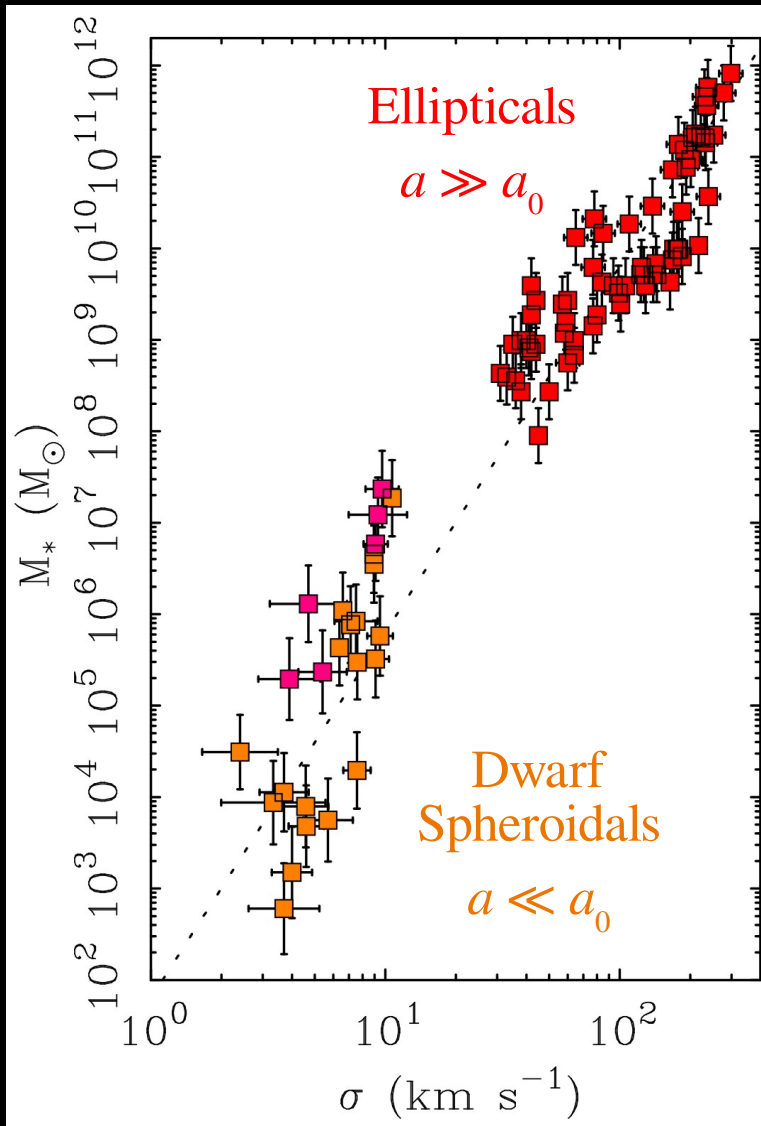
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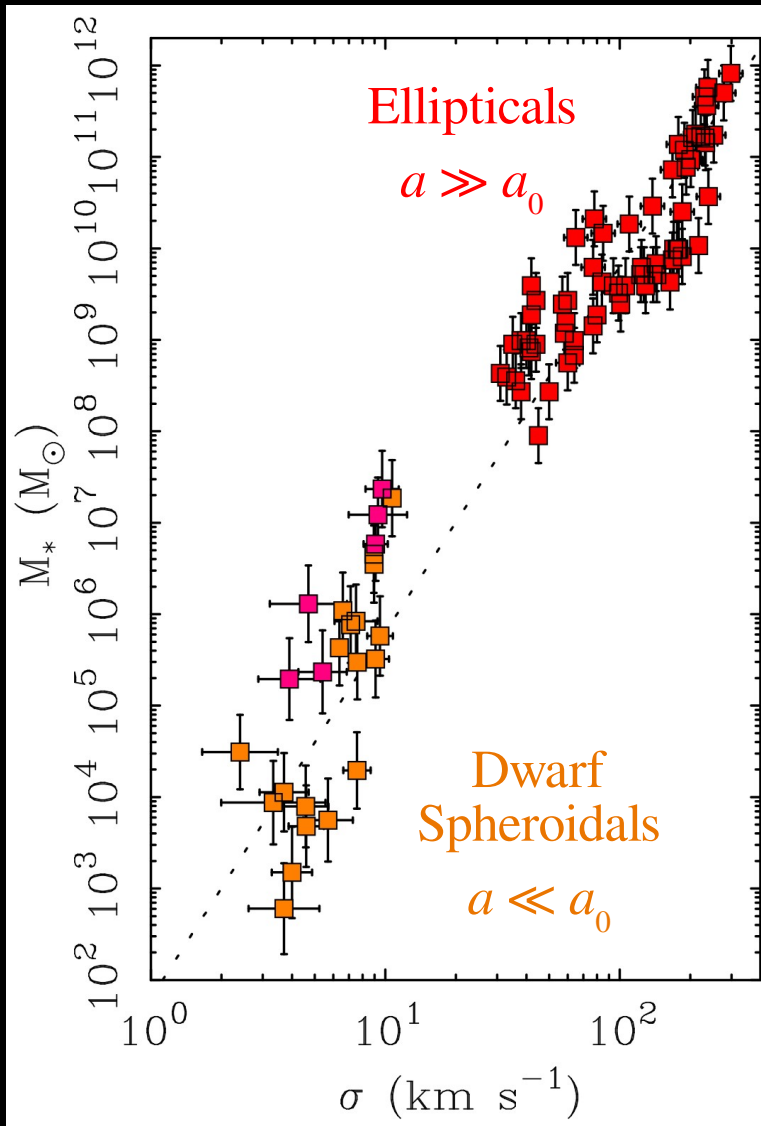
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For giant ellipticals:  $a \gg a_0$  at  $R < R_e \rightarrow$  Newtonian regime

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$$\frac{\sigma_V^2}{R} \simeq \frac{G M}{R^2} \quad \rightarrow \quad M \simeq \sigma_V^2 R_e \quad \text{Fundamental plane of ellipticals}$$

(Djorgovski & Davis 1987; Dressler 1987)

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$$a\mu(x) = g_N \begin{cases} \lim_{x \rightarrow \infty} \mu \rightarrow 1 \quad \Rightarrow \quad a = g_N & \text{Newtonian regime} \\ \lim_{x \rightarrow 0} \mu \rightarrow x \quad \Rightarrow \quad \frac{a^2}{a_0} = g_N \quad \Rightarrow \quad a = \sqrt{a_0 g_N} & \text{MOND regime} \end{cases}$$

(3) **Rotation curves** can be predicted from the baryon distribution

We introduce an **interpolation function**  $\mu(x)$  with  $x = a/a_0$ :

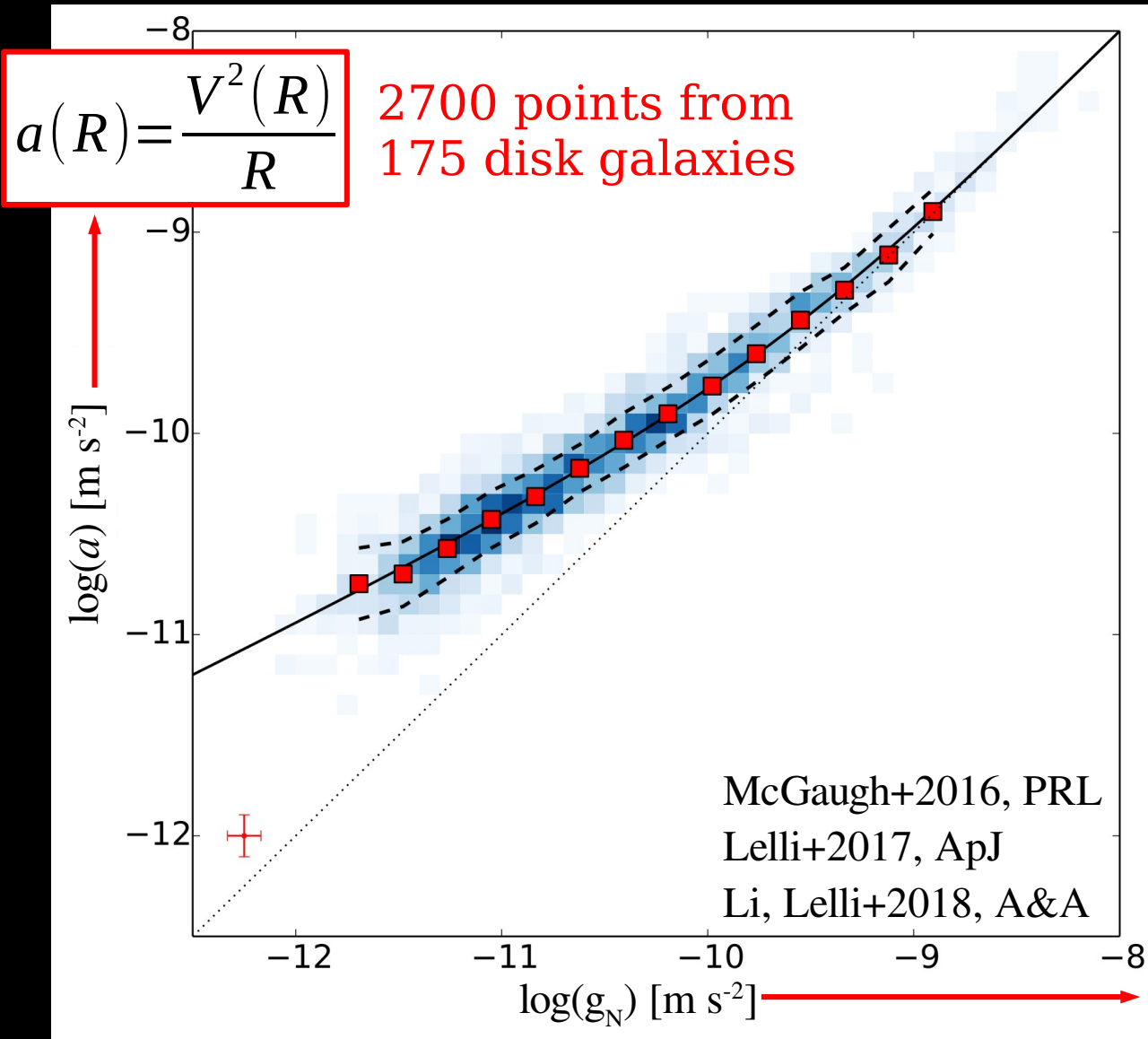
$$a\mu(x) = g_N \left\{ \begin{array}{l} \lim_{x \rightarrow \infty} \mu \rightarrow 1 \quad \Rightarrow \quad a = g_N \quad \text{Newtonian regime} \\ \lim_{x \rightarrow 0} \mu \rightarrow x \quad \Rightarrow \quad \frac{a^2}{a_0} = g_N \quad \Rightarrow \quad a = \sqrt{a_0 g_N} \quad \text{MOND regime} \end{array} \right.$$

Interpolation functions are common in Physics. Examples:

- **Lorentz factor  $\gamma$  (via  $c$ )**: Newton's second law  $\leftrightarrow$  special relativity
- **Planck's law for the blackbody radiation (via  $\hbar$ )**: Rayleigh-Jeans  $\leftrightarrow$  Wein regimes
- **Probability for quantum tunnelling (via  $\hbar$ )**: classical mechanics  $\leftrightarrow$  quantum theory

MOND postulates specify only asymptotic limits of  $\mu$ . Which function to choose?

### (3) Rotation curves can be predicted from the baryon distribution

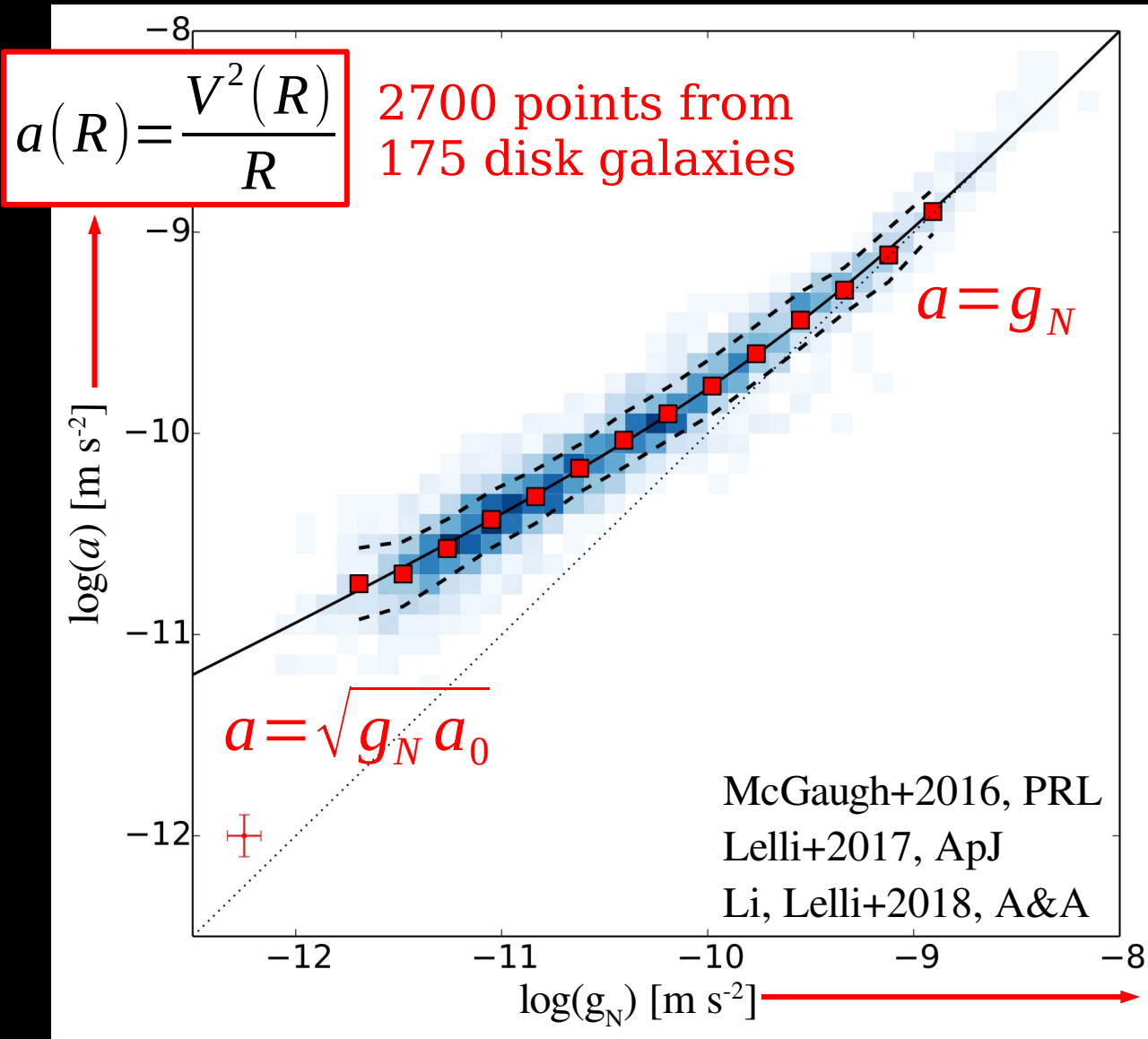


### Radial Acceleration Relation (RAR)

- Fully empirical - independent of MOND

$$\nabla^2 \Phi_N(R, z) = 4 \pi G \rho_b(R, z)$$
$$g_N(R, z=0) = -\nabla \Phi_N(R, z=0)$$

### (3) Rotation curves can be predicted from the baryon distribution



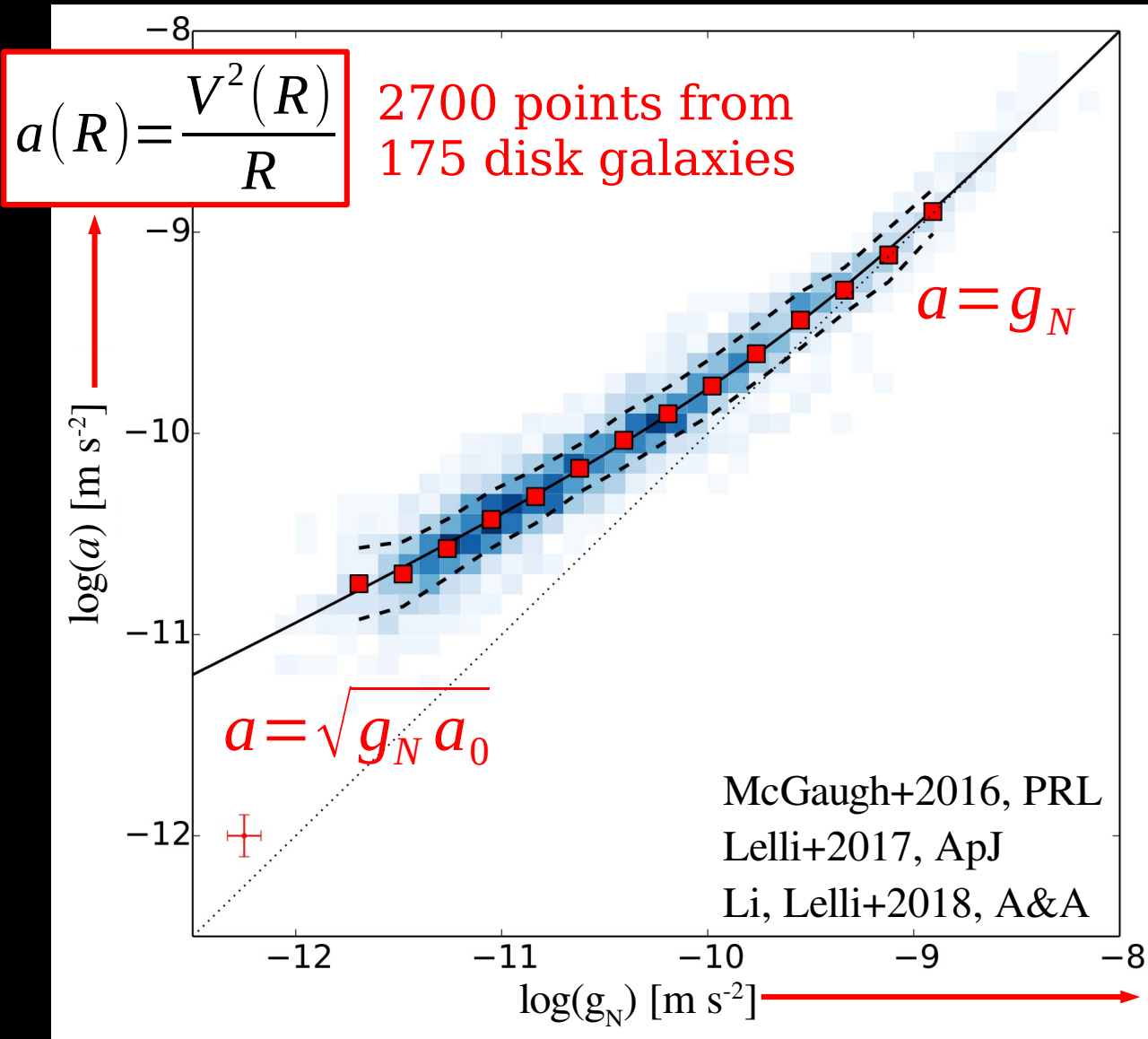
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- RAR shape specifies the form of  $\mu(x)$

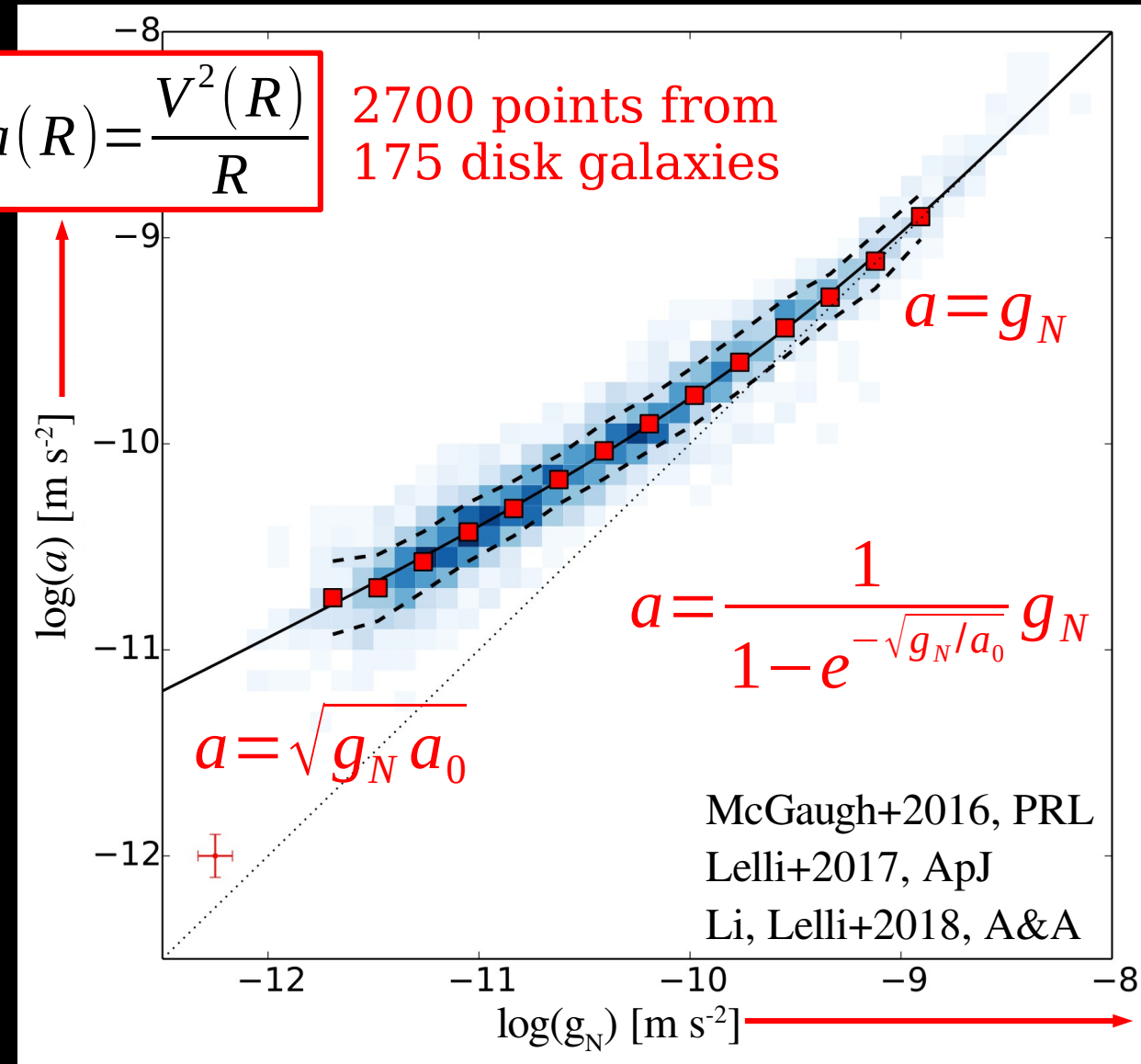
$$a \mu\left(\frac{a}{a_0}\right) = g_N \iff a = \nu\left(\frac{g_N}{a_0}\right) g_N$$

$$\nu = \mu^{-1}$$

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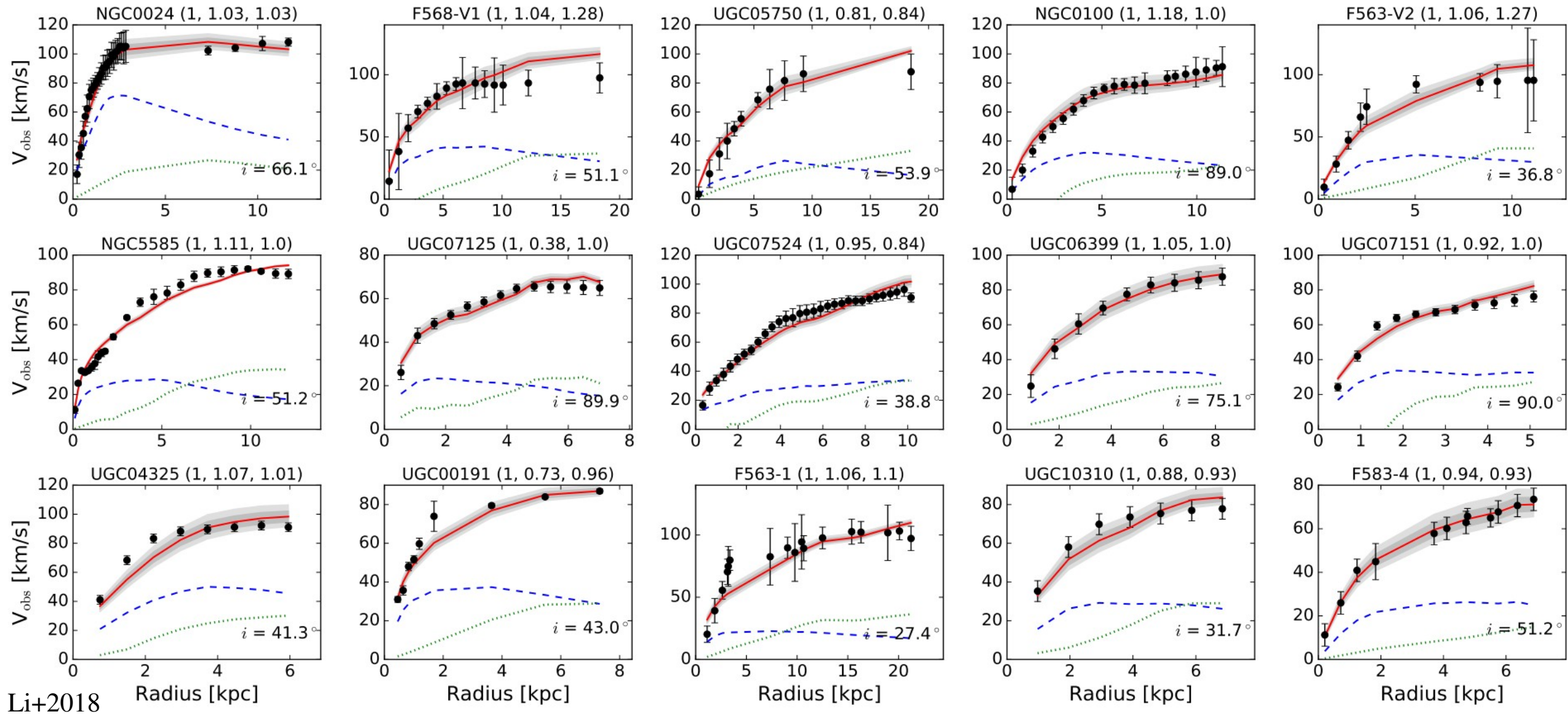
$$\nu = \mu^{-1}$$

We can now assume  $\nu(g_N/a_0)$  and predict rotation curves given  $\rho_b$  (within the errors)

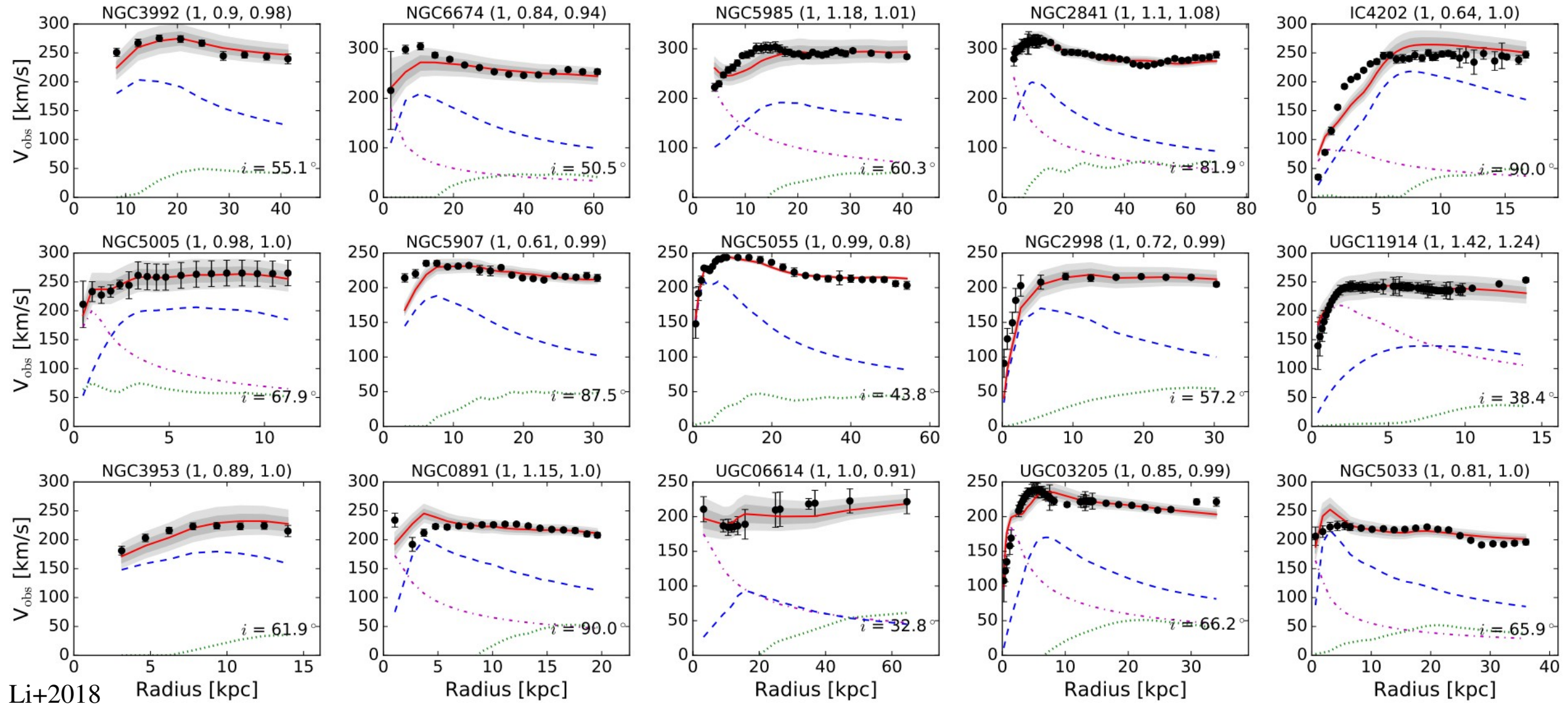
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### (3) Rotation curves can be predicted from the baryon distribution

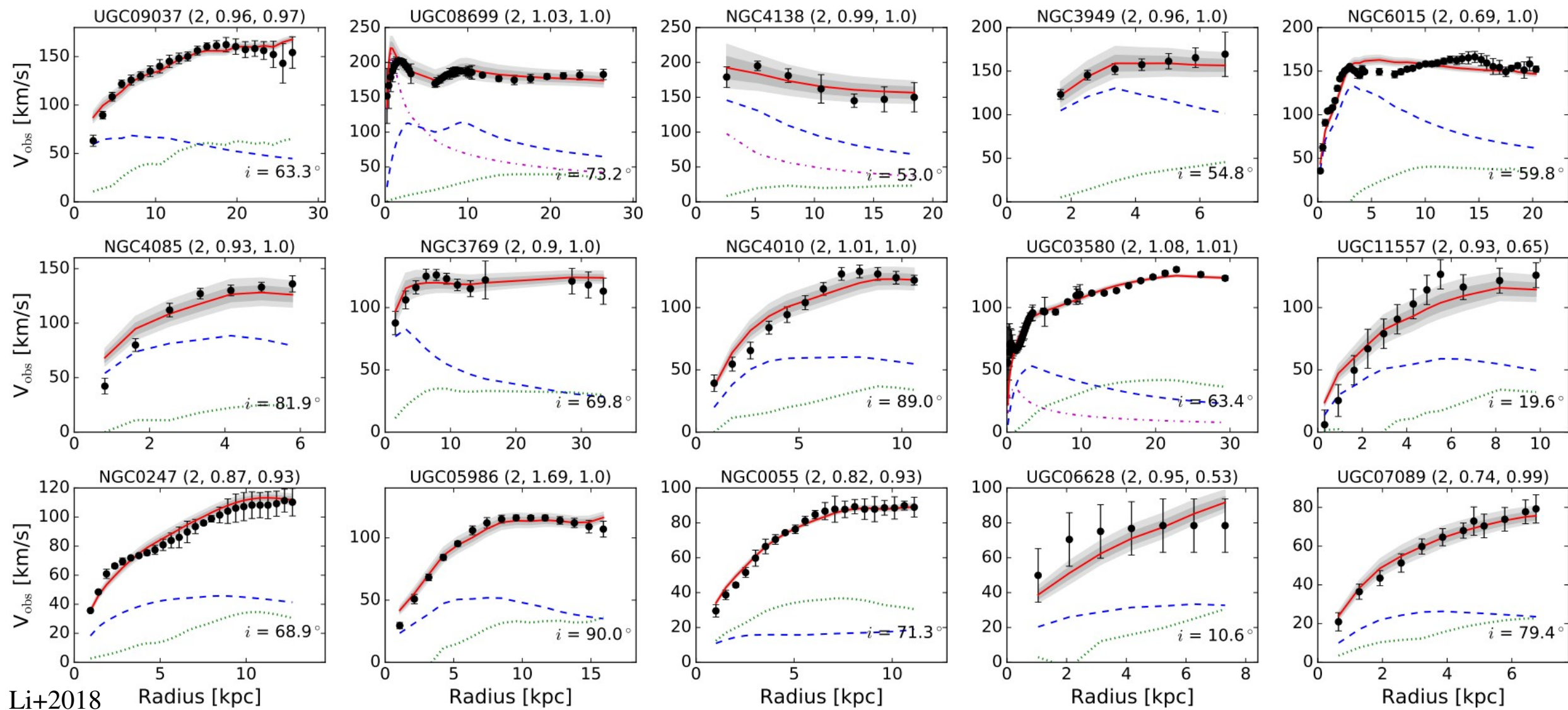


# (3) Rotation curves can be predicted from the baryon distribution





### (3) Rotation curves can be predicted from the baryon distribution

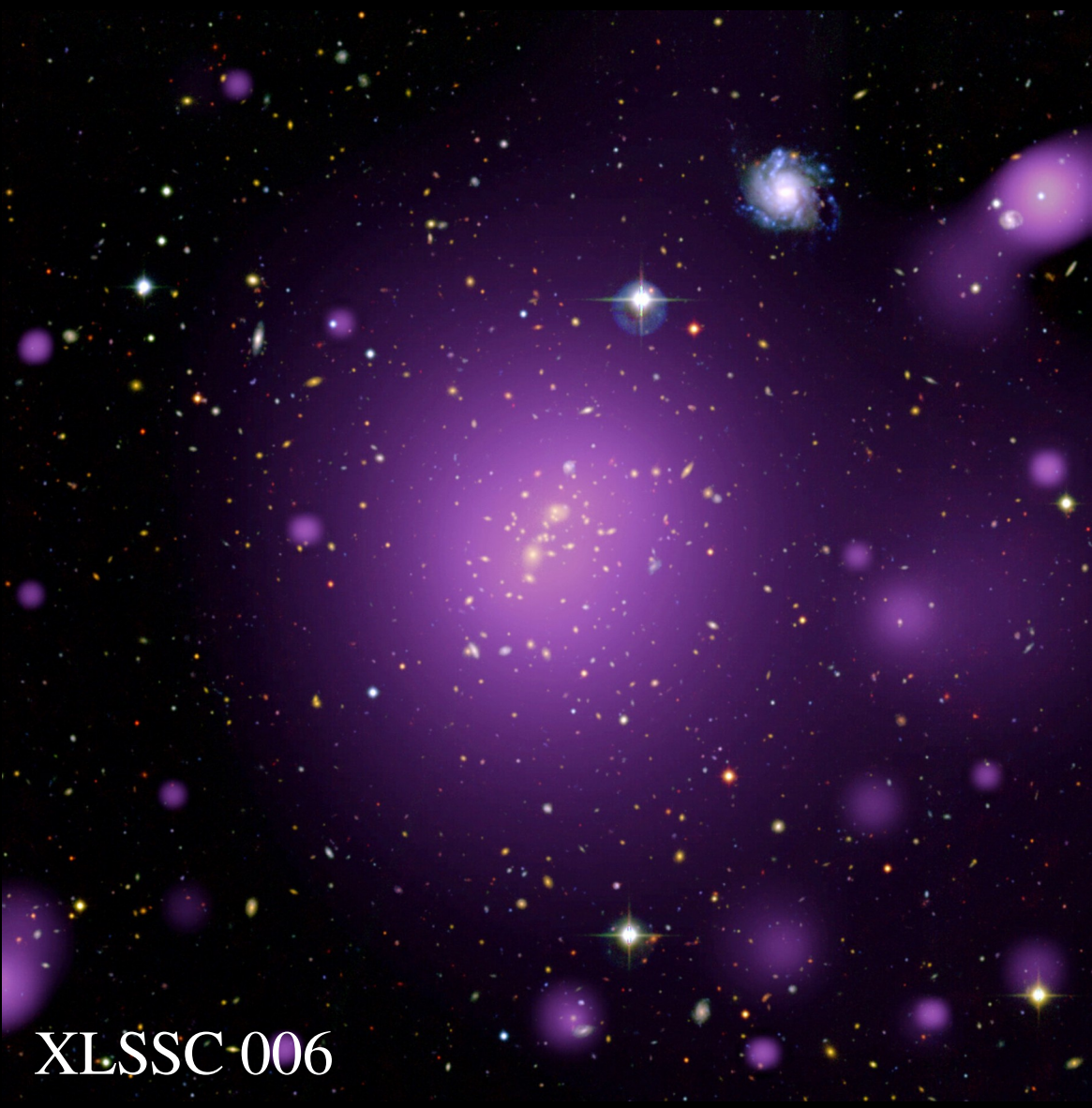


# Galaxy Clusters: bound systems with $\sim 100$ - $1000$ galaxies

Observed baryon budget:

$\sim 10\%$  galaxies (optical & NIR)

$\sim 90\%$  hot ionized gas (X rays)



XLSSC-006

# Galaxy Clusters: bound systems with ~100-1000 galaxies

Observed baryon budget:

~10% galaxies (optical & NIR)

~90% hot ionized gas (X rays)

Sphere in Hydrostatic Equilibrium

$$\frac{\partial P_{gas}}{\partial r} = \rho_{gas} \frac{\partial \Phi}{\partial r} \quad P_{gas} = \frac{k_B T_{gas}}{w m_p}$$

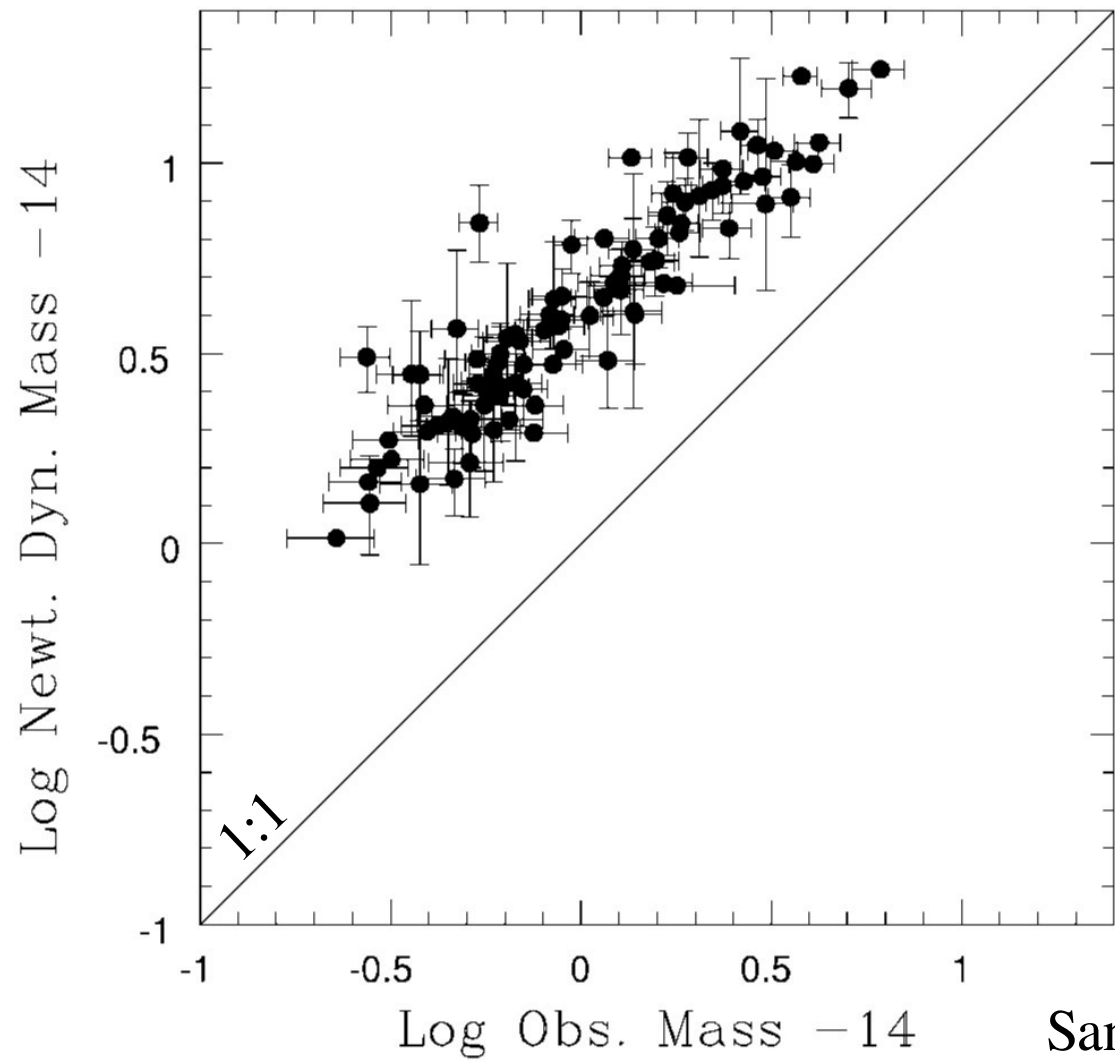
$$\Rightarrow \frac{\partial \Phi}{\partial r} = \frac{k_B}{w m_p} \frac{1}{\rho_{gas}} \frac{\partial}{\partial r} (\rho_{gas} T_{gas})$$

XLSSC-006

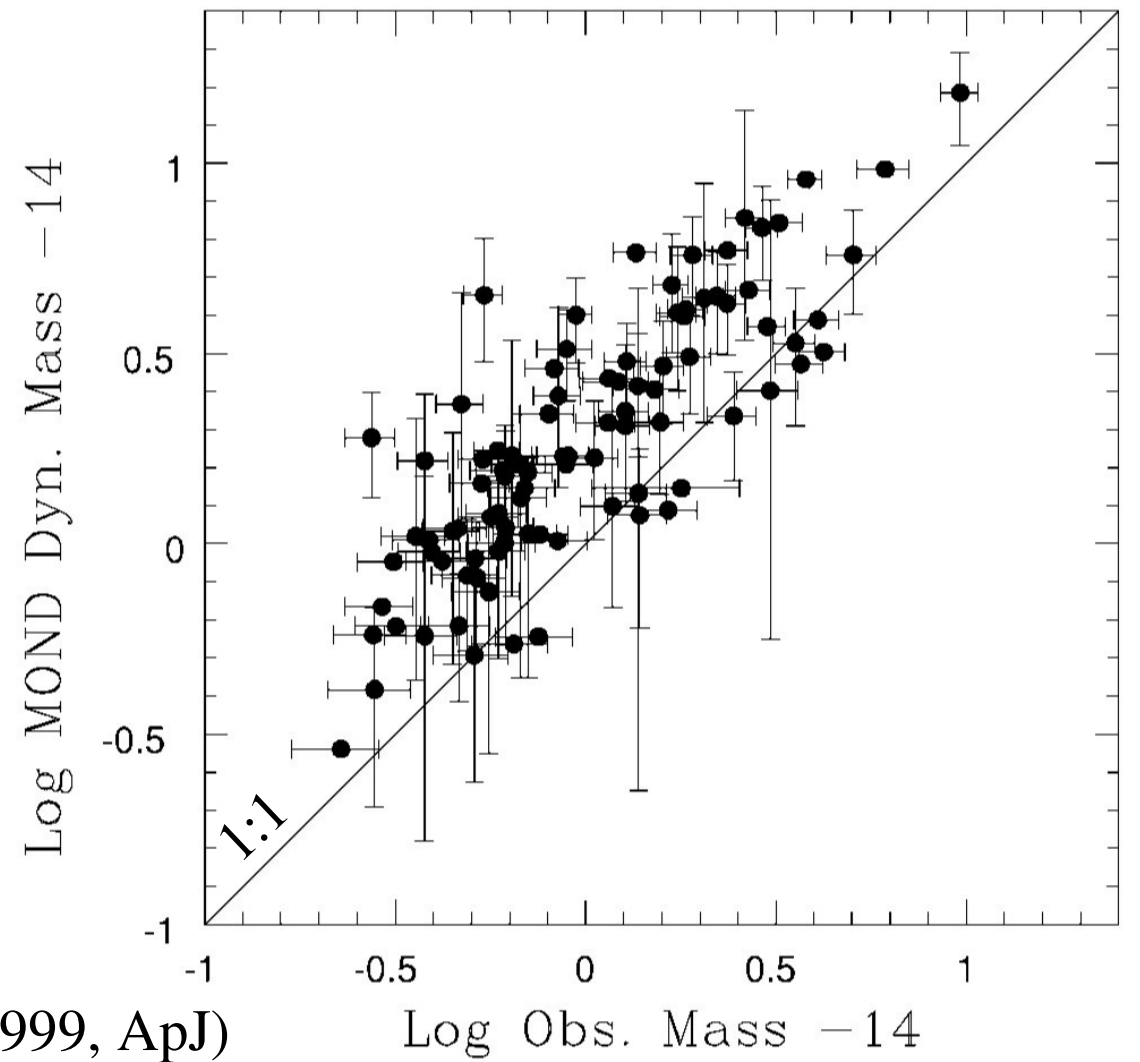
# Galaxy Clusters: Long-standing problem for MOND

Newtonian analysis:  $M_{\text{dyn}}/M_{\text{bar}} \simeq 4-5$

MOND analysis:  $M_{\text{dyn}}/M_{\text{bar}} \simeq 2$



Sanders (1999, ApJ)



Log Obs. Mass -14

# Galaxy Clusters: Long-standing problem for MOND

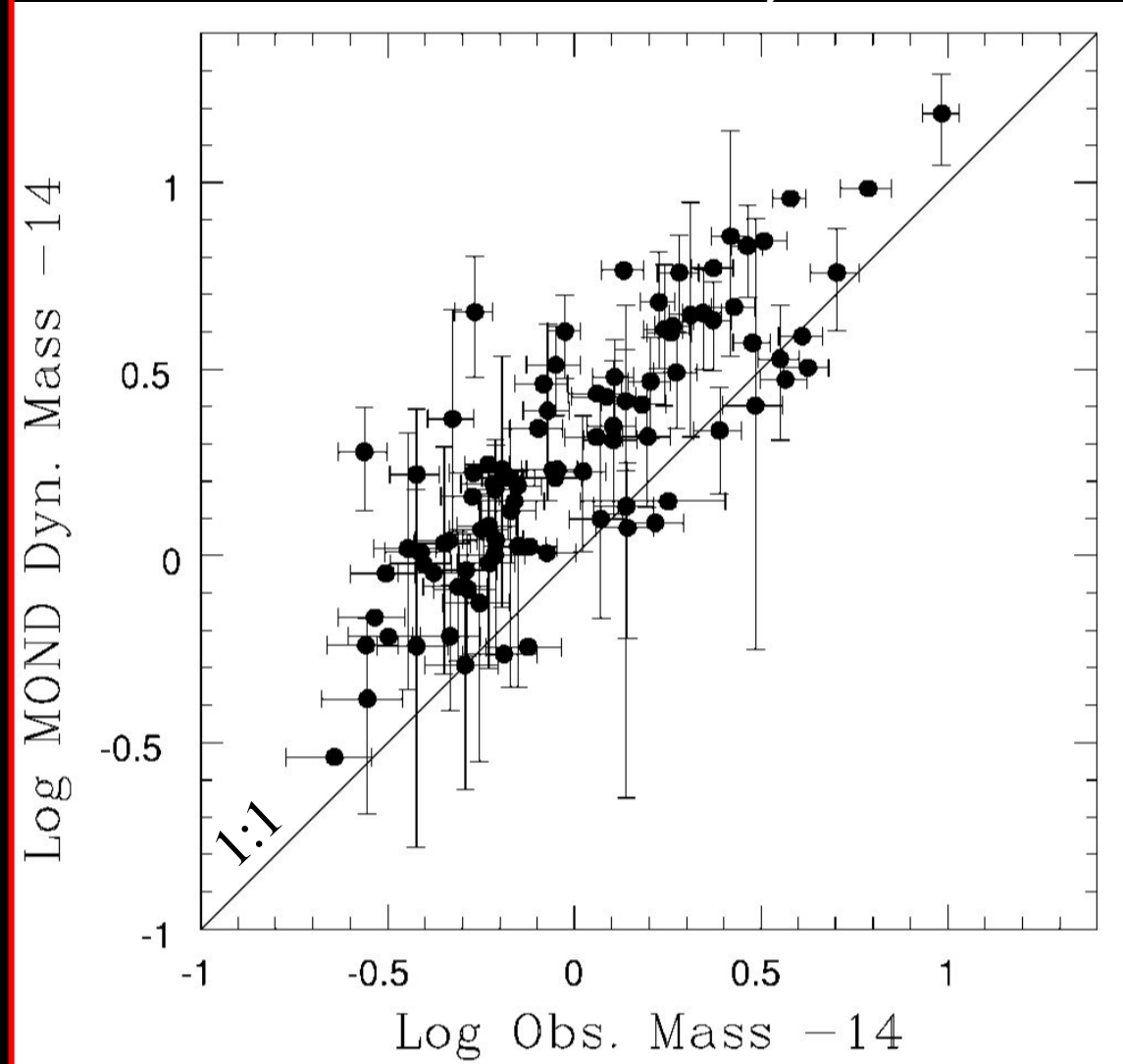
## Proposed Solutions:

- **Undetected, missing baryons ?**

Expected from BBN:  $\Omega_{\text{bar}} > \Omega_{\text{gals}} + \Omega_{\text{clusters}}$

Compact clouds of cold gas? (Milgrom 2008)

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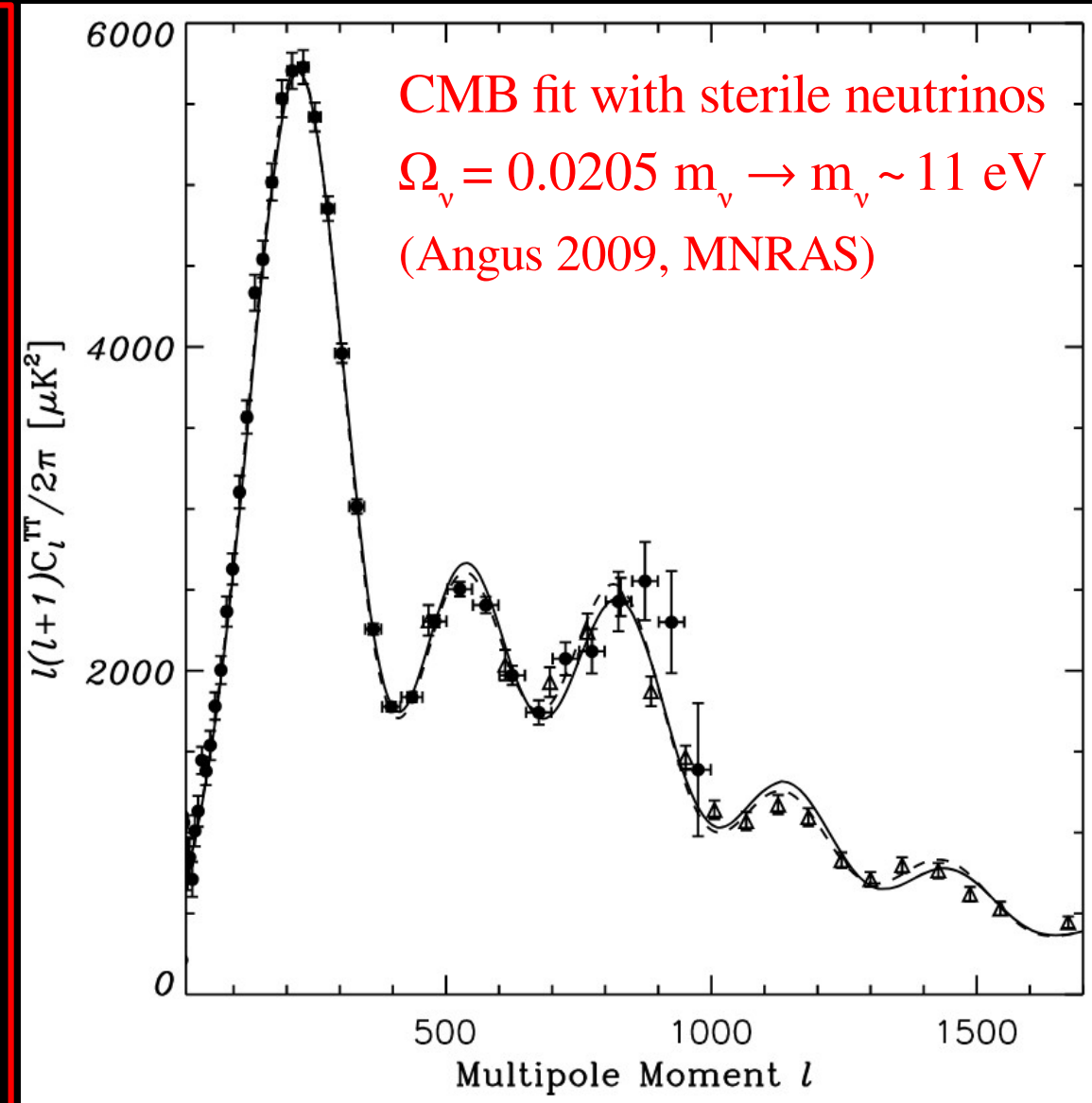
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- **Sterile neutrinos with  $m_{\nu} \approx 11 \text{ eV}$  ?**

Bound in clusters, not in galaxies

HDM fits the CMB as good as CDM

(Sanders 2007, MNRAS; Angus+2010, MNRAS)



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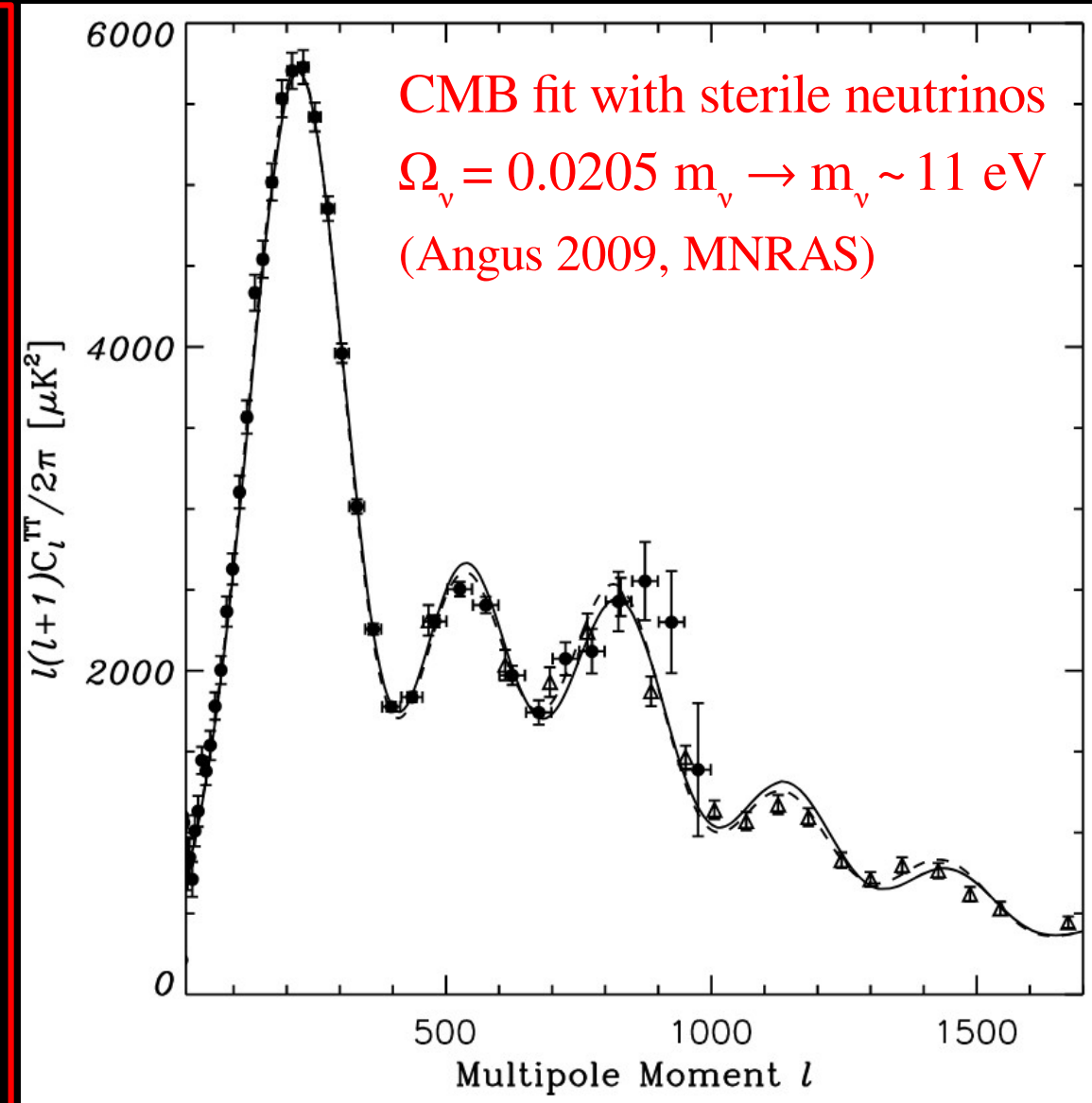
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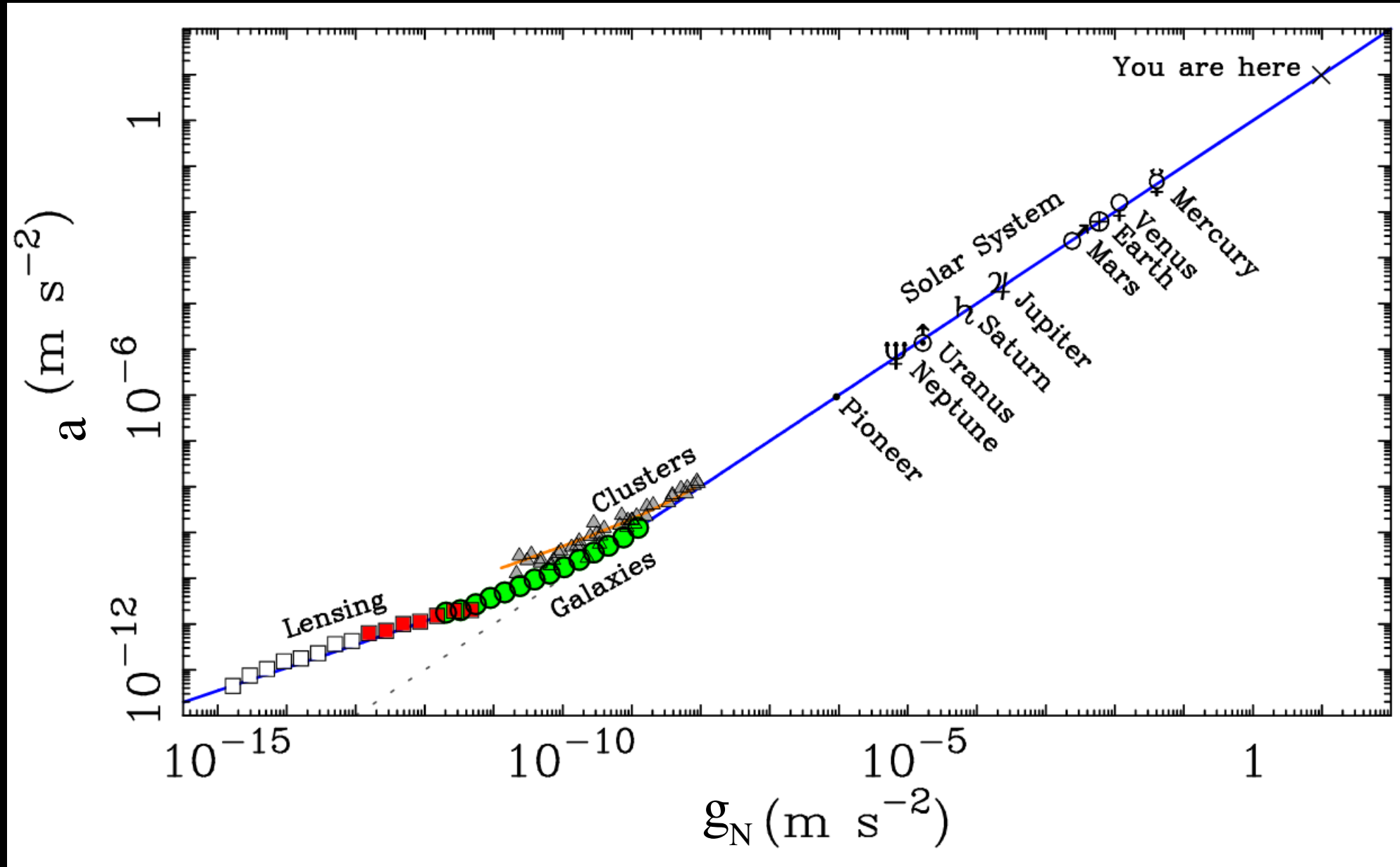
- **Extended MOND:  $a_0 \propto \Phi$  ?**

Deeper theory? But more freedom!

(Zhao & Famaey 2012, PRD)



# Galaxy Clusters on the Radial Acceleration Relation





# II. Specific MOND theories

# MOND paradigm

```
graph TD; A[MOND paradigm] --> B["Modified Gravity (→ ∇²Φ = 4πGρ)"]; A --> C["Modified Inertia (→ F = ma)"]; style A fill:#fff,stroke:#fff,stroke-width:2px; style B fill:#fff,stroke:#fff,stroke-width:2px; style C fill:#fff,stroke:#fff,stroke-width:2px;
```

**Modified Gravity** (→  $\nabla^2\Phi = 4\pi G\rho$ )

**Modified Inertia** (→  $F = ma$ )

# MOND paradigm

**Modified Gravity** ( $\rightarrow \nabla^2\Phi = 4\pi G\rho$ )

**Modified Inertia** ( $\rightarrow F = ma$ )

## Non-relativistic theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010)

## Relativistic theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

TeVes: Tensor-Vector-Scalar (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

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**Modified Inertia** ( $\rightarrow F = ma$ )

**Time non-local theories:**

$a = v(g_N/a_0)g_N$  holds for circular orbits

(Milgrom 1994, 1999, 2022)

**Heuristic ideas:**

Mach's principle for inertia ?

$a_0 \simeq c \Lambda^{1/2} \rightarrow$  quantum vacuum?

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**Hybrid MOND/DM models:**

Bipolar DM (Blanchet+2008, 2009, 2015, 2017)

Superfluid DM (Berezhiani & Khoury 2015)

Baryon-interacting DM (Famaey+2018, 2020)

# AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984, ApJ)

$$S_N = \int dt L_N = \int dt d^3x \left( \rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right)$$
$$- \frac{a_0^2}{8\pi G} F \left( \frac{|\vec{\nabla} \Phi|^2}{a_0^2} \right)$$

Lagrangian is quadratic in  $\nabla\Phi$

$$F(z) \rightarrow z \text{ for } z = |\nabla\Phi|^2/a_0^2 \gg 1$$

$$F(z) \rightarrow z^{3/2} \text{ for } z = |\nabla\Phi|^2/a_0^2 \ll 1$$

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$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla} \Phi|^2}{a_0^2}\right) \quad \begin{array}{l} F(z) \rightarrow z \text{ for } z = |\nabla \Phi|^2/a_0^2 \gg 1 \\ F(z) \rightarrow z^{3/2} \text{ for } z = |\nabla \Phi|^2/a_0^2 \ll 1 \end{array}$$

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla \cdot \left[ \mu \left( \frac{|\vec{\nabla} \Phi|^2}{a_0^2} \right) \vec{\nabla} \Phi \right] = 4\pi G \rho \quad \text{Modified Poisson's Equation}$$

$$\mu(x) = \frac{dF(z)}{dz} \quad z = x^2 \quad F(z) \text{ provides the interpolation function } \mu = v^{-1}$$

# Interacting & Merging Galaxies: The Antennae

## Observations

Blue = Gas

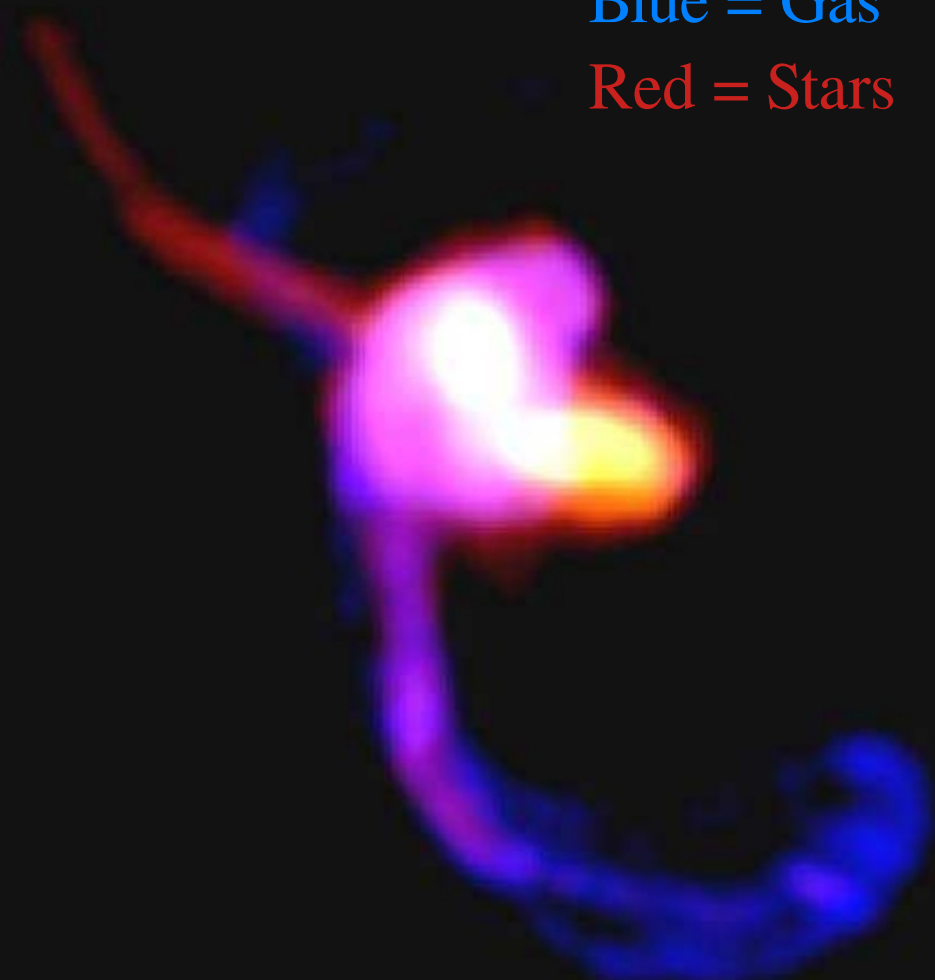
Green = Stars



## Simulations

Blue = Gas

Red = Stars

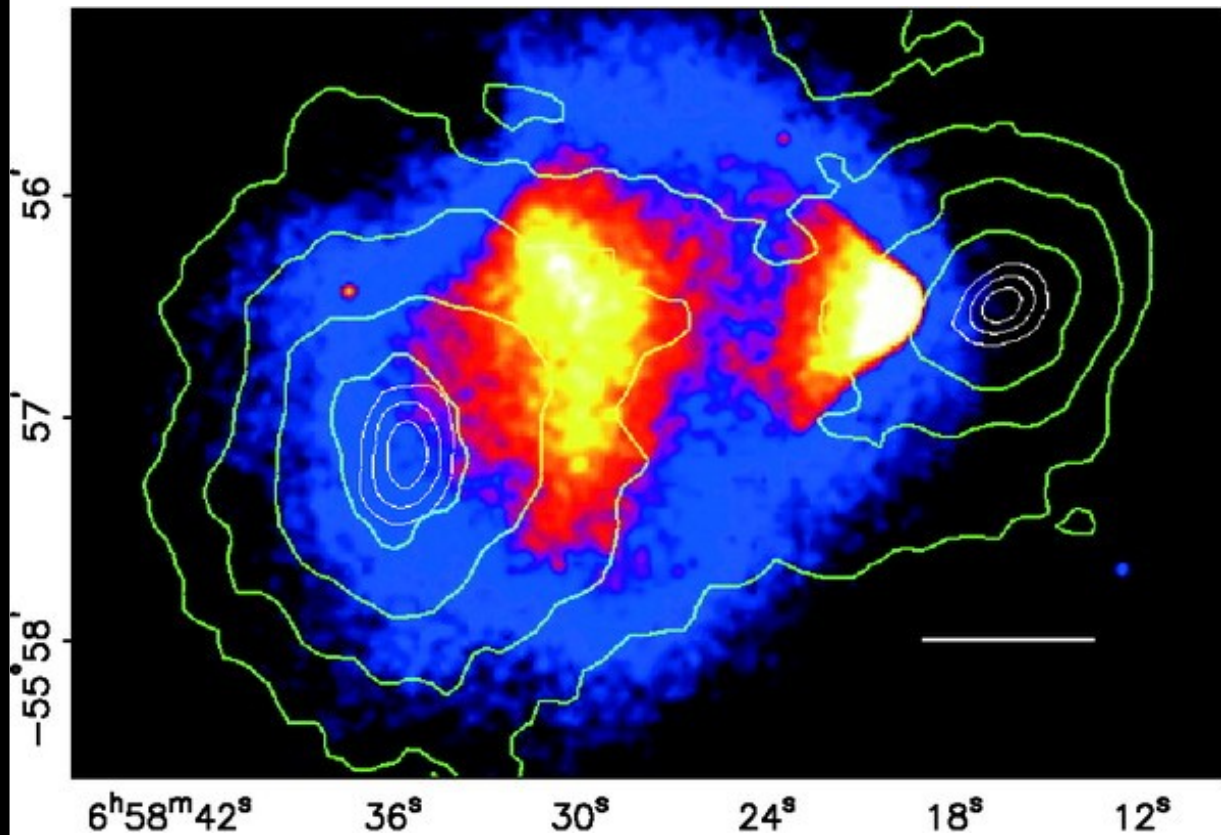


Tiret & Combes (2008, ASPC)



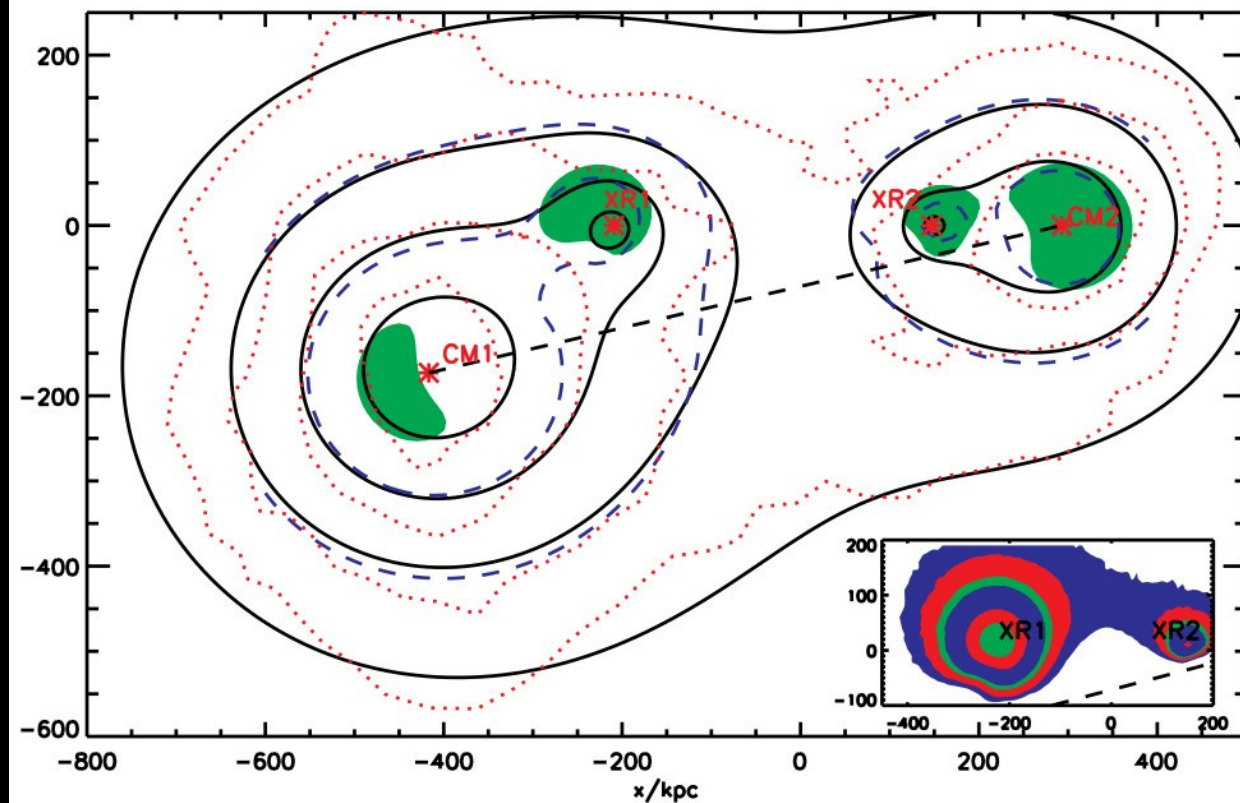
# Interacting & Merging Clusters: The Bullet Cluster

## OBSERVATIONS (Clowe+2004, ApJ)



**Green:** Observed lensing map (total mass)  
**Blue/Red/Yellow:** X-ray emission (hot gas)

## MOND (Angus+2006, MNRAS; Angus+2007, ApJ)



**Red:** Observed lensing convergence map  
**Black:** MOND model with 2eV neutrinos  
**Blue:** total surface densities (baryons +  $\nu$ )

# QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S_N = \int dt L_N = \int dt d^3 x \left( \rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Single gravitational potential } \Phi$$

$$\frac{-1}{8\pi G} \left[ 2 \vec{\nabla} \Phi \cdot \vec{\nabla} \Phi_N - a_0^2 Q \left( \frac{|\vec{\nabla} \Phi_N|^2}{a_0^2} \right) \right] \quad \text{Two potentials: } \Phi \text{ and } \Phi_N!$$

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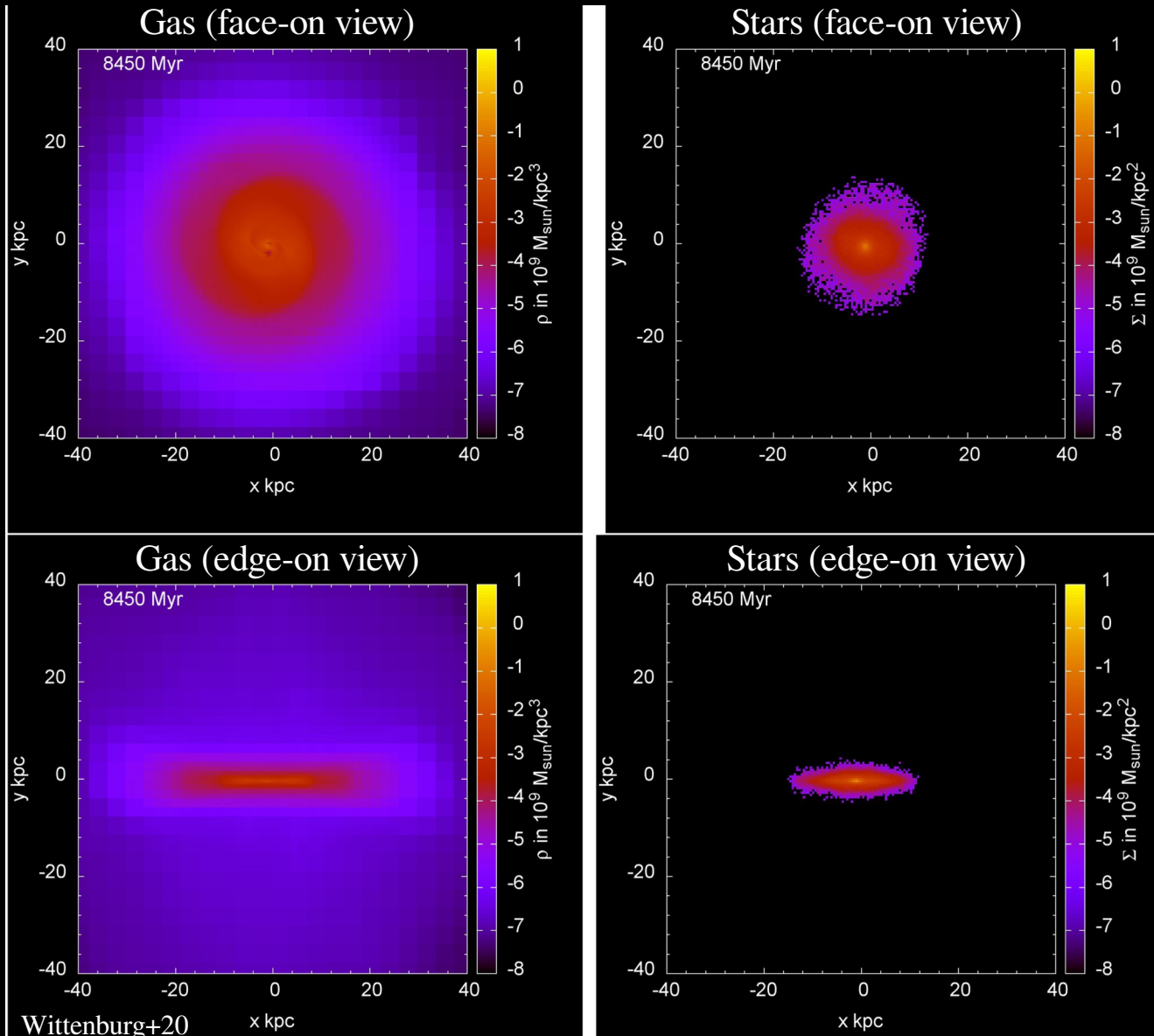
Principle of least Action varying  $\Phi$ ,  $\Phi_N$  and  $\vec{x} \rightarrow$  set of 3 equations

$$\nabla^2 \Phi_N = 4\pi G \rho \longrightarrow \text{Standard, linear Poisson's equation for } \Phi_N$$

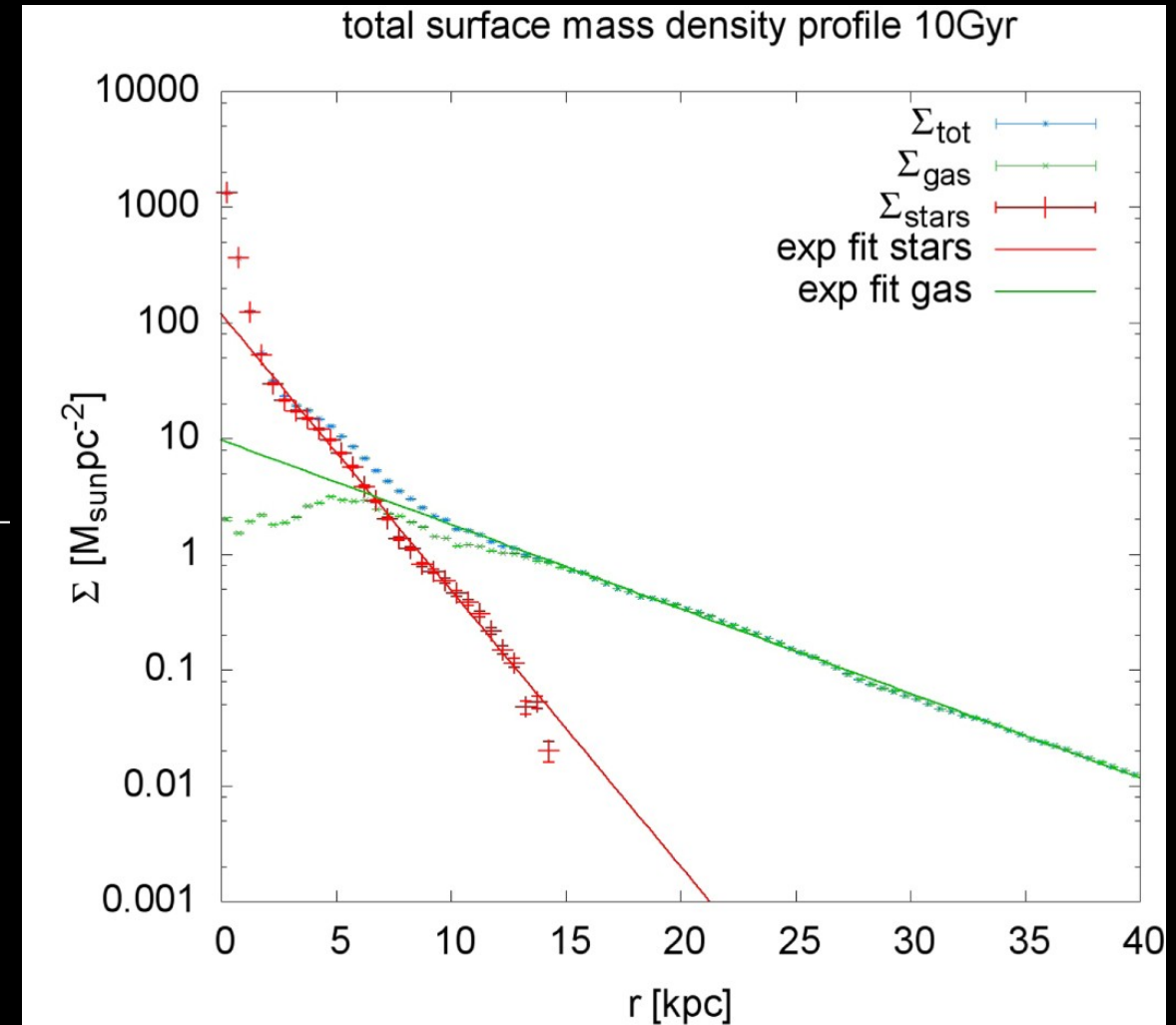
$$\nabla^2 \Phi = \vec{\nabla} \cdot \left[ v \left( \frac{|\vec{\nabla} \Phi_N|}{a_0} \right) \vec{\nabla} \Phi_N \right] \longrightarrow \text{Non-linear step: get } \Phi \text{ from } \Phi_N \quad v(\sqrt{x}) = \frac{dQ(x)}{dx}$$

$$\vec{a} = -\vec{\nabla} \Phi \longrightarrow \text{Acceleration/force set by second potential } \Phi$$

# Hydrodynamic Simulations of Galaxy Formation in QUMOND



Monolithic collapse  $\rightarrow$  realistic galaxy



Wittenburg+20, Nagesh+21, Roshan+21, Eappen+22, Banik+22, Srikanth+23

# Relativistic MOND → add degrees of freedom (new fields) to GR

- Tensor  $g_{\mu\nu}$  → Einstein's metric
- Scalar  $\phi$  → for the DM effect in the NR limit (Bekenstein & Milgrom 1984, ApJ)
- Vector  $A^\mu$  → for gravitational lensing (Sanders 1997, ApJ; Bekenstein 2004, PRD)
- Free function(s) → interpolation function(s) (non-fundamental effective theories?)

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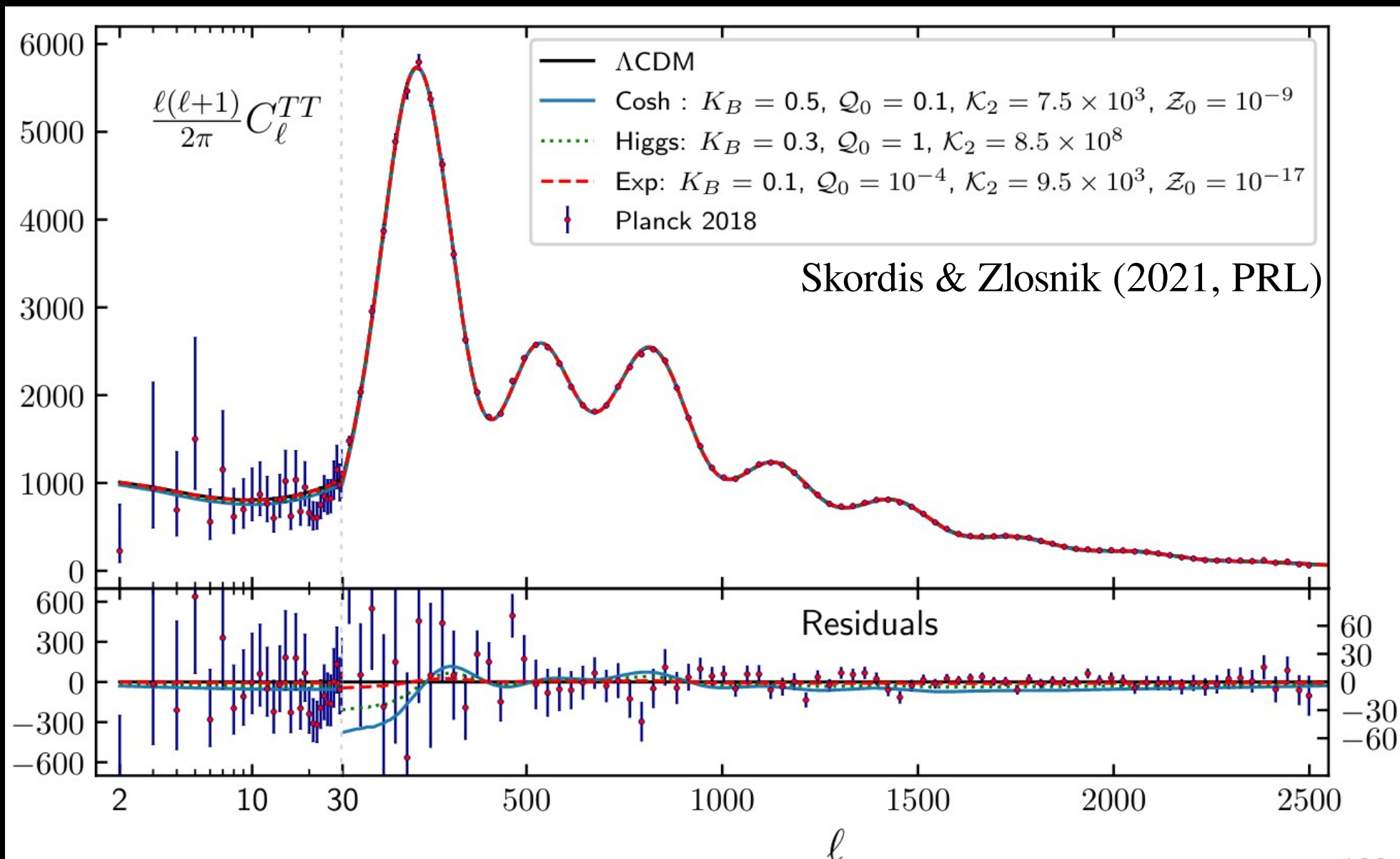
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## AeST: Aether Scalar-Tensor theory (Skordis & Zlosnik 2019, PRD; 2021, PRL; 2022, PRD)

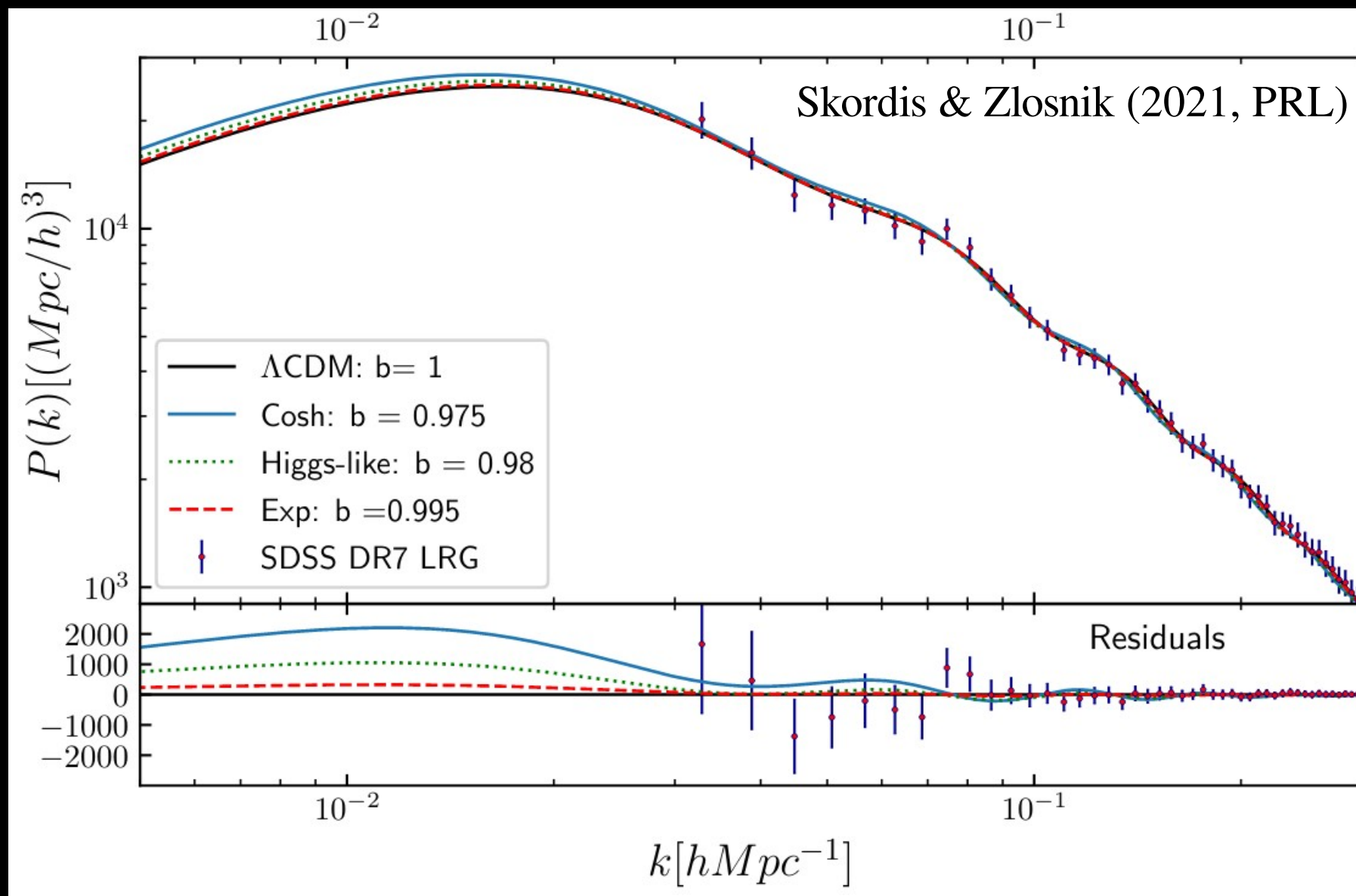
(1) Grav. Waves:  $B^\mu = e^{-2\phi} A^\mu$  (timelike)  $\rightarrow$  theories  $\{B_\mu, g_{\mu\nu}\} \rightarrow c_{\text{GW}} = c_{\text{EM}}$

(2) Cosmology:  $k$ -essence term for  $\phi \rightarrow \rho_\phi \propto a(t)^{-3}$  (like dust or CDM)

# AeST theory: Fit to the CMB Power Spectrum



# AeST theory: Fit to the Matter Power Spectrum at $z=0$

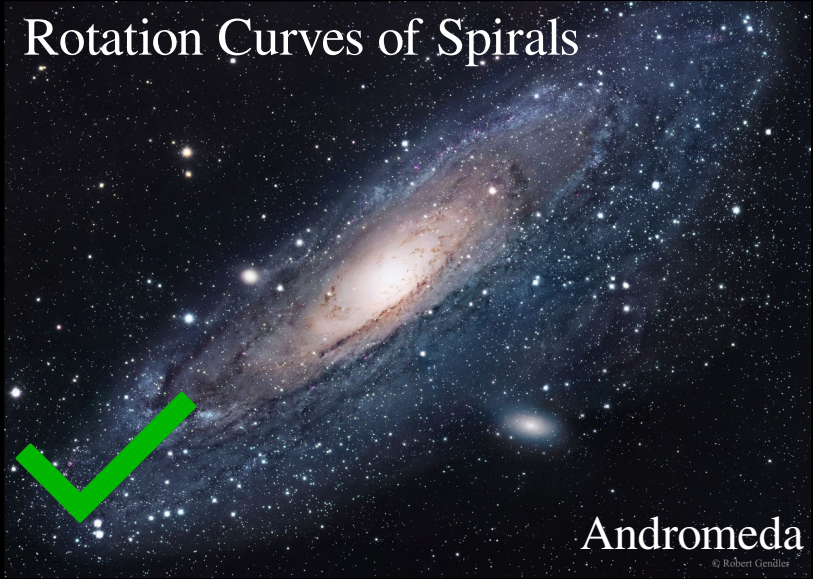




# Conclusions: Status of MOND at Various Scales

## Galaxy Scales (~1-100 kpc)

Rotation Curves of Spirals



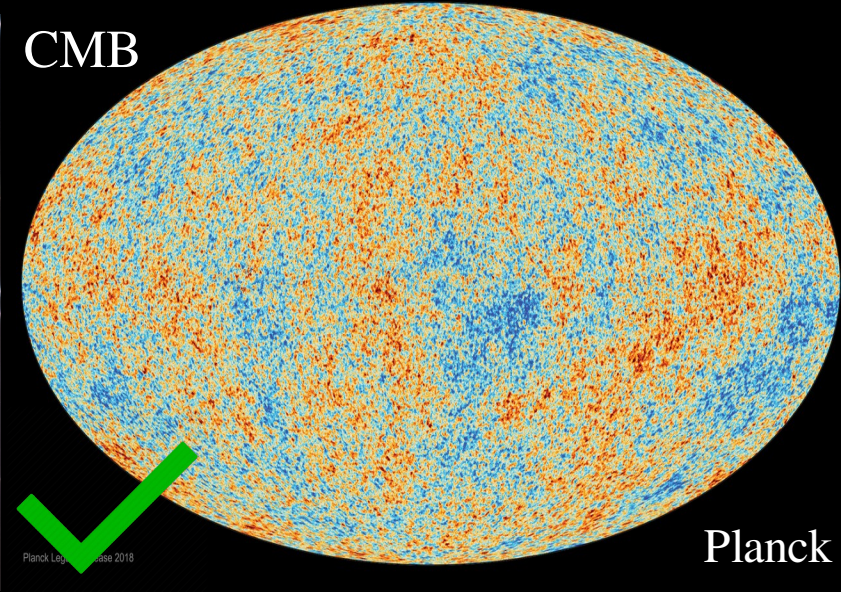
## Groups/Clusters Scales (~1-5 Mpc)

Interactions & Mergers in Groups

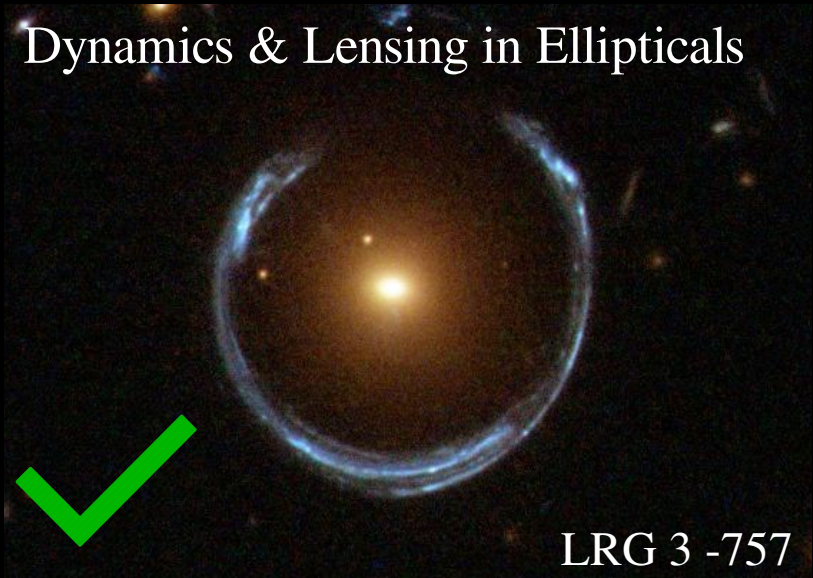


## Cosmological Scales (>100 Mpc)

CMB



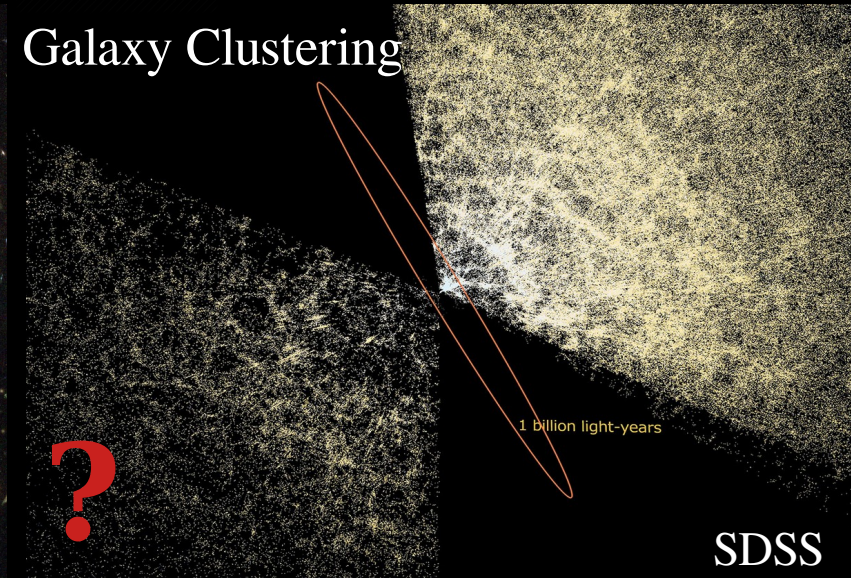
Dynamics & Lensing in Ellipticals



Dynamics & Lensing in Clusters



Galaxy Clustering



# More Slides

# MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \simeq \frac{H_0 \cdot c}{2\pi} \quad H_0 = \text{Hubble constant} \rightarrow \text{maybe } a_0(t) \sim H(t) ?$$

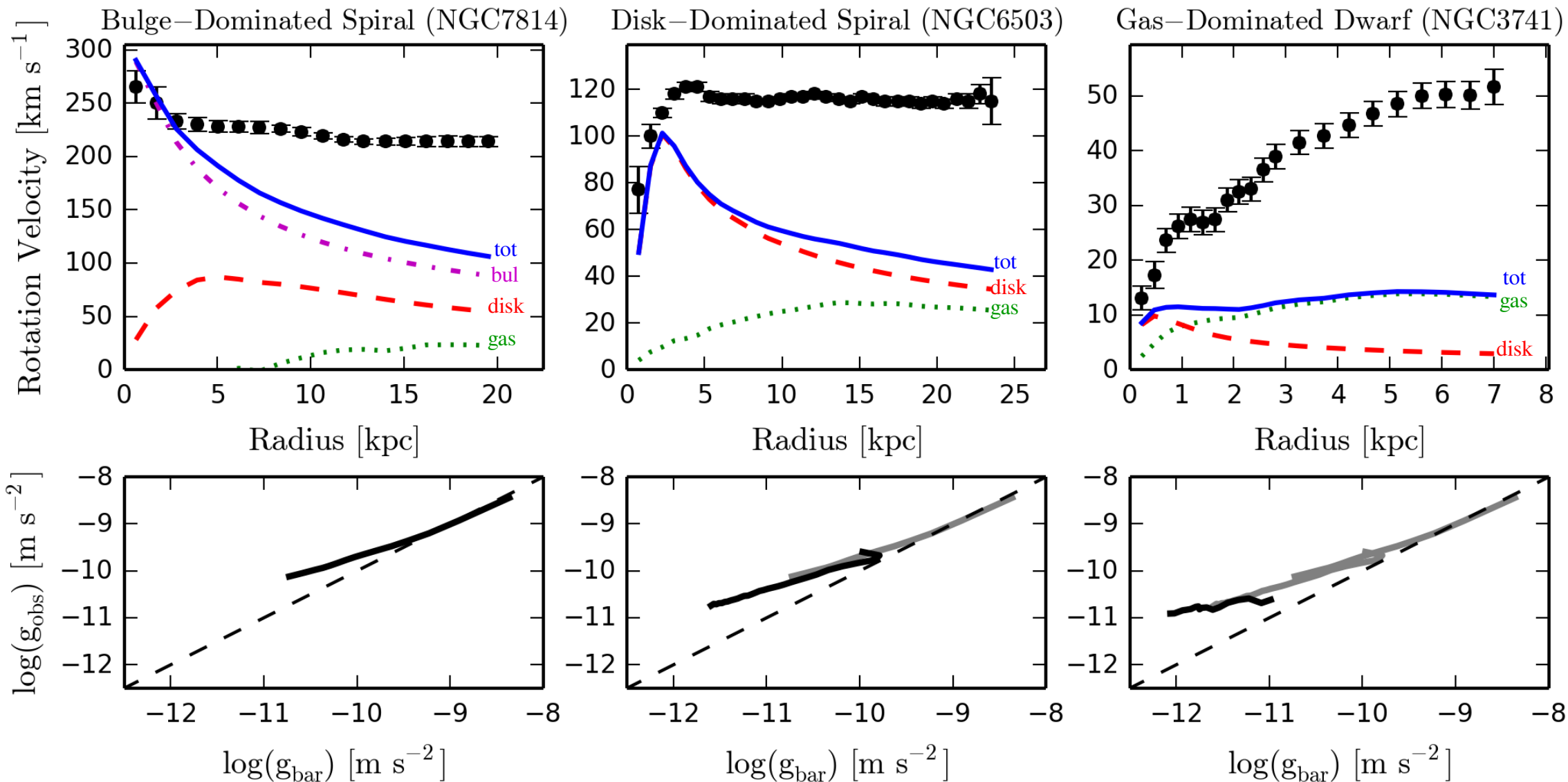
$$a_0 \simeq \frac{c^2 \sqrt{\Lambda/3}}{2\pi} \quad \Lambda = \text{Cosmological constant} \rightarrow \text{relation to Dark Energy?}$$

IF this numerology has some deeper, fundamental meaning:

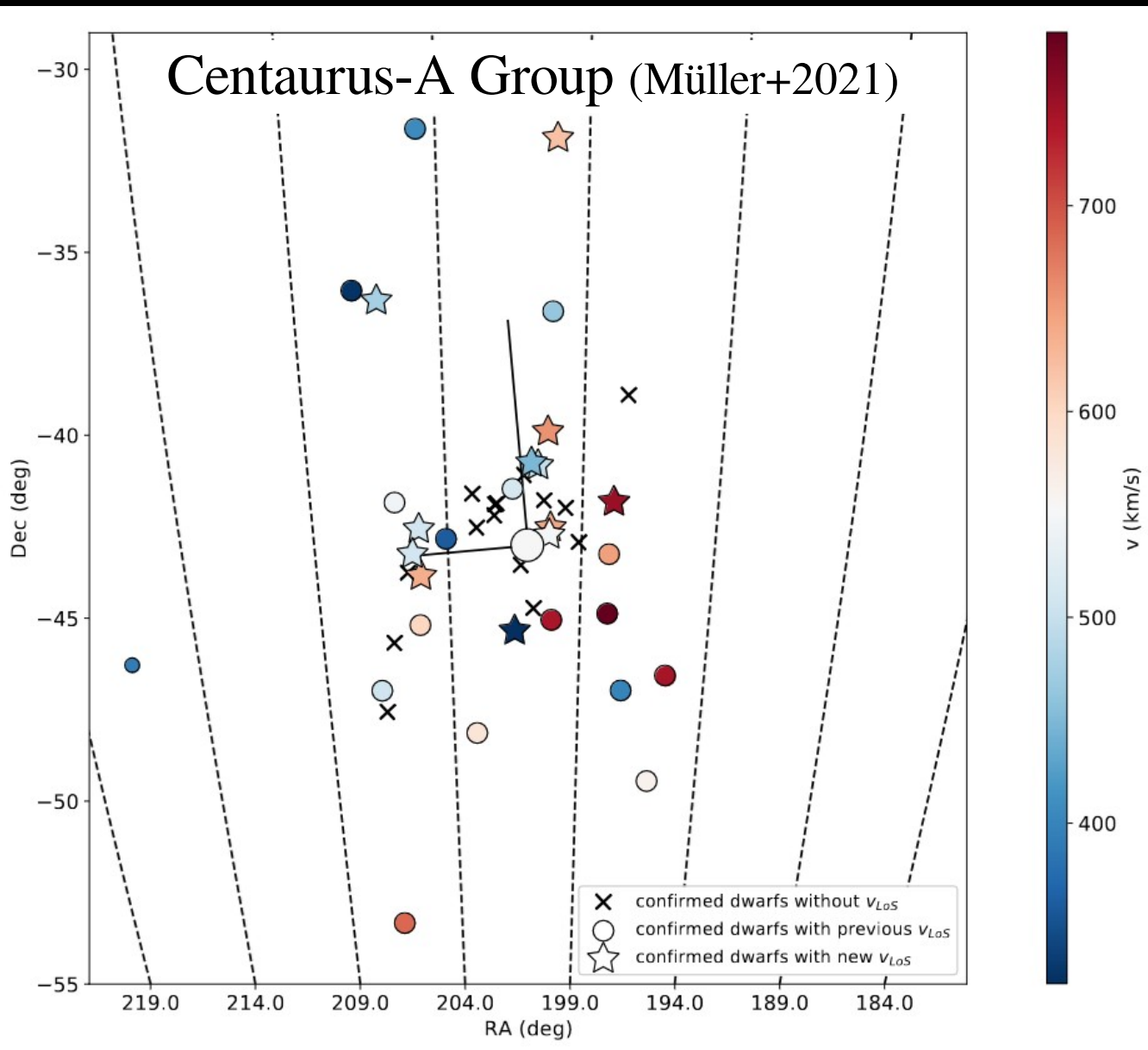
either the state of the Universe at large enters in local dynamics, or

the same parameters enters both Cosmology ( $\Lambda$ ) and local dynamics ( $a_0$ ).

# Galaxies lie on the same RAR *despite* their diversity



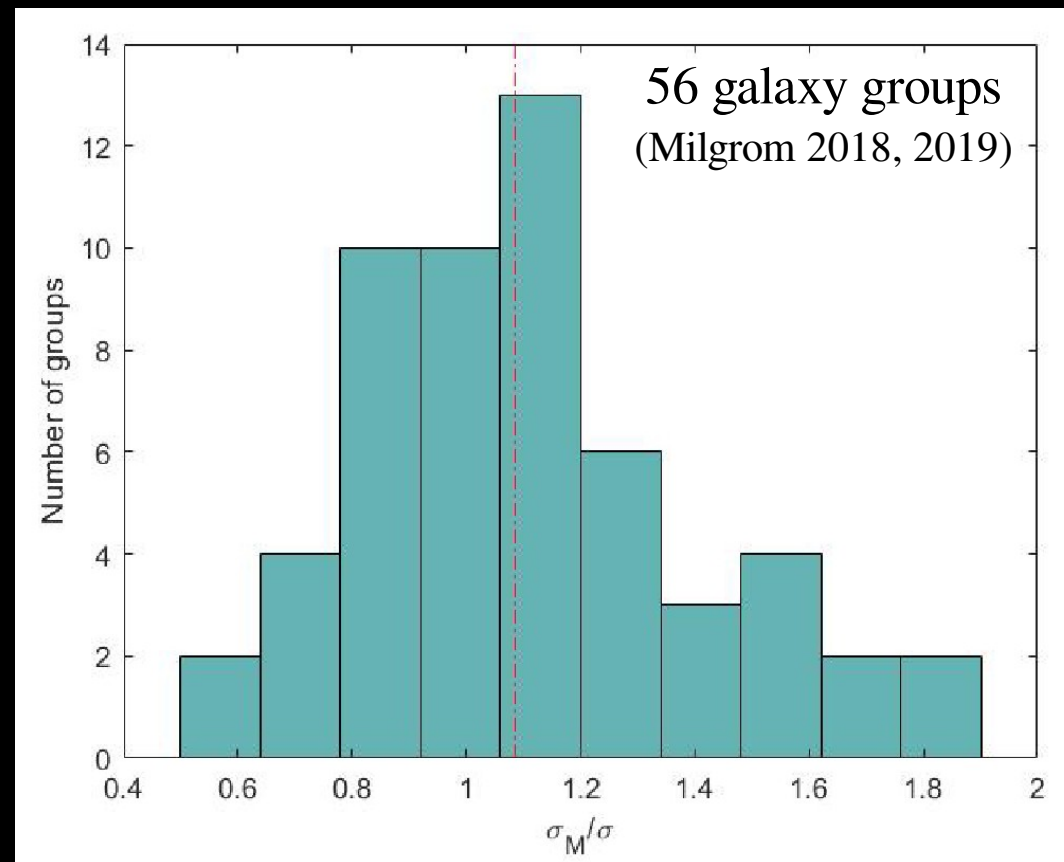
# Galaxy Groups: bound systems with $\sim 10$ -100 galaxies



$\sigma_{obs} \rightarrow$  group velocity dispersion

$$M_b = (M/L) \sum_i L_i \rightarrow \sigma_M^4 \simeq a_0 G M_b$$

$\rightarrow \sigma_{obs}/\sigma_M \simeq 1$  (within uncertainties)



# MOND as Modified Inertia (Milgrom 1994, 1999, 2022)

$$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla} \Phi_N \quad \bar{A} \text{ is a functional of the full trajectory } \bar{x}(t)$$

For  $a \gg a_0$ ,  $A \rightarrow a = d^2x/dt^2$  (Newton's 2<sup>nd</sup> law)

No full theory yet, but two general results:

(1) Imposing Newtonian and MOND limits + Galilei Invariance  $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

Theory is **time non-local**:  $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i \vec{x}}{dt^i}; i=1, 2, \dots, N\right)$

Accelerations at  $(\bar{x}, t)$  depend on the **full orbital history**!

(2) For purely circular orbits:  $\vec{a} \mu\left(\frac{a}{a_0}\right) = \vec{g}_N$  holds exactly (RAR for disk galaxies)

The **interpolation function** is a **derived concept** valid for circular orbits!

## Weak Equivalence Principle (WEP):

→ Universality of free fall (center-of-mass motion)

## Einstein Equivalence Principle (EEP):

→ WEP + Lorentz invariance (spacetime rotations)

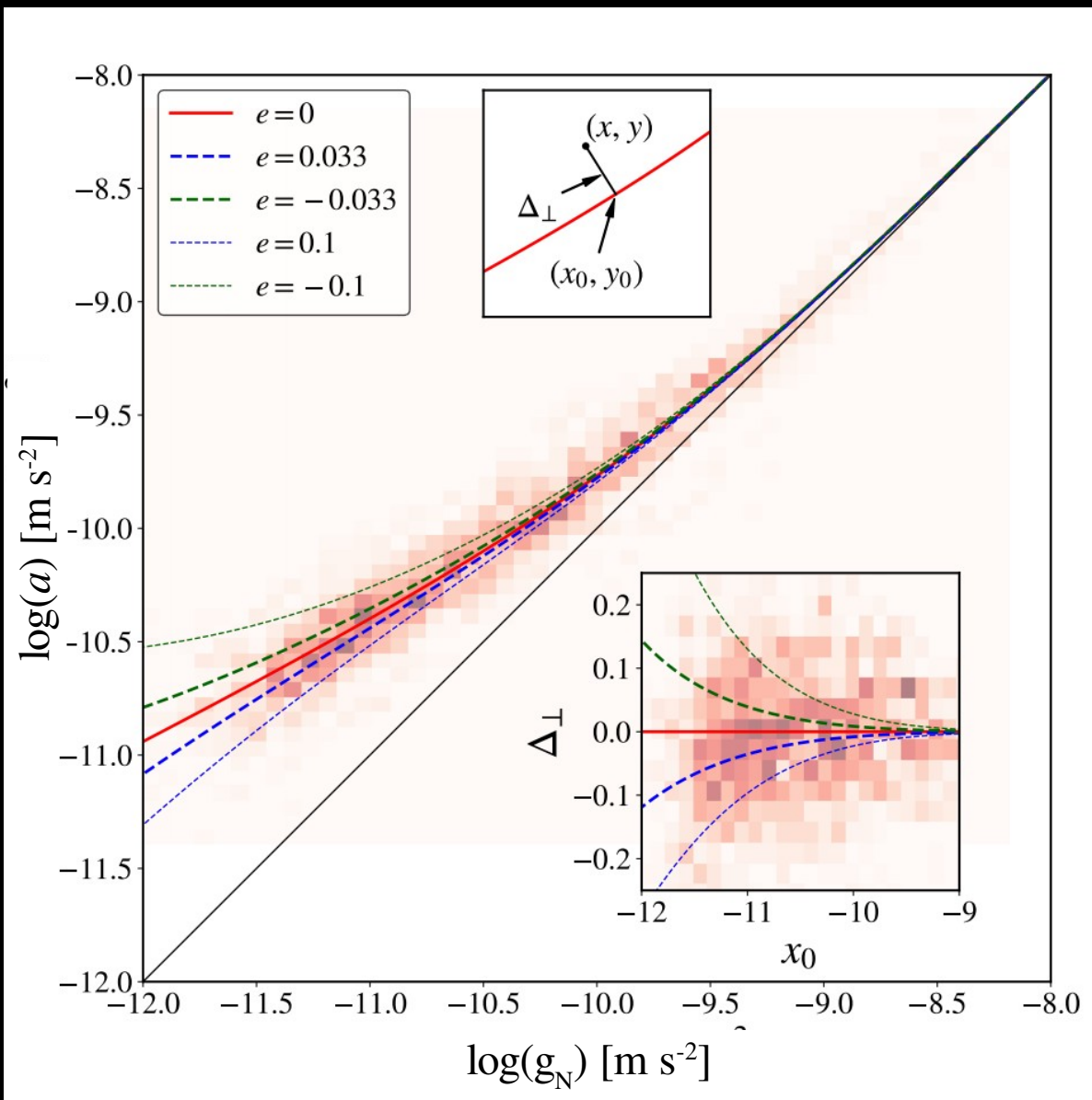
+ Local Position Invariance (LPI) for non-gravitational experiments:  
the results of experiments do not depend on where/when they are done

## Strong Equivalence Principle (SEP):

→ EEP + ~~LPI for gravitational experiments too~~

**Broken by MOND:**  
**External Field Effect**  
(Bekenstein & Milgrom 1984, ApJ)

# External field effect (EFE): implications for the RAR



- For truly *isolated* galaxies:  

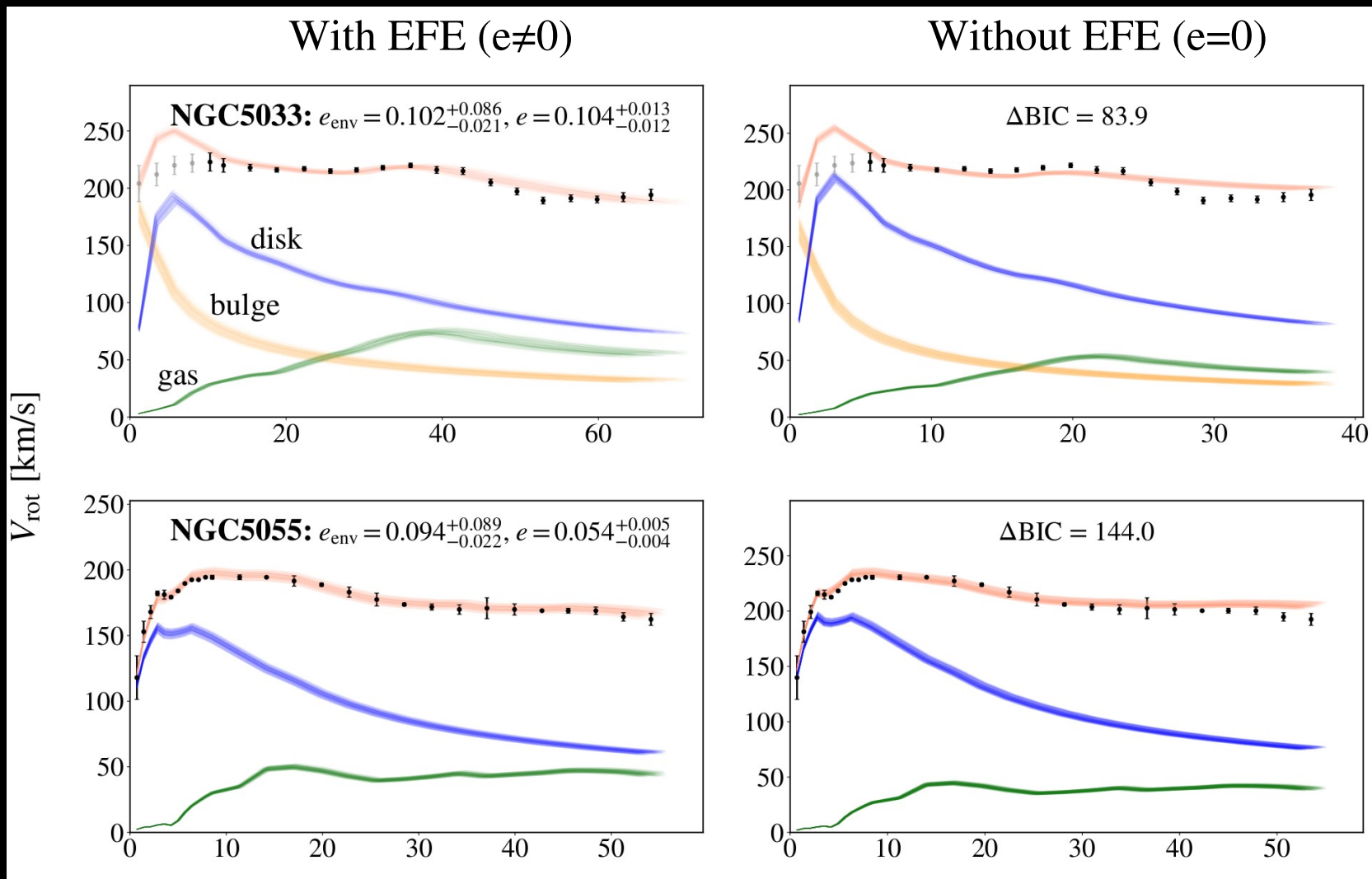
$$a = v_0 (g_{N,\text{int}}/a_0) g_{N,\text{int}}$$
- For galaxies subjected to  $e = g_{N,\text{ext}}/a_0$ :  

$$a = v_e (g_{N,\text{int}}/a_0; e) g_{N,\text{int}}$$
- RAR should be a family of curves depending on the galaxy environment
- We can fit RCs to infer the value of  $e$  and independently estimate  $e_{\text{env}}$  from the galaxy large-scale environment.

Chae, Lelli, Desmond et al. (2020)



# EFE is weak: individual detections only in extreme cases



NGC 5033



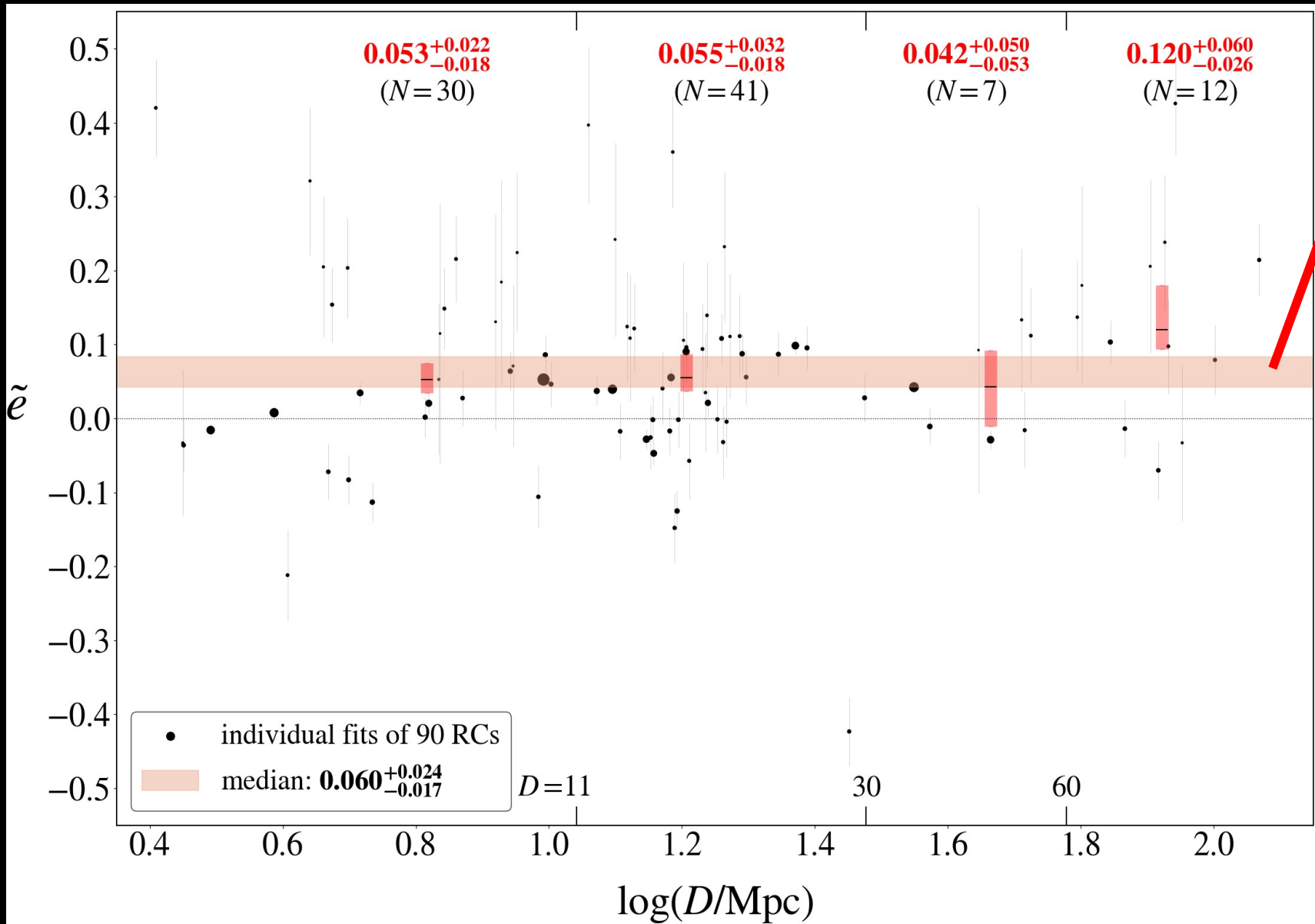
NGC 5055



Chae+2020, 2021

# Statistical approach: $EFE > 0$ at $>4\sigma$ and agrees with LSS

$$\frac{g_{ext}}{a_0} \leftarrow \tilde{e}$$

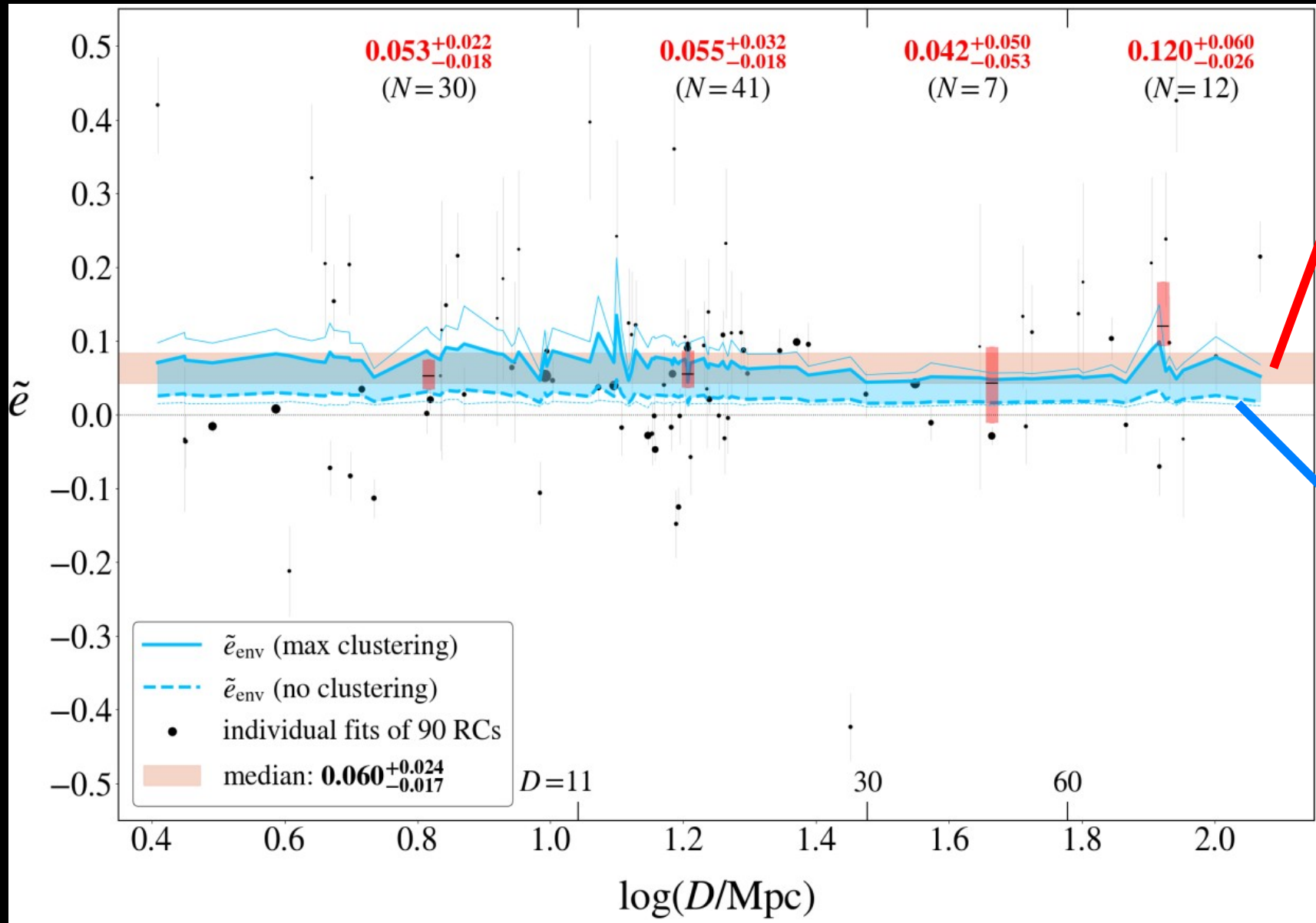


From Rotation  
Curve Fits

Chae+2020, 2021

# Statistical approach: $EFE > 0$ at $>4\sigma$ and agrees with LSS

$$\frac{g_{ext}}{a_0}$$



From Rotation  
Curve Fits

From Baryon  
Large Scale  
Structure

Chae+2020, 2021

# MOND External Field effect (EFE)

**MOND** is non-linear  $\rightarrow$  both internal ( $g_{N,int}$ ) and external ( $g_{N,ext}$ ) fields

For non-isolated systems, three possibilities:

(1)  $g_{N,ext} \ll g_{N,int} \ll a_0$

$\rightarrow$  MOND regime (e.g. nearly isolated galaxies)

(2)  $g_{N,int} \ll a_0 \ll g_{N,ext}$

$\rightarrow$  Newtonian regime (e.g. star clusters in the inner MW)

(3)  $g_{N,int} \ll g_{N,ext} \ll a_0$

$\rightarrow$  Newton with  $G_{eff} \sim G a_0 / g_{N,ext}$  (e.g. some satellites of MW)

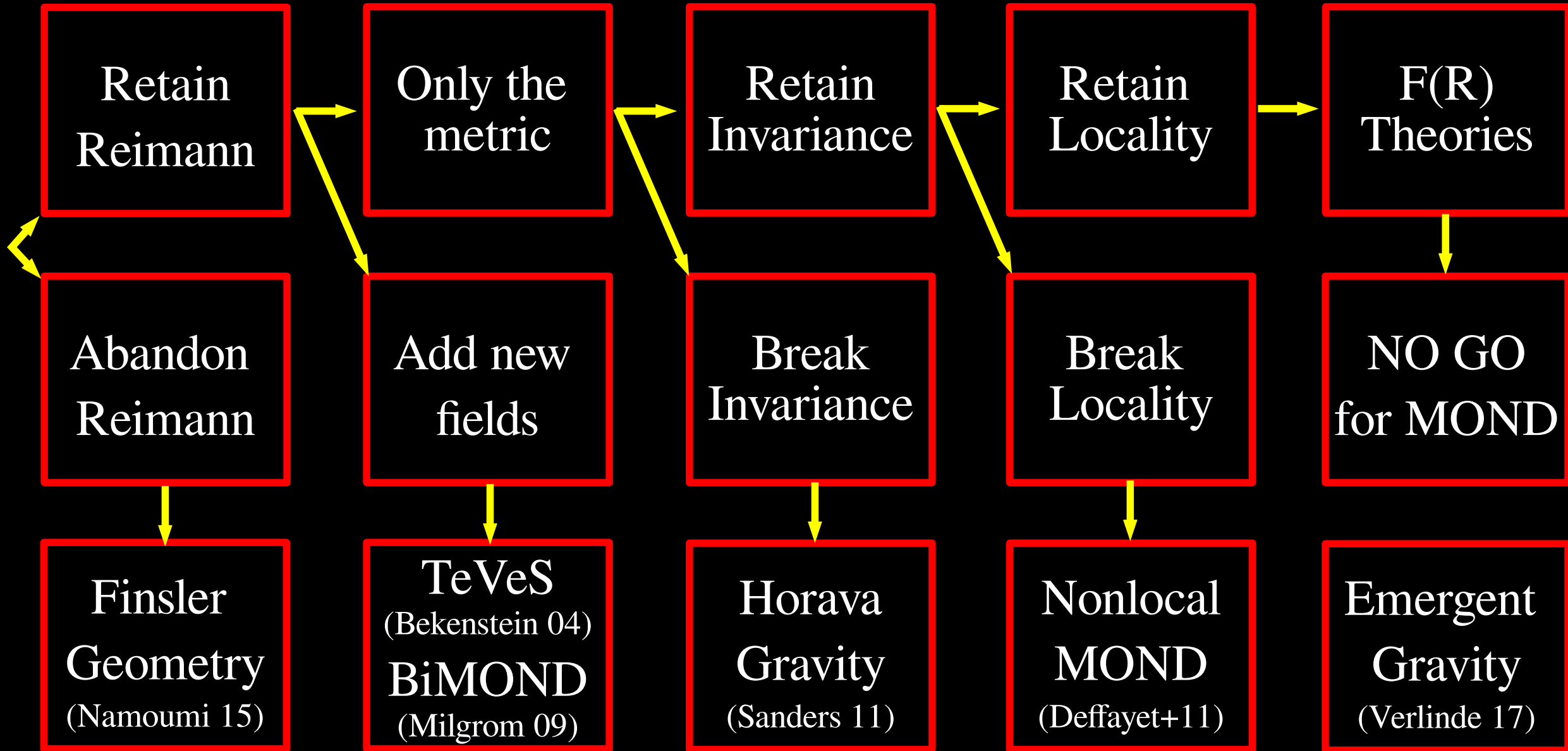
**EFE is a general MOND prediction but details depend on the specific theory**

# Relativistic Theories: Lovelock-Grigore Theorem

GR (+ $\Lambda$ ) is the only theory that satisfy these assumptions:

- 1- Geometry is Riemannian
- 2- The Action depends only on  $g_{\mu\nu}$
- 3- It is diffeomorphism invariant
- 4- It is local
- 5- It leads to 2<sup>nd</sup> order field equations

# Paths towards a relativistic version of MOND



# Intuitive Cartoon: Scale Invariance = Flat Rotation Curves

