Alternatives To Dark Matter

Federico Lelli

INAF – Arcetri Astrophysical Observatory

Need of Dark Matter at Different Scales

Small Scales (~1-100 kpc)

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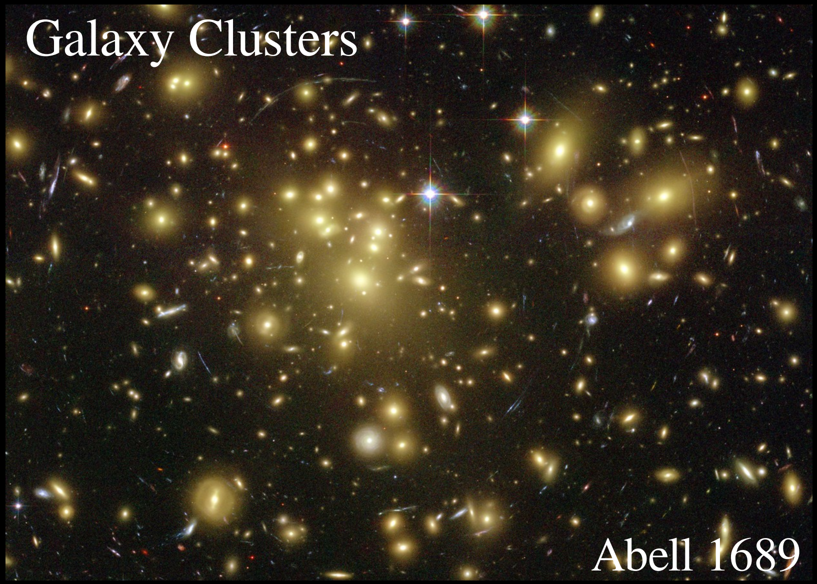
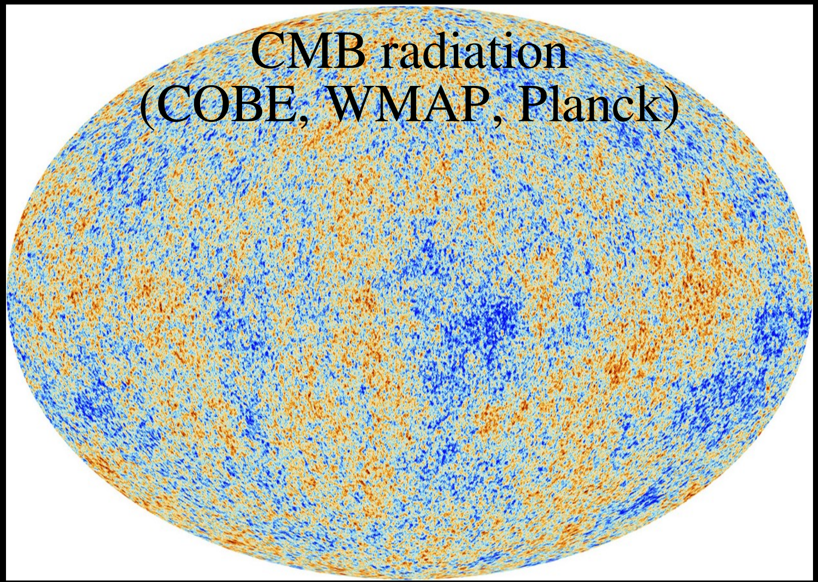
Small Scales (~1-100 kpc)

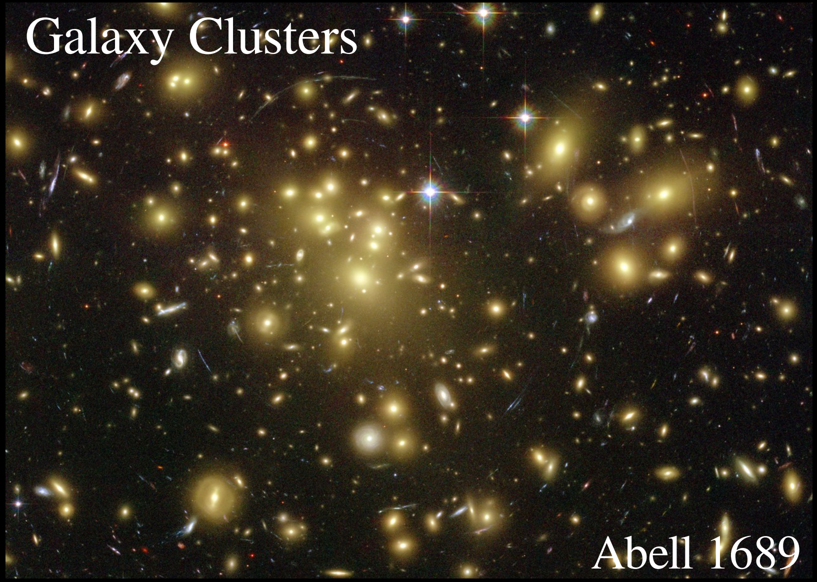
Intermediate Scales (~1-5 Mpc)

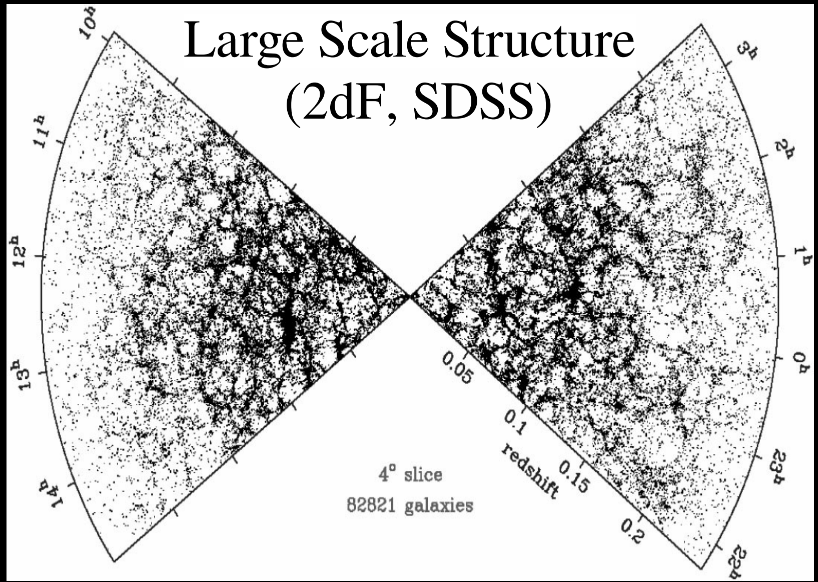


Need of Dark Matter at Different Scales

Small Scales (~1-100 kpc) | **Intermediate Scales (~1-5 Mpc)** | **Large Scales (>100 Mpc)**







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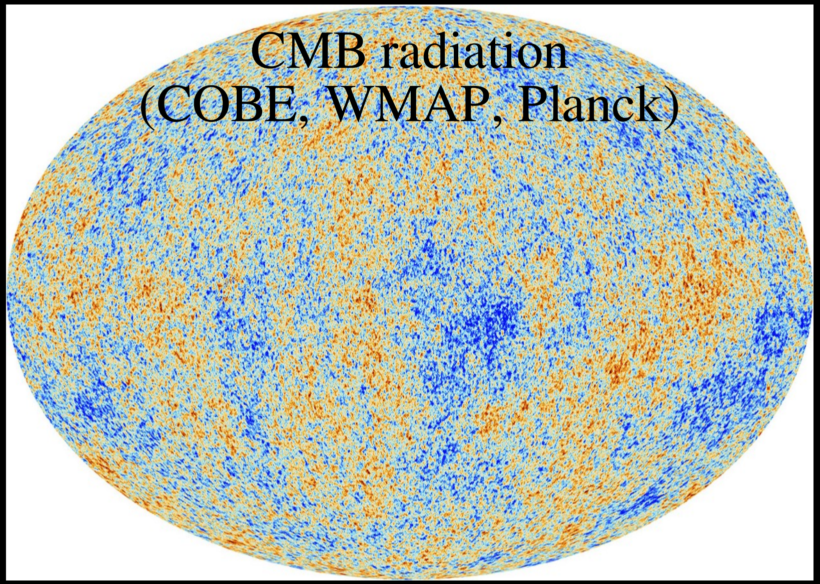
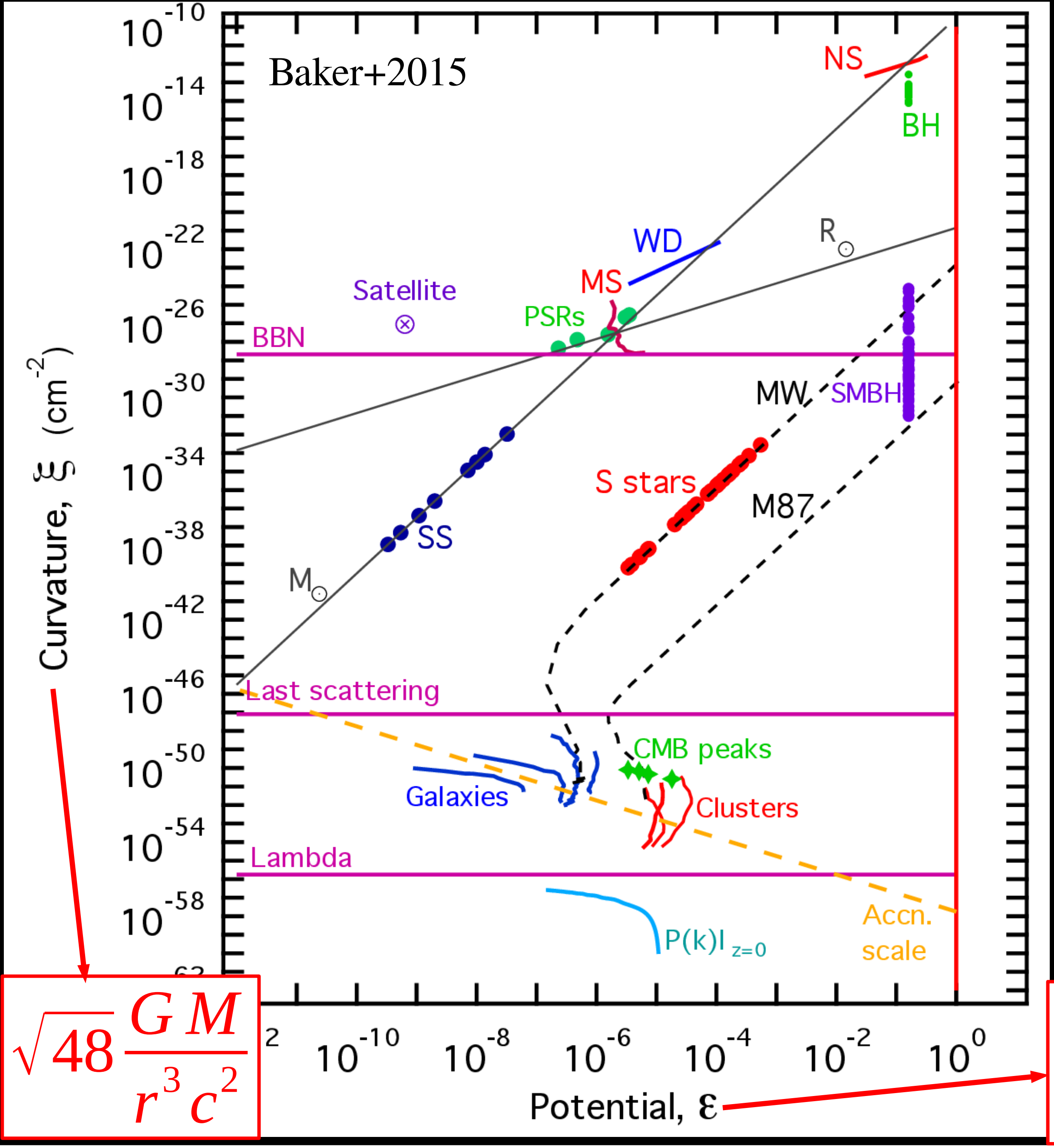


This is *not* direct evidence for *particle* dark matter!

Current Gravitational Laws (Einstein & Newton) + Standard Model of Particle Physics = Do NOT work

Probing Gravity at All Scales



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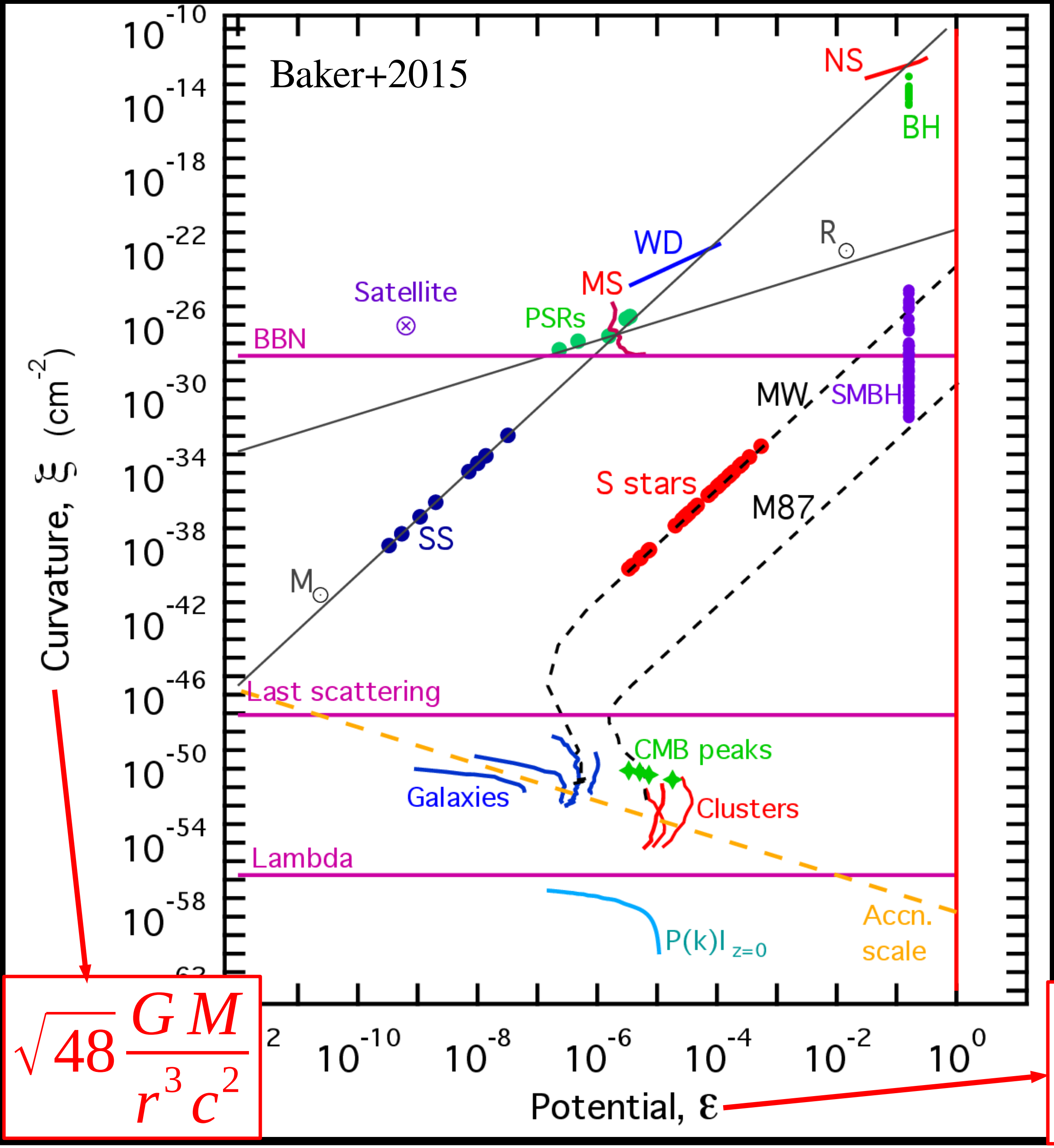

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For particles orbiting a point mass M:

$$\xi(M) = \frac{\sqrt{48}\,c^4}{G^2 M^2}\,\varepsilon^3$$

In a log-log plot, this is a line with fixed slope of 3 and normalization given by M.

(Exercise 1: derive this Eq. given ξ and ε)

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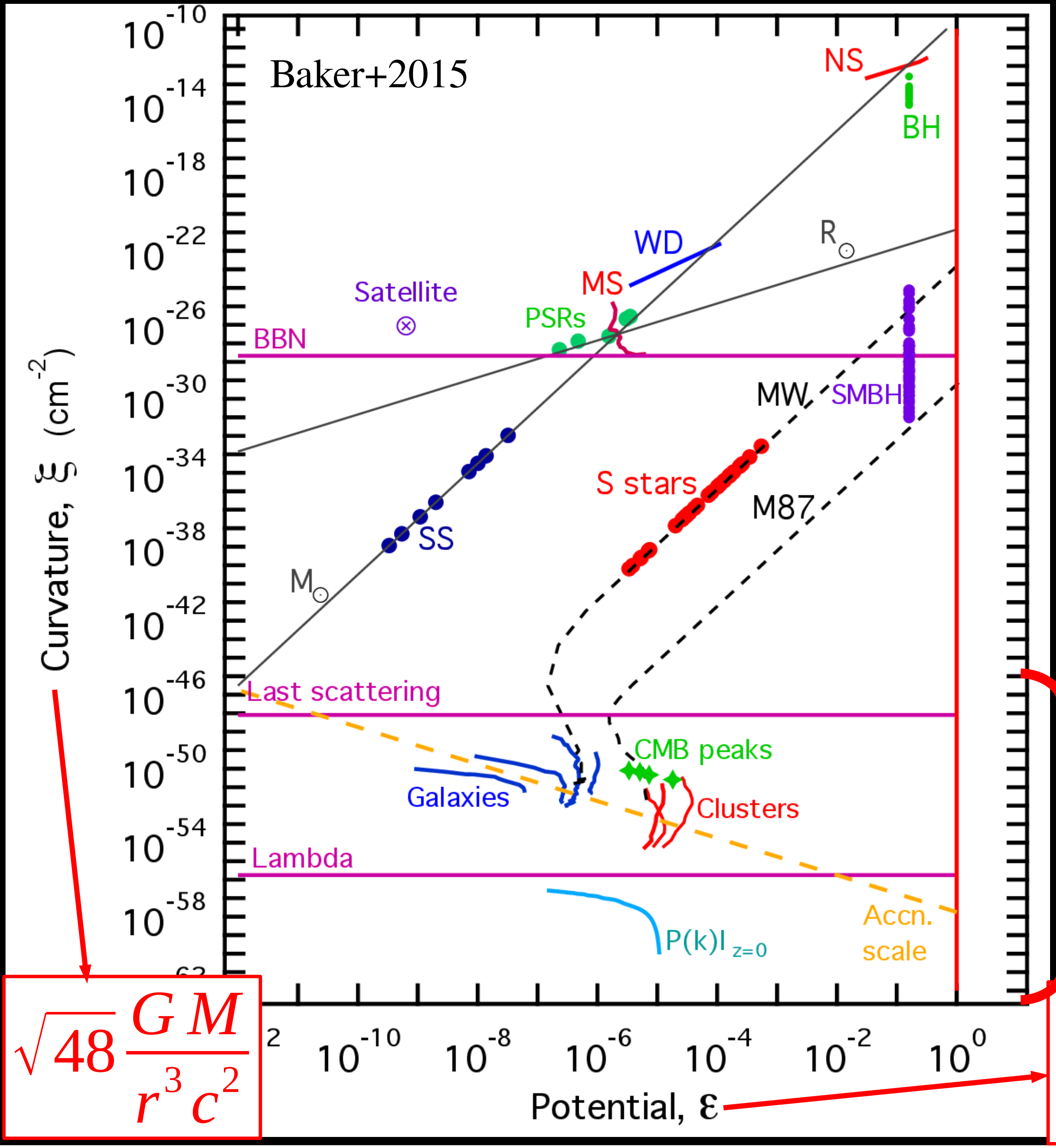

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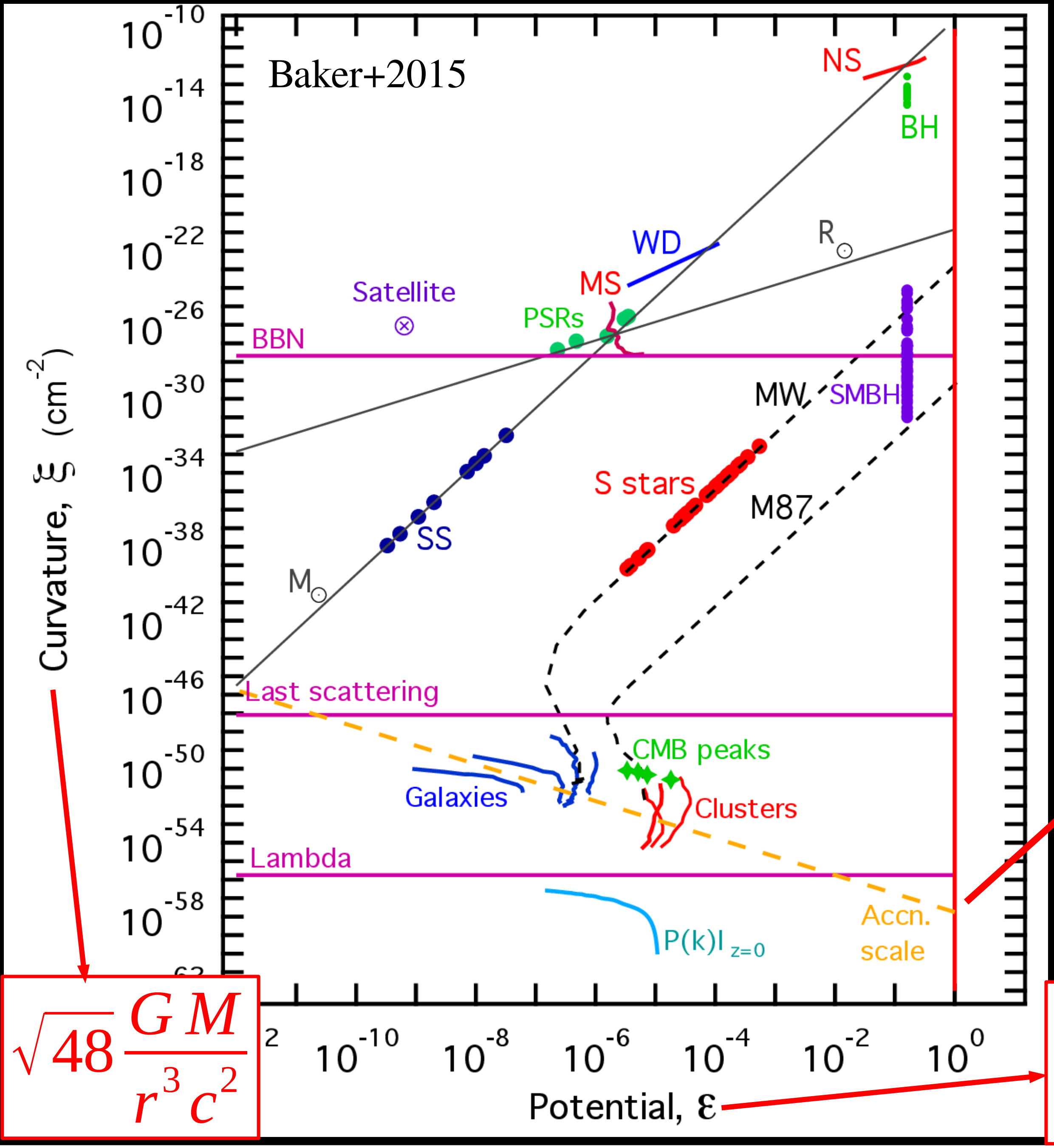
General Relativity has been well tested at high curvatures (strong Gravity).

Dark Matter and Dark Energy arise at low curvatures (weak Gravity).



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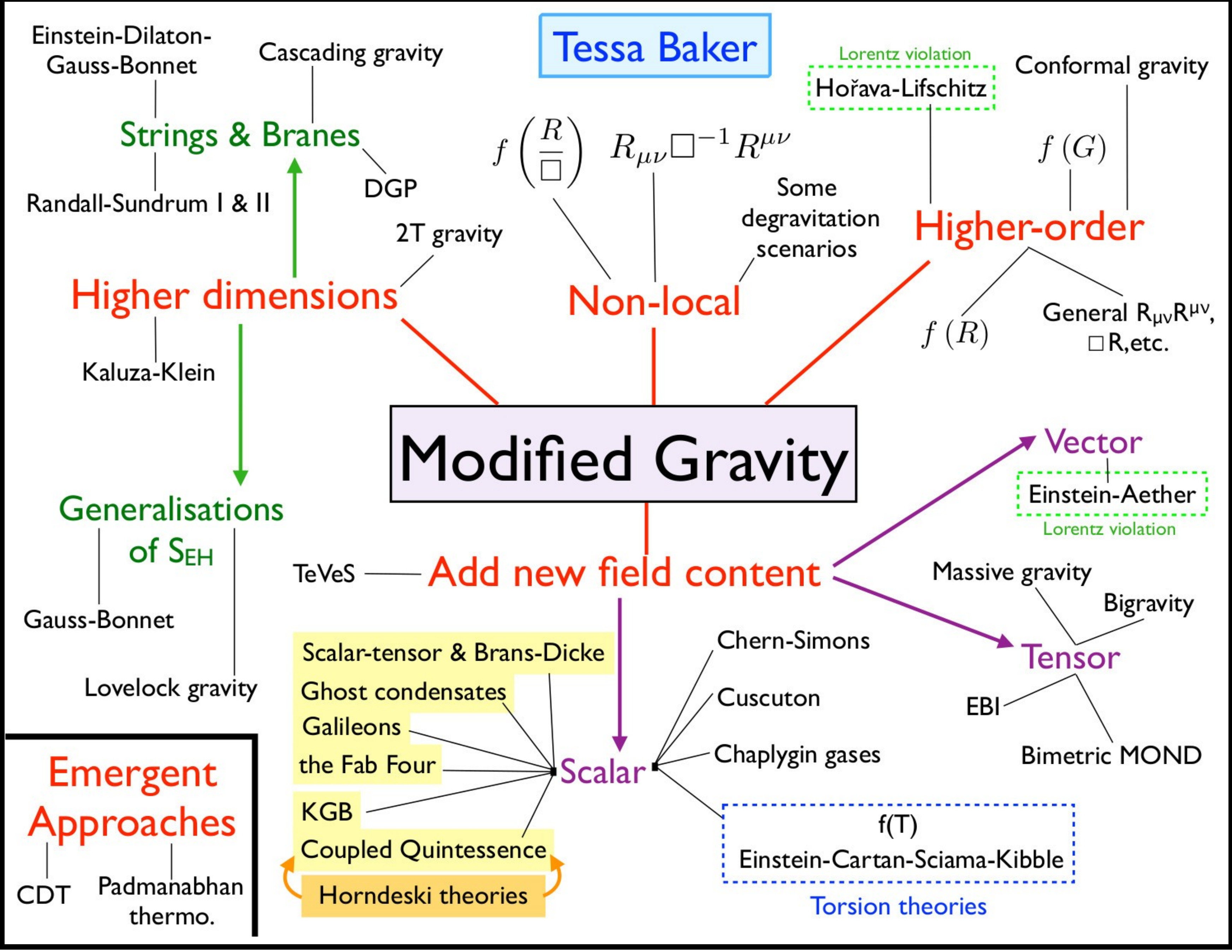
DM appears below a characteristic acceleration $a_0 = GM/r^2 \sim 10^{-10}$ m s^{-2}

$$\xi(a_0) = \frac{\sqrt{48}\,a_0^2}{c^4}\,\varepsilon^{-1}$$

(Exercise 2: derive this Eq. given a_0)



Many versions of Modified Gravity to explain DM or DE (each one with its own serious problems).

This lecture will NOT cover all this.

I will focus on Milgromian Dynamics (aka MOND). Empirically motivated paradigm with no DM.

Roadmap of the Lecture

1. The general MOND paradigm

2. Non-relativistic MOND theories

3. Relativistic MOND theories

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MOND = Modified Newtonian Dynamics

or MilgrOmiaN Dynamics

Proposed by Moderhai Milgrom (1983a, b, c)

- MOND is a general paradigm that includes several theories (at both the non-relativistic and relativistic level)
- Key distinguishing general predictions of the general MOND paradigm from specific predictions of specific MOND theories

General MOND postulates (at the non-relativistic level)

1) New constant of Physics: a_0 (~10^{-10} m/s^2)

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

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2) For $a \gg a_0 \rightarrow \bar{a} = \bar{g}_N$ (correspondence principle as in Quantum Mechanics)

$\vec{a} = \frac{d^2 \vec{x}}{dt^2}$ kinetic (observed) acceleration of a particle

$\vec{g}_N = -\vec{\nabla}\phi_N$ Newtonian gravitational field (from the Poisson's equation)

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Circular orbit at large R

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Circular orbit at large R — Flat rotation curve!

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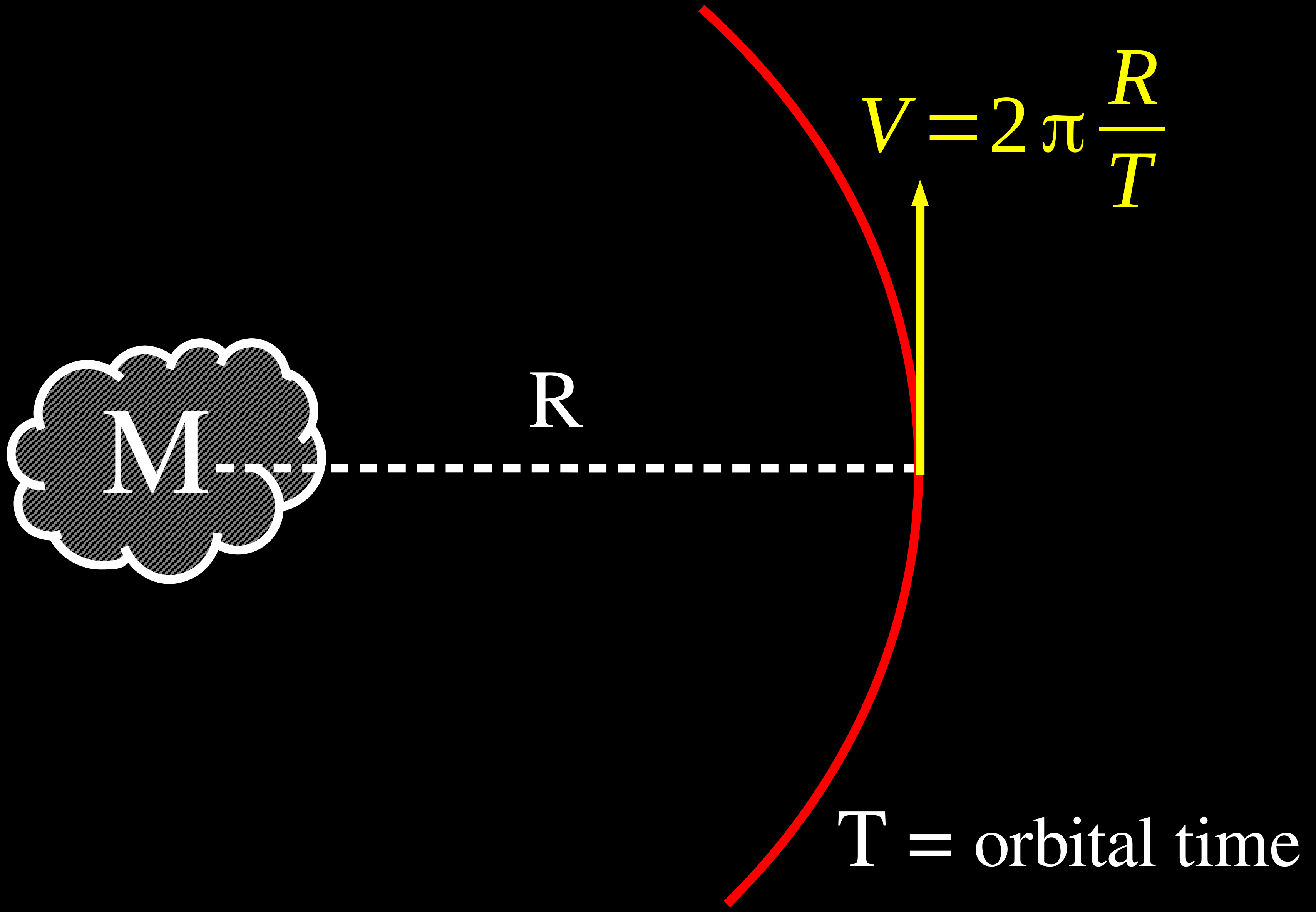
Circular orbit at large R — Flat rotation curve! — Baryonic Tully-Fisher Relation

Intuitive Cartoon: Scale Invariance = Flat Rotation Curves



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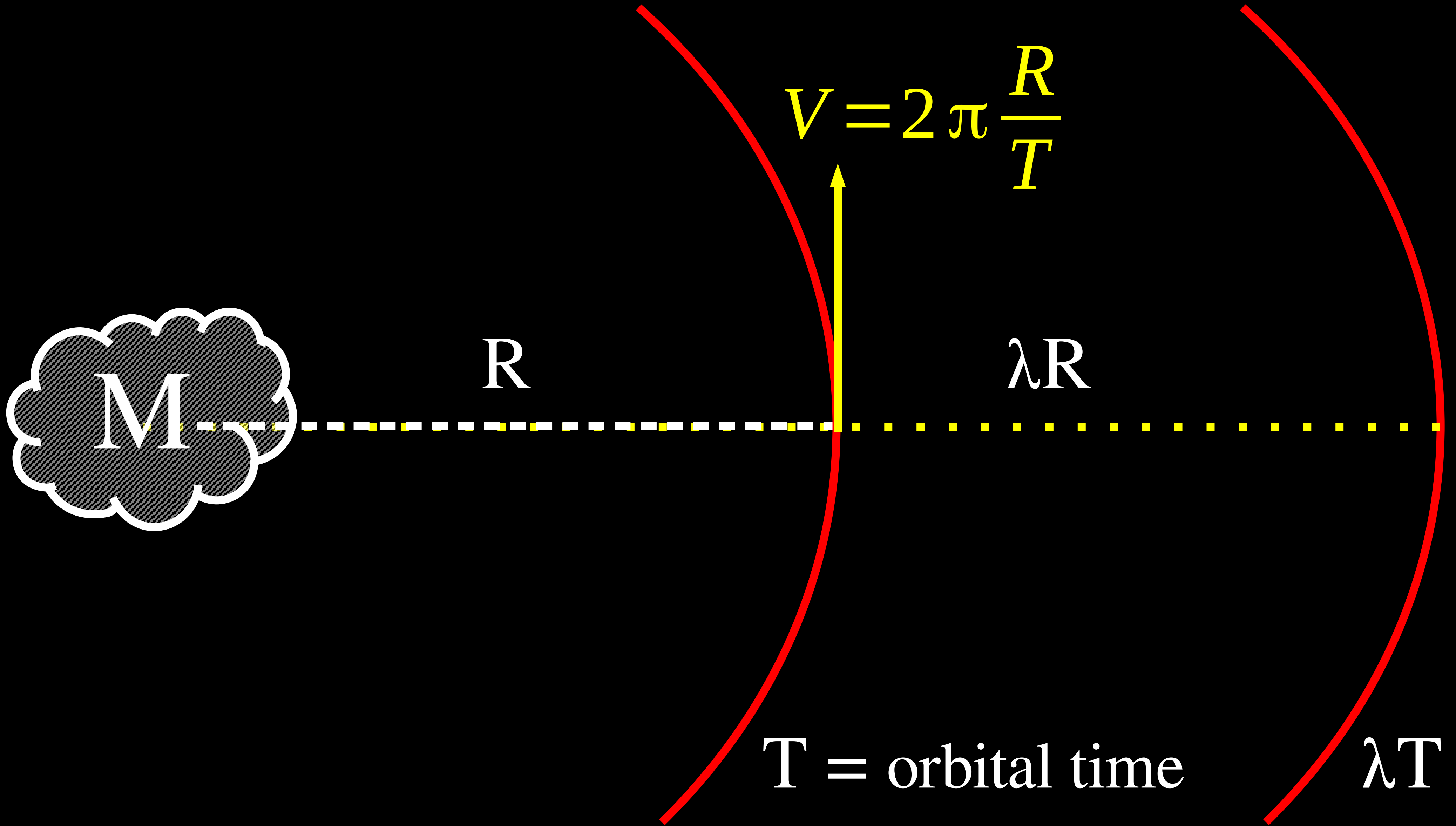

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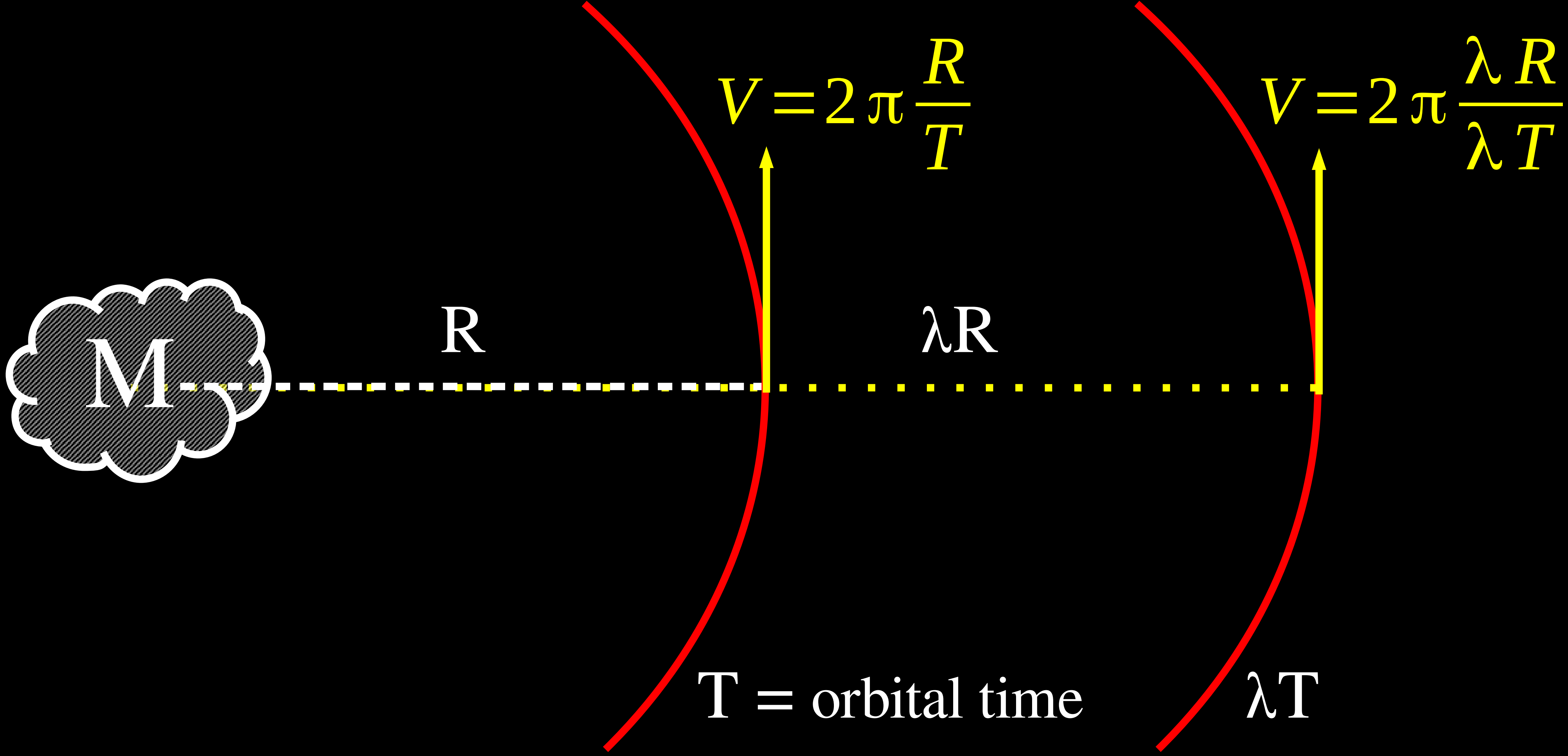

A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES[1]

M. MILGROM
Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study
Received 1982 February 4; accepted 1982 December 28

ABSTRACT

I use a modified form of the Newtonian dynan
of bodies in the gravitational fields of galaxies, a
the following main results.

1. The Keplerian, circular velocity around a fi
thus resulting in asymptotically flat velocity curv

2. The asymptotic circular velocity (V_∞) is det
$V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant
is consistent with the observed Tully-Fisher rela
proportional to the observable mass.

3. The discrepancy between the dynamically
and the density of observed matte

4. The rotation curve of a galax
galaxy's average surface density Σ
and Freeman laws. For smaller va

5. The value of the acceleration
mately $2 \times 10^{-8}(H_0/50$ km s^{-1} M
km s^{-1} Mpc^{-1}) cm s^{-2}.

The main predictions are:

1. Rotation curves calculated
dynamics should agree with the ob

2. The $V_\infty^4 = a_0 GM$ relation sho

3. An analog of the Oort discr
increasing r in a predictable way.

Trilogy of papers in 1983 on ApJ, 270

A MODIFICATION OF THE NEWTONIAN DYNAMICS AS A POSSIBLE ALTERNATIVE TO THE HIDDEN MASS HYPOTHESIS[1]

M. MILGROM
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ABSTRACT

I consider the possibility that there is not, in fact, much hidden mass in galaxies and galaxy

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ABSTRACT

I consider the implications of a modification of the Newtonian dynamics to galaxy systems. Masses and mass-to-light ratios are rederived, on the basis of existing data, for binary galaxies, small groups, clusters of galaxies, and the Virgo Supercluster. For each type of galaxy system, the average M/L values come out to be a few solar units. These results eliminate the need to assume large amounts of hidden mass in galaxy systems, if the modified dynamics applies.

General MOND predictions (most dating 1983-1984):

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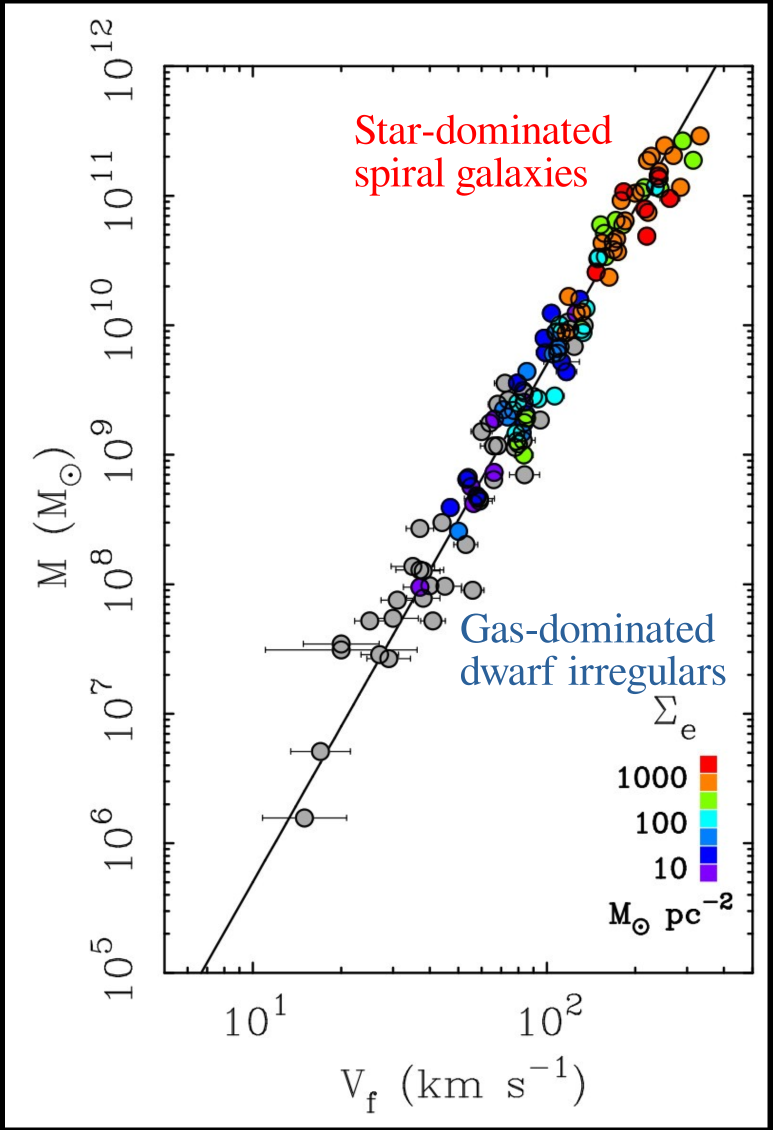

Four predictions in one equation:

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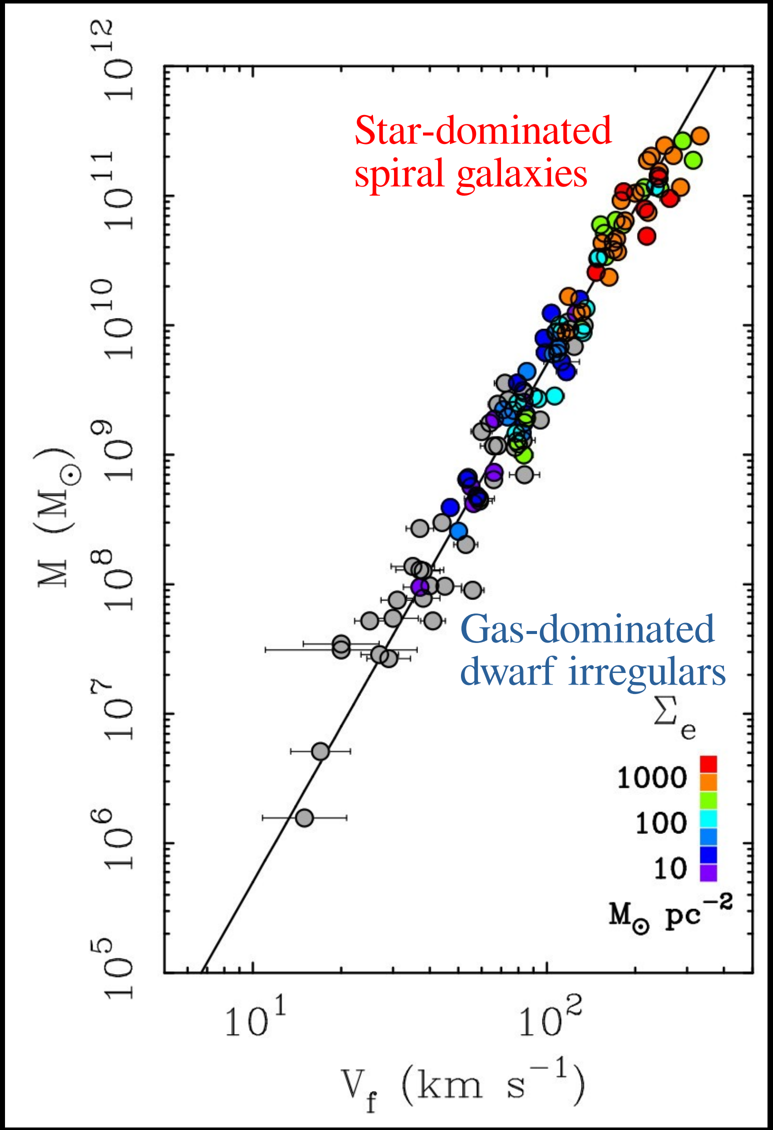

Four predictions in one equation:

(i) The relevant quantities are V_∞ and M_b → OK

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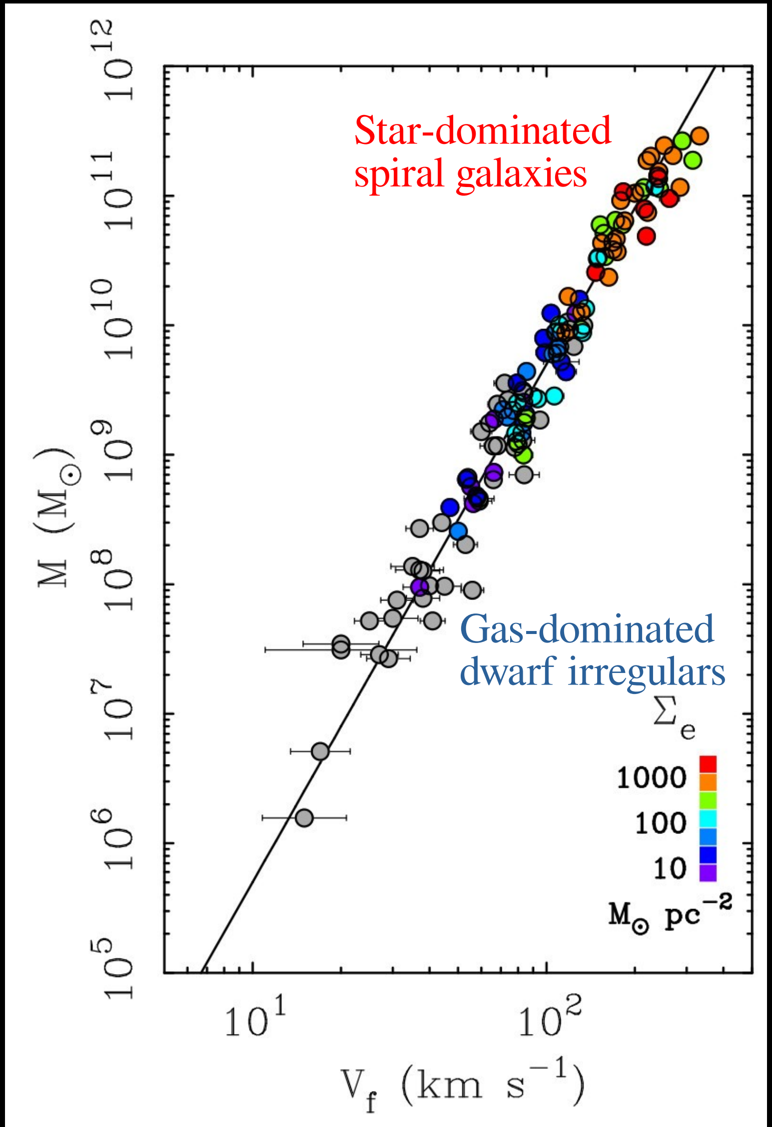

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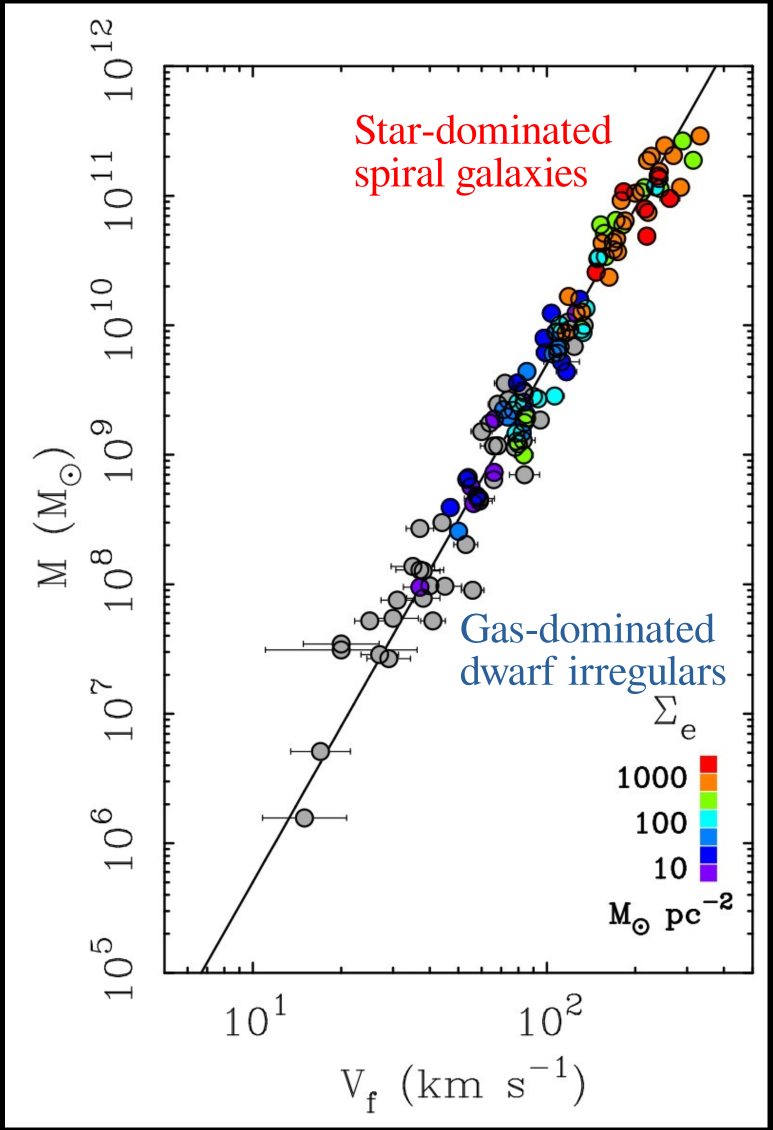

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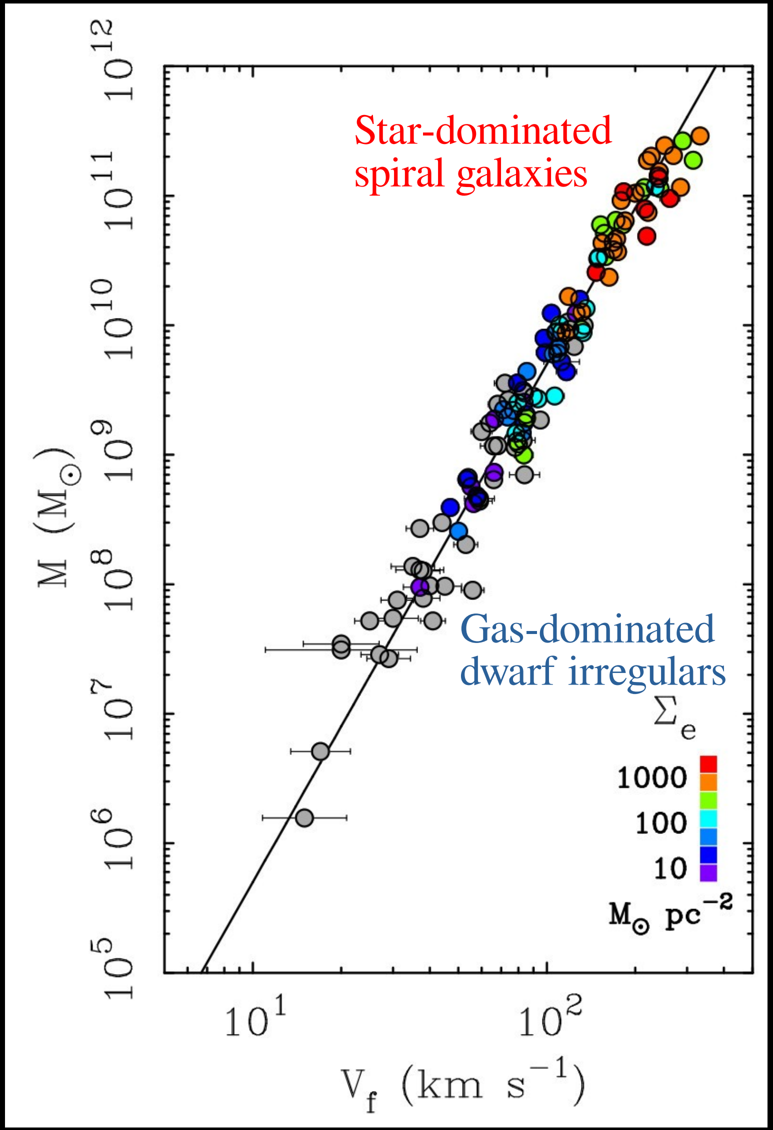

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(iv) No dependence on other quantities → OK

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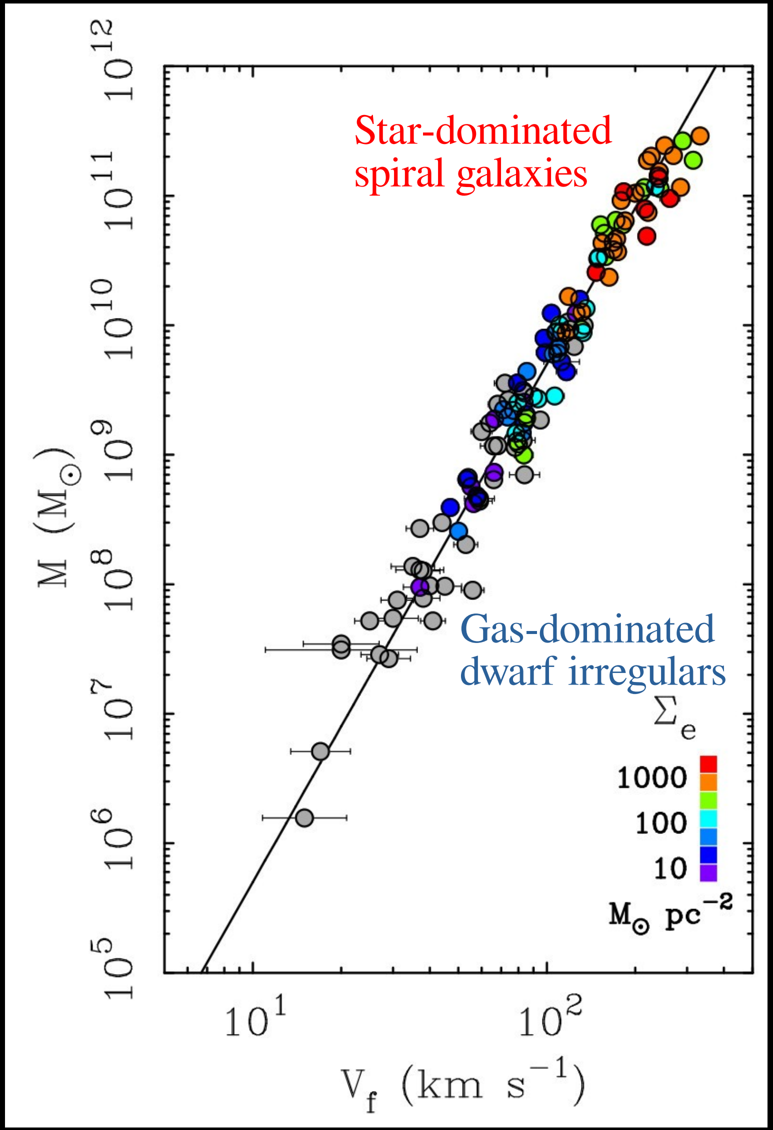

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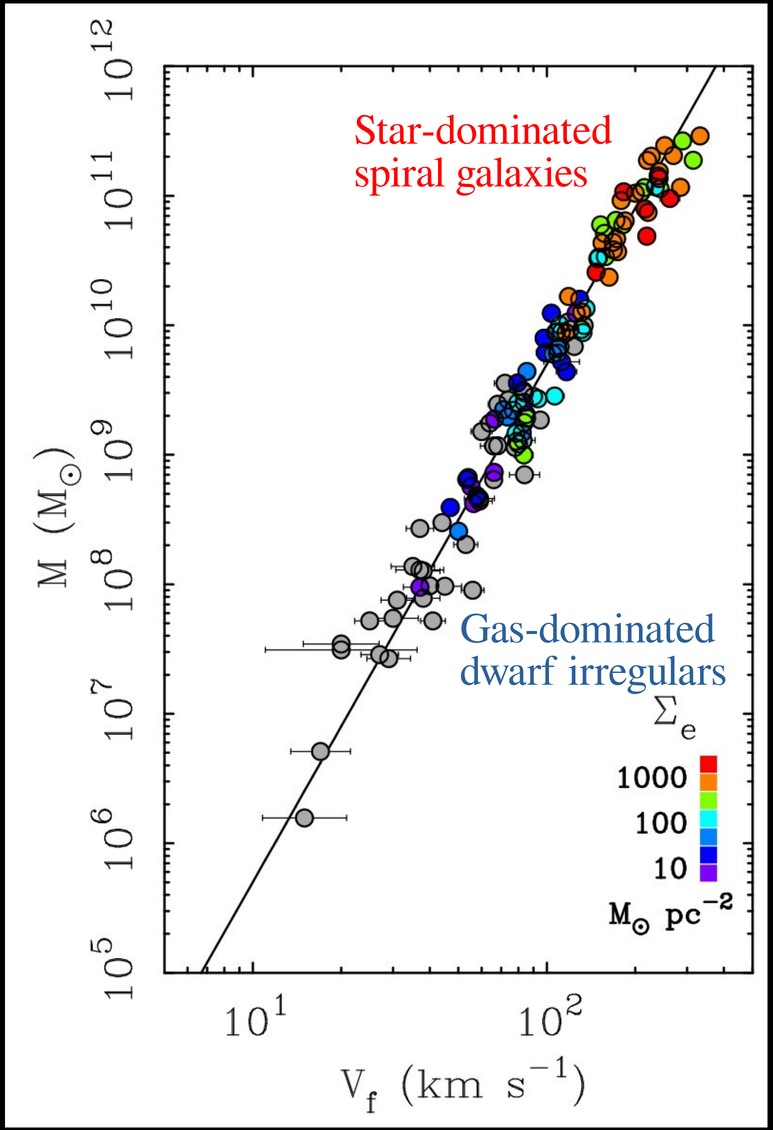

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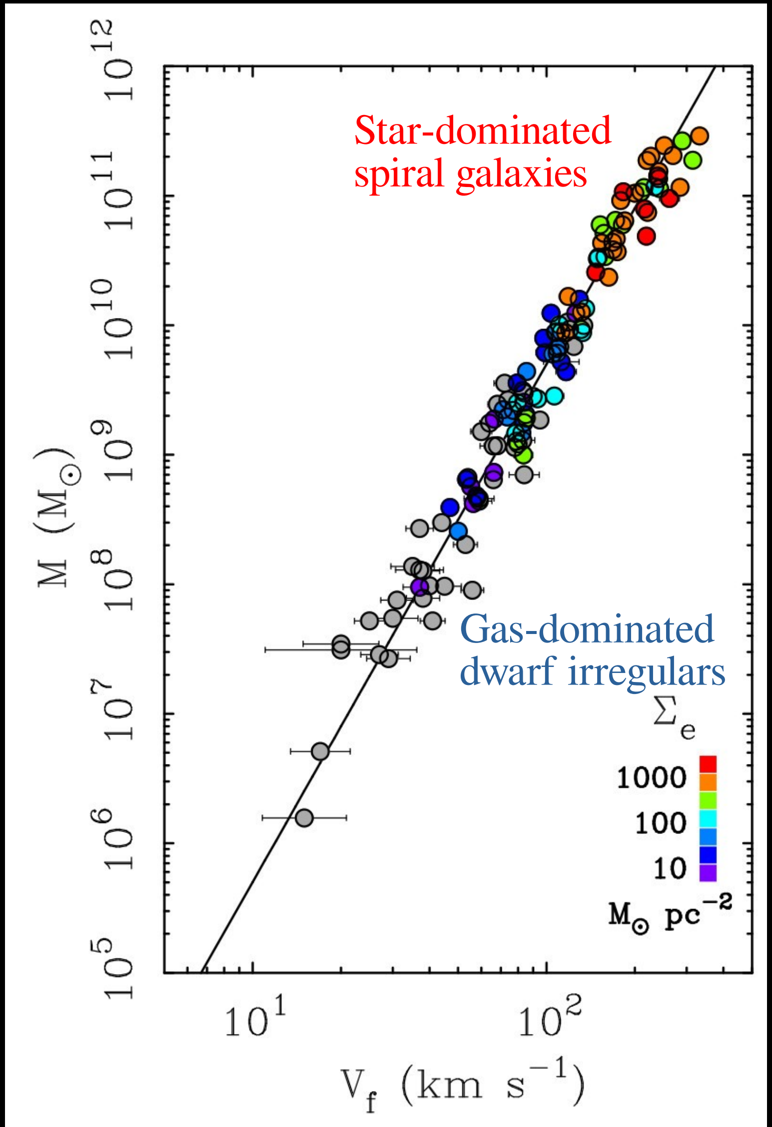

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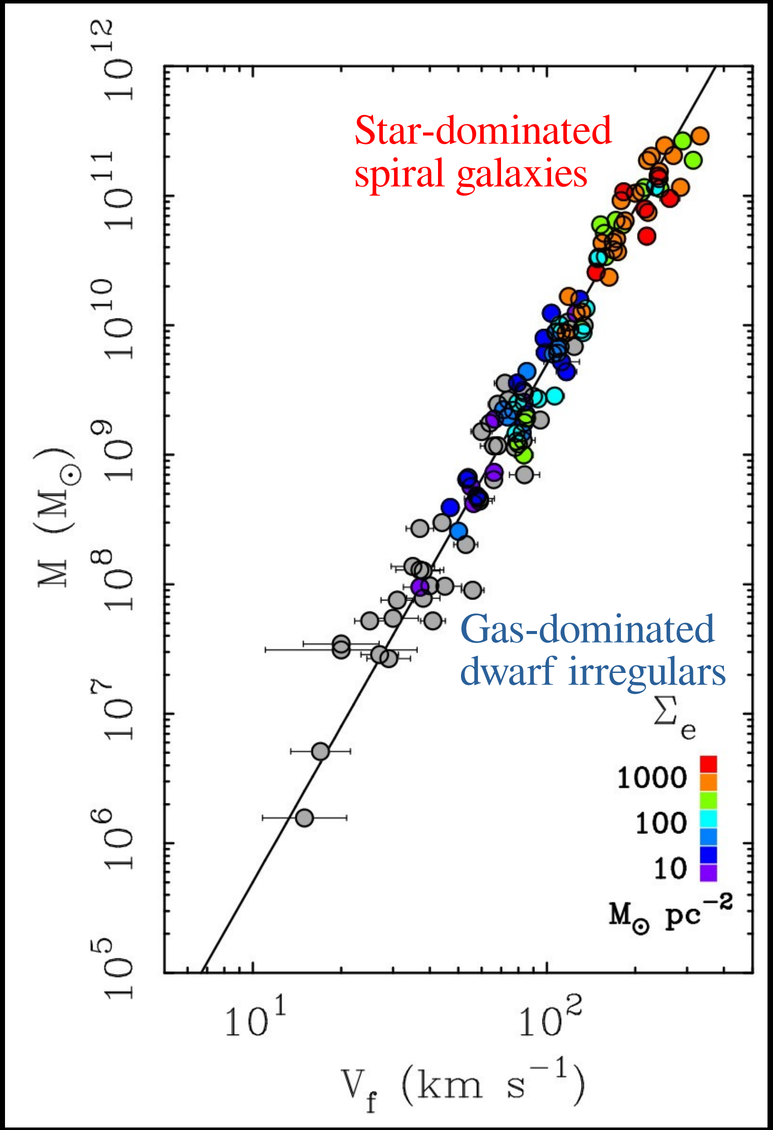

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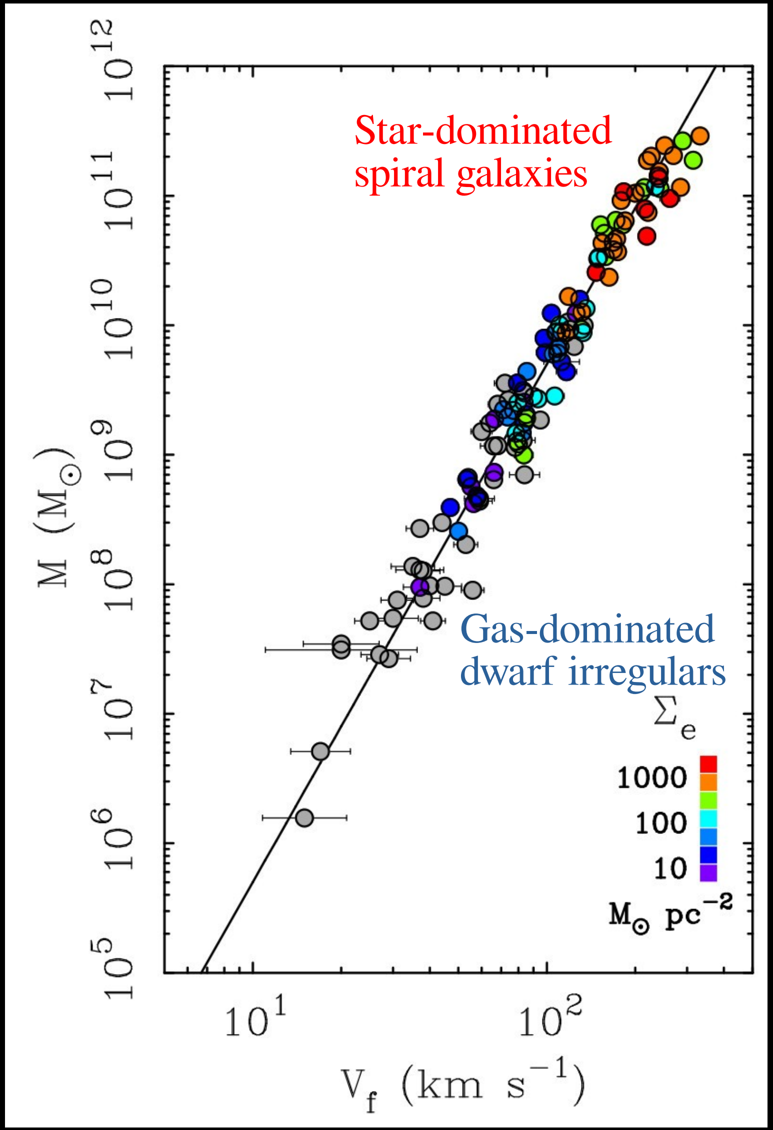

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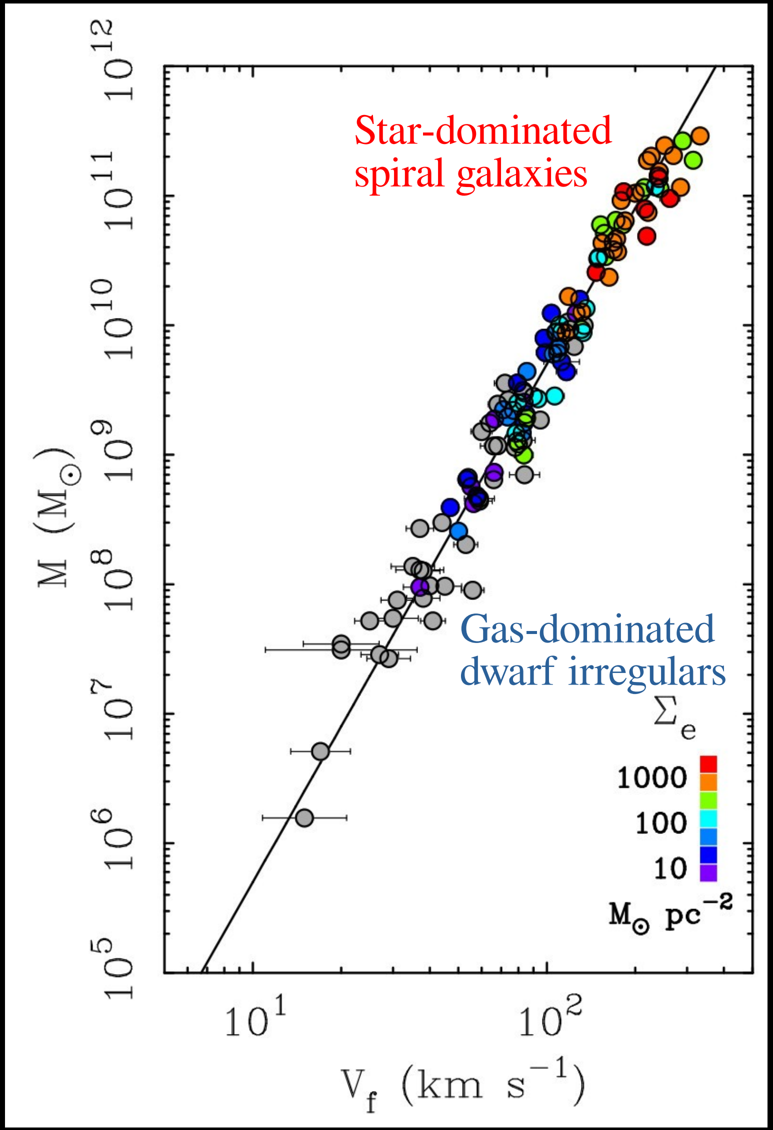

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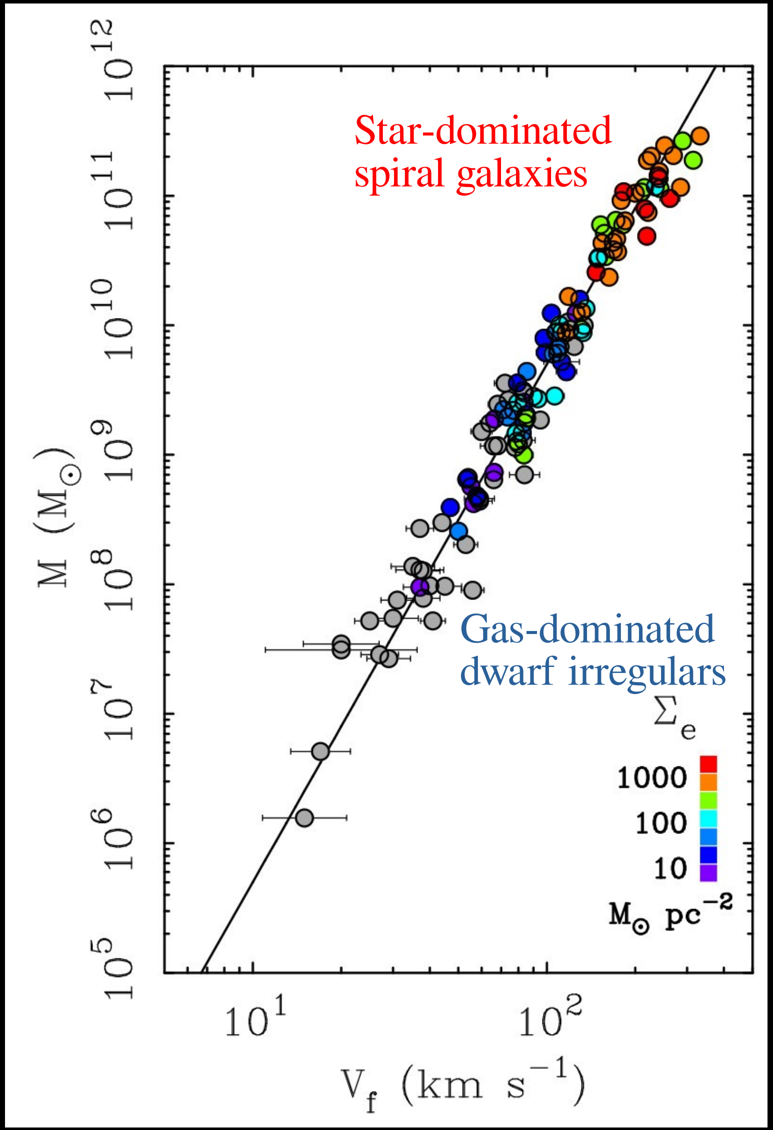

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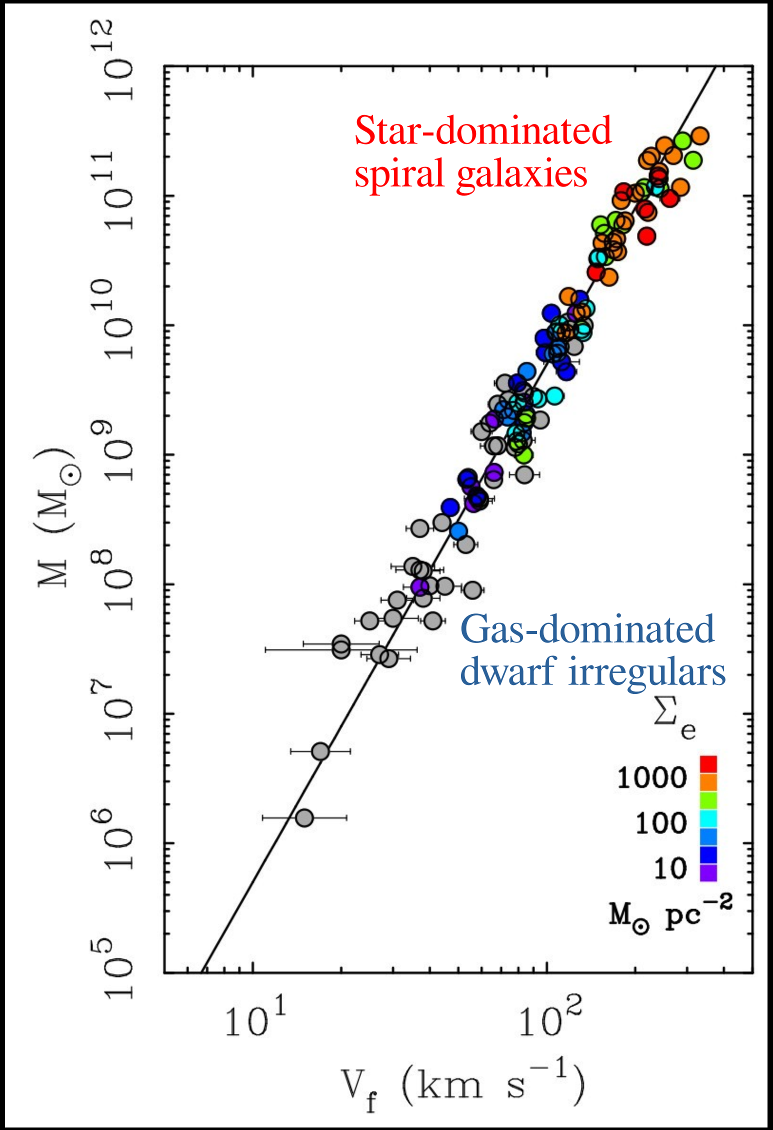

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$$\Sigma_b = \frac{M_b}{\pi R^2} \rightarrow V^4 = \frac{\pi^2 G^2}{f_b^2} \Sigma_b M_b$$

Galaxy disks with different Σ_b should follow different BTFRs (but they don't...)

General MOND predictions (most dating 1983-1984):

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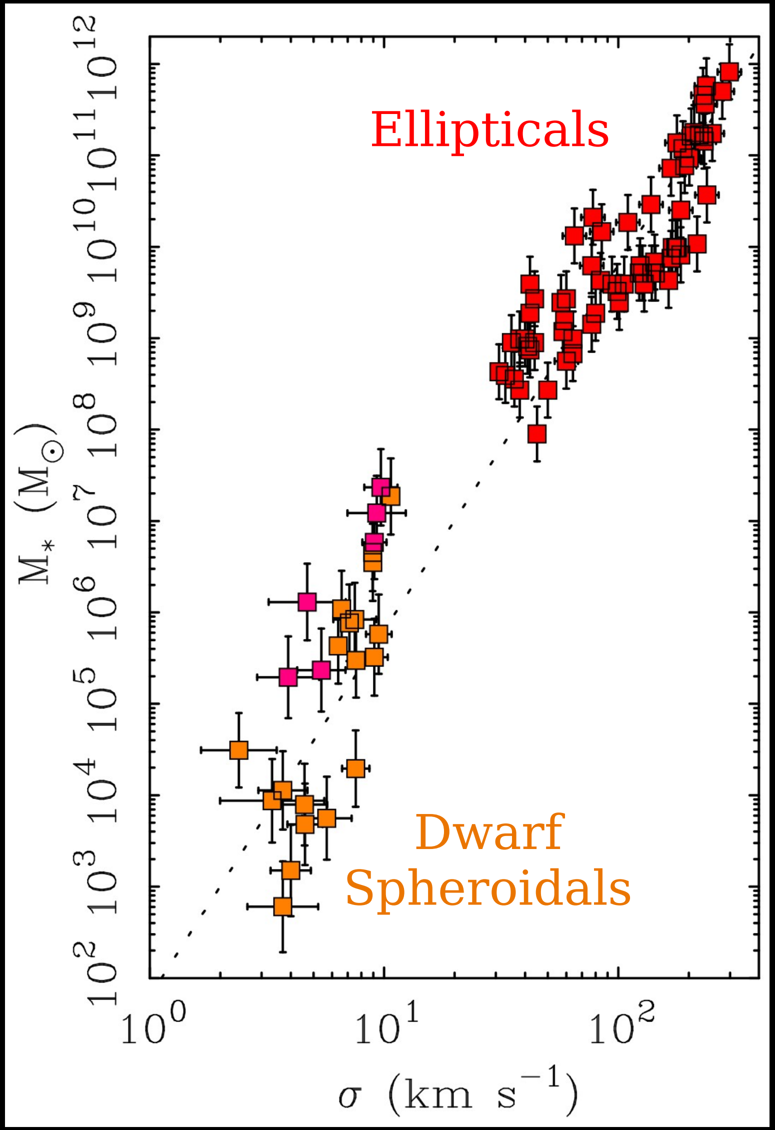

Faber-Jackson (1976) relation for elliptical galaxies

Three predictions in one equation:

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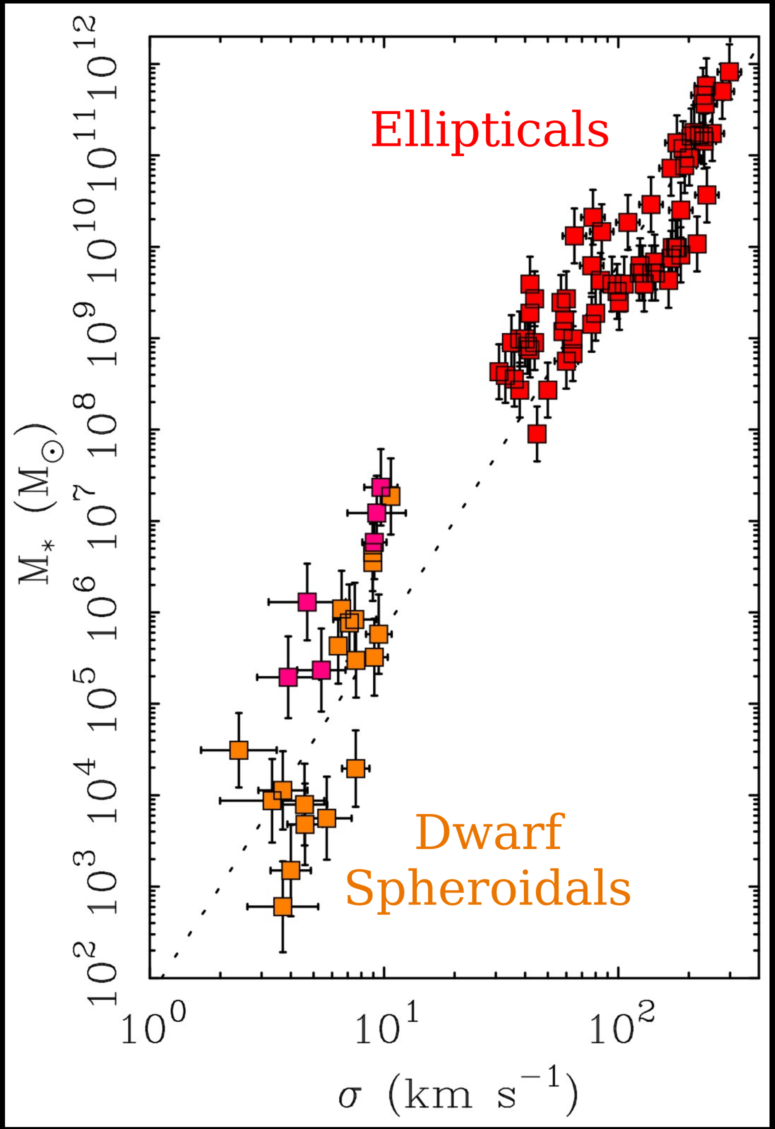

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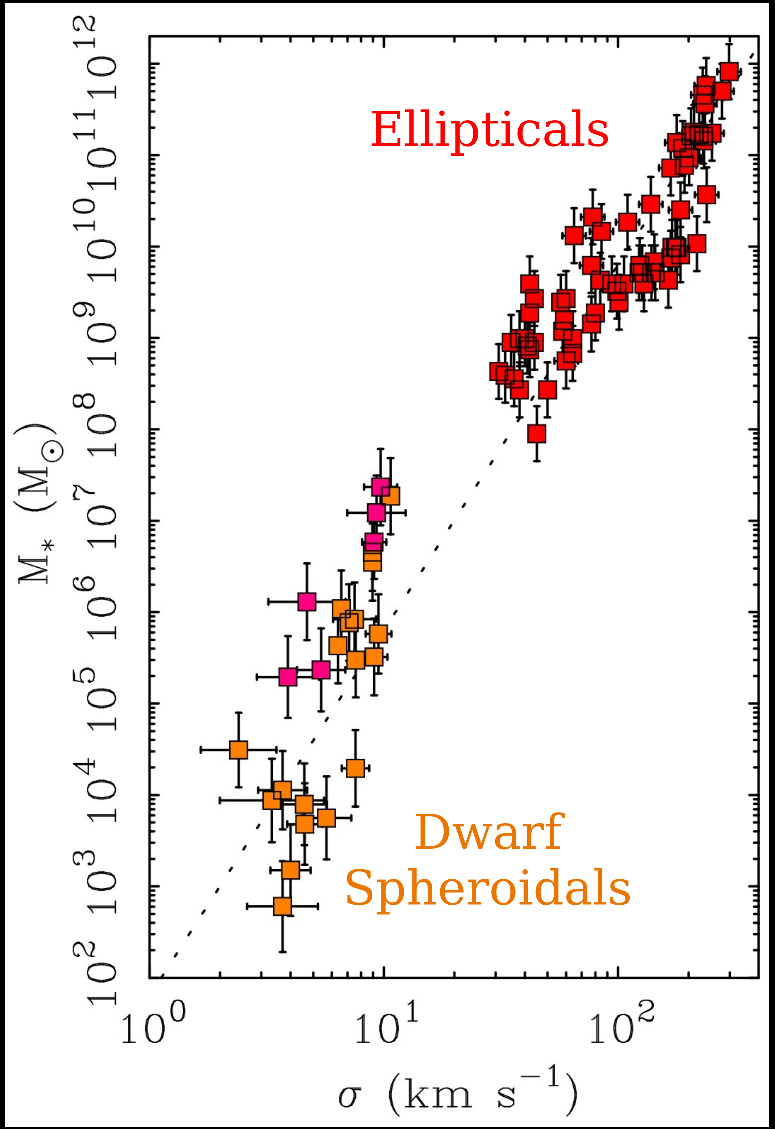

Faber-Jackson (1976) relation for elliptical galaxies

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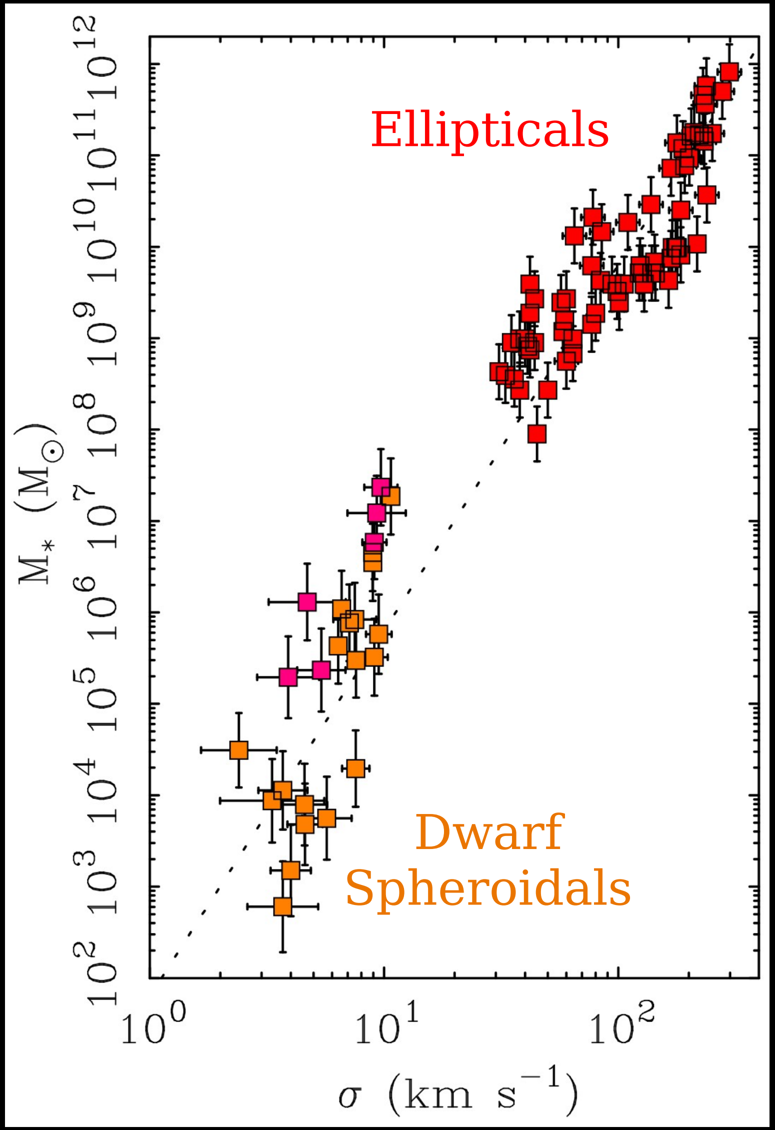

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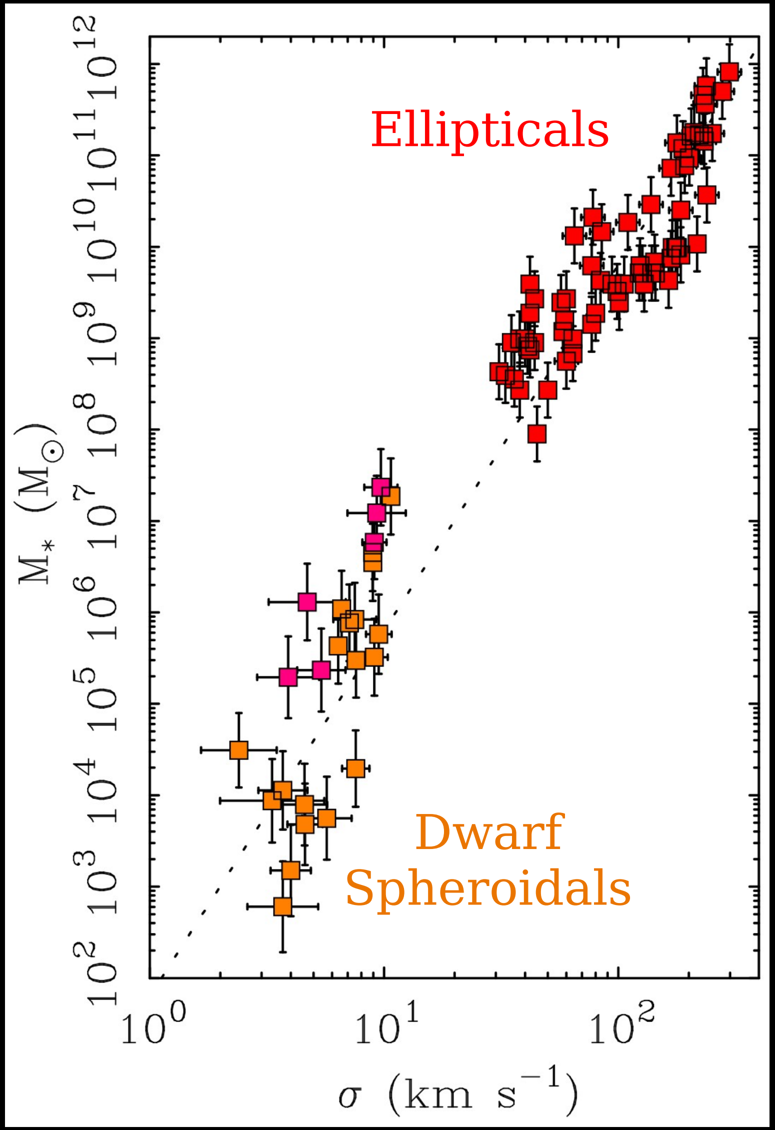

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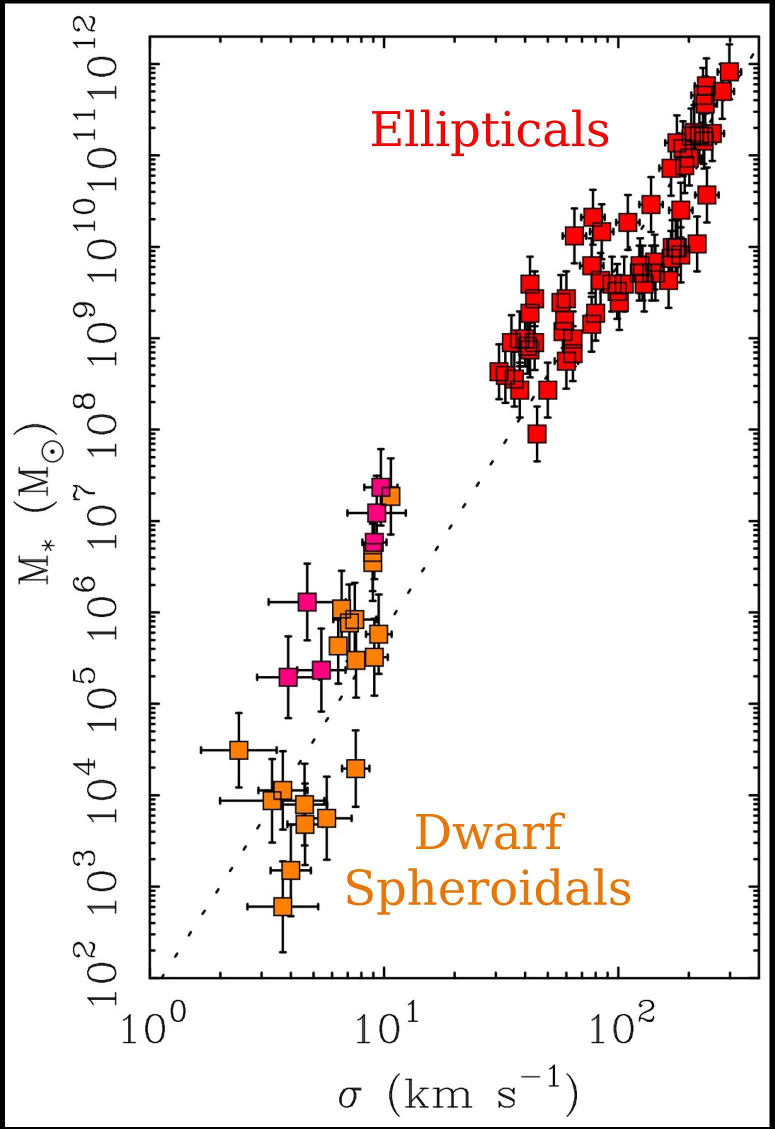

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$$\frac{\sigma_V^2}{R} \sim \frac{G\,M}{R^2} \Rightarrow M \sim \sigma_V^2\, R_e$$

Fundamental plane of ellipticals
(Djorgovski & Davis 1987; Dressler 1987)

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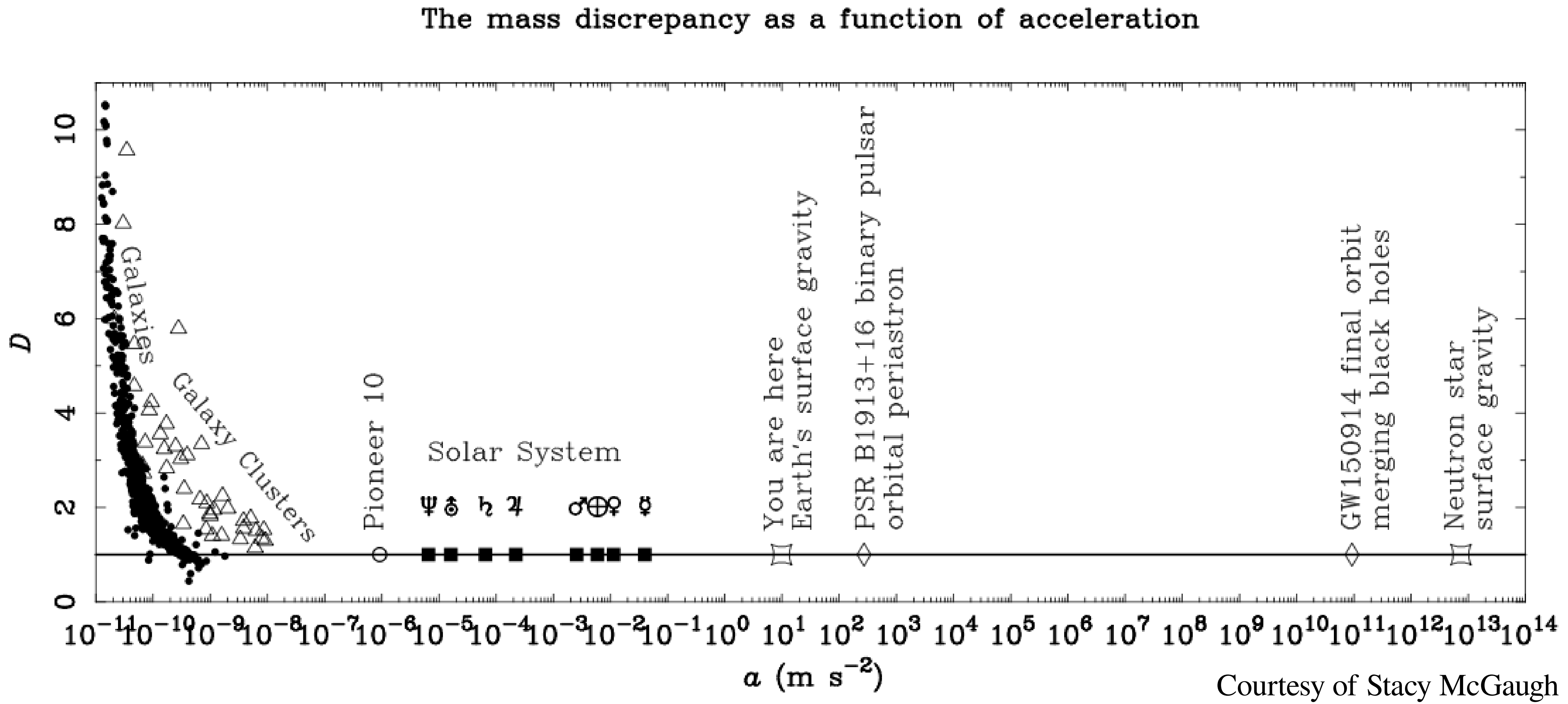
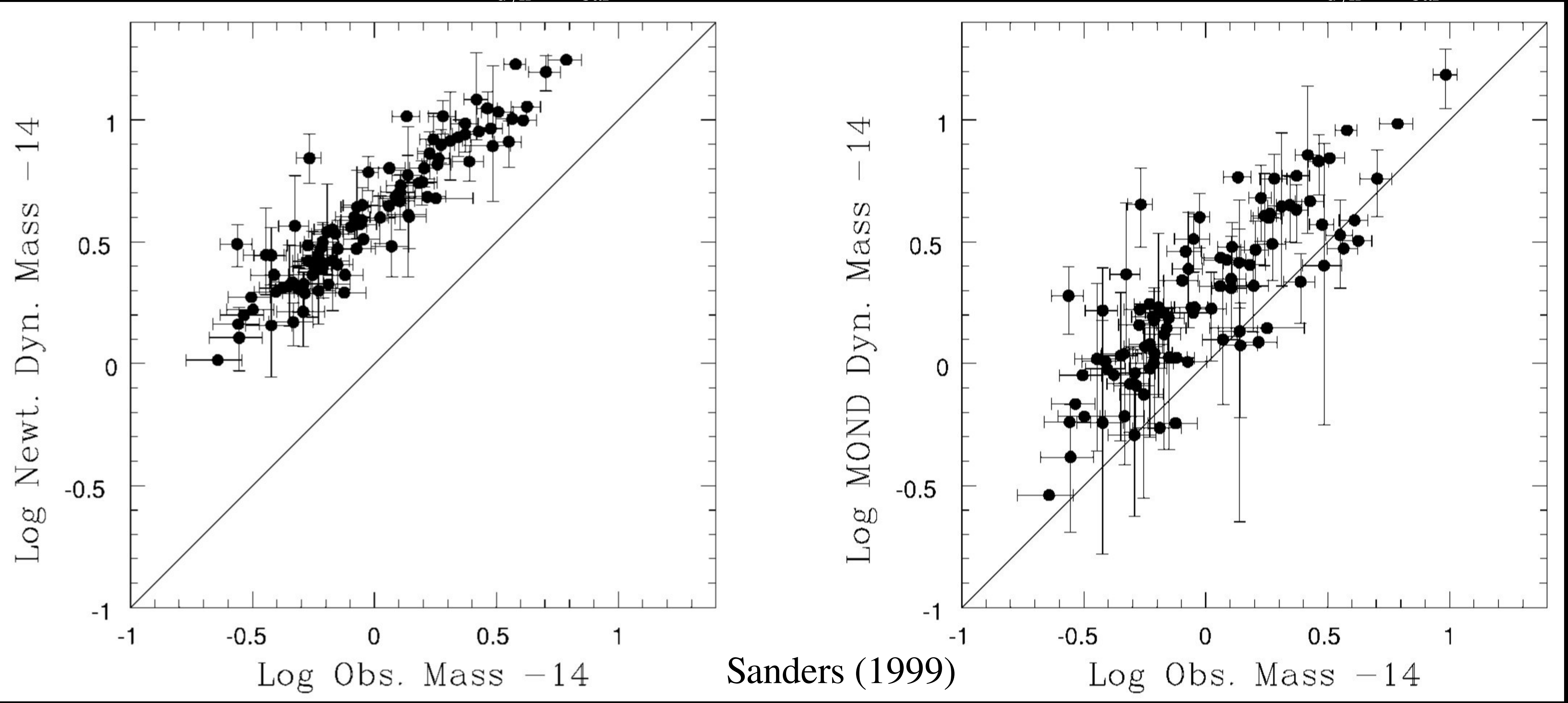


Courtesy of Stacy McGaugh

Problem for MOND: Galaxy Clusters

Newtonian analysis: M_{dyn}/M_{bar}~4-10

MOND analysis: M_{dyn}/M_{bar}~2



Sanders (1999)

Problem for MOND: Galaxy Clusters

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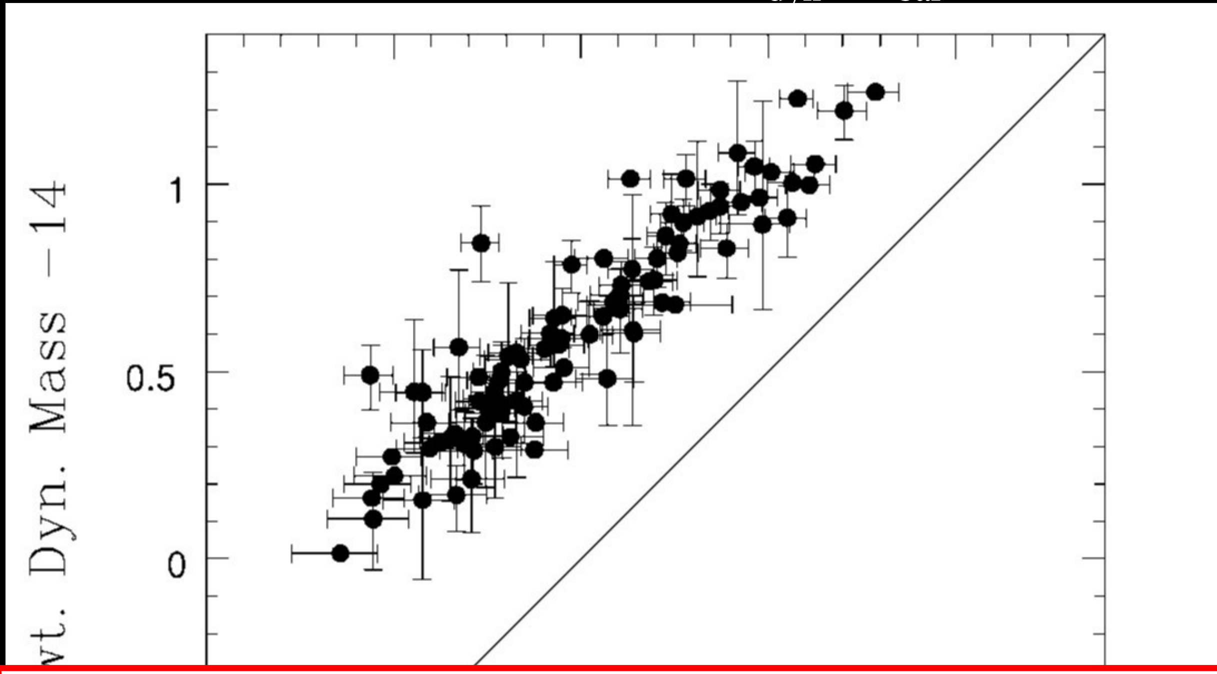

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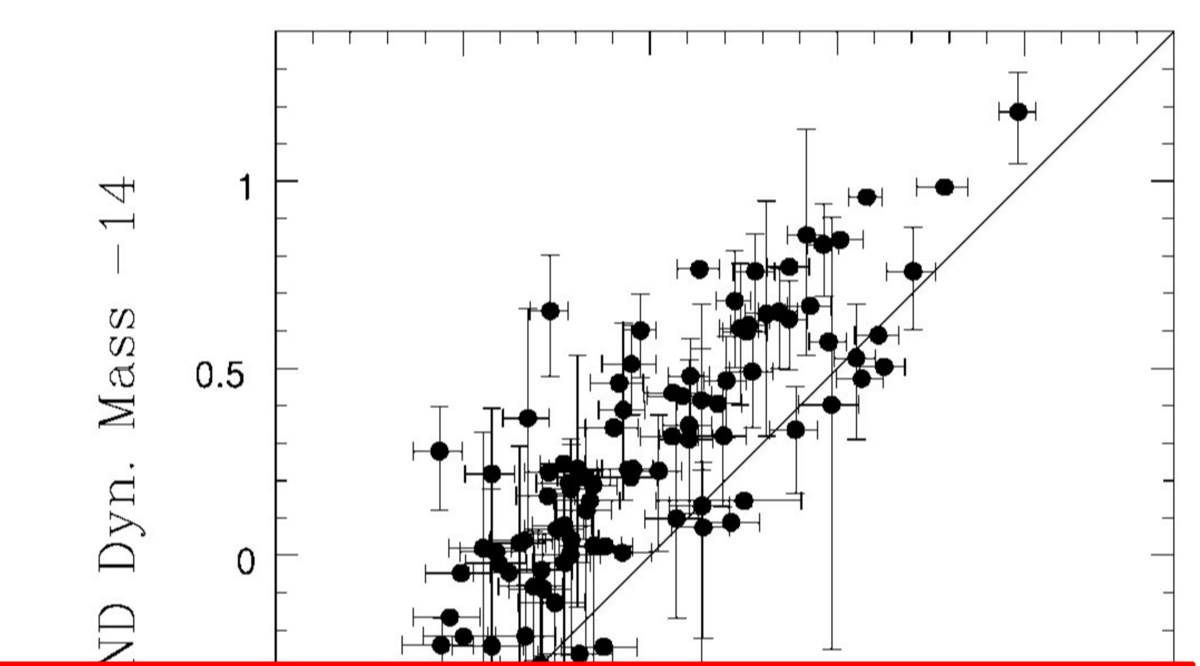

Proposed solutions:

- Undetected baryons (Milgrom 2008) → BBN implies ~30% missing baryons
- Sterile neutrinos with m~10 eV (Angus 2008) → υ oscillations and masses
- Extended MOND: $a_0 \propto \Phi$ (Zhao & Famaey 2012) → deeper theory?

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(4) Rotation curves can be predicted from the baryon distribution

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We need to introduce an interpolation function $\mu(x)$ with $x = a/a_0$:

$$a\mu(x) = g_N \begin{cases} \lim_{x \to \infty} \mu \to 1 \\ \lim_{x \to 0} \mu \to x \end{cases}$$

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Interpolation functions are *very common* in Physics. Examples:

- Lorentz factor γ (via c): Newton's second law ↔ special relativity
- Planck's Law for the blackbody radiation (via $\hbar$): Rayleight-Jeans ↔ Wein regimes
- Probability for quantum tunneling (via $\hbar$): classic mechanics ↔ quantum theory

MOND postulates do NOT specify μ, only asymptotic limits. Which function to choose?

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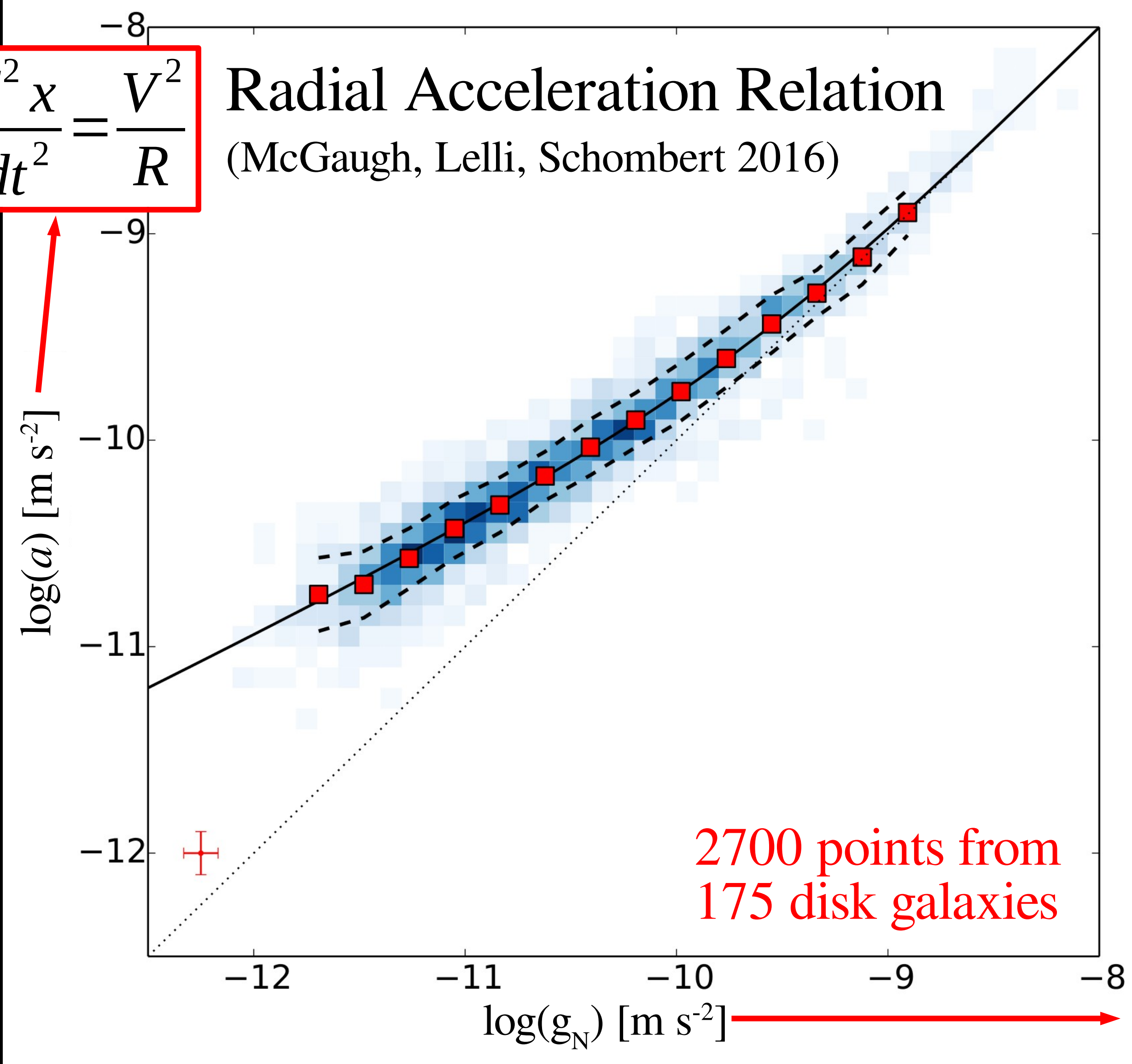
$$\frac{d^2x}{dt^2} = \frac{V^2}{R}$$



$$g_N = -\nabla \Phi_N$$
$$\nabla^2 \Phi_N = 4\pi G \rho_b$$

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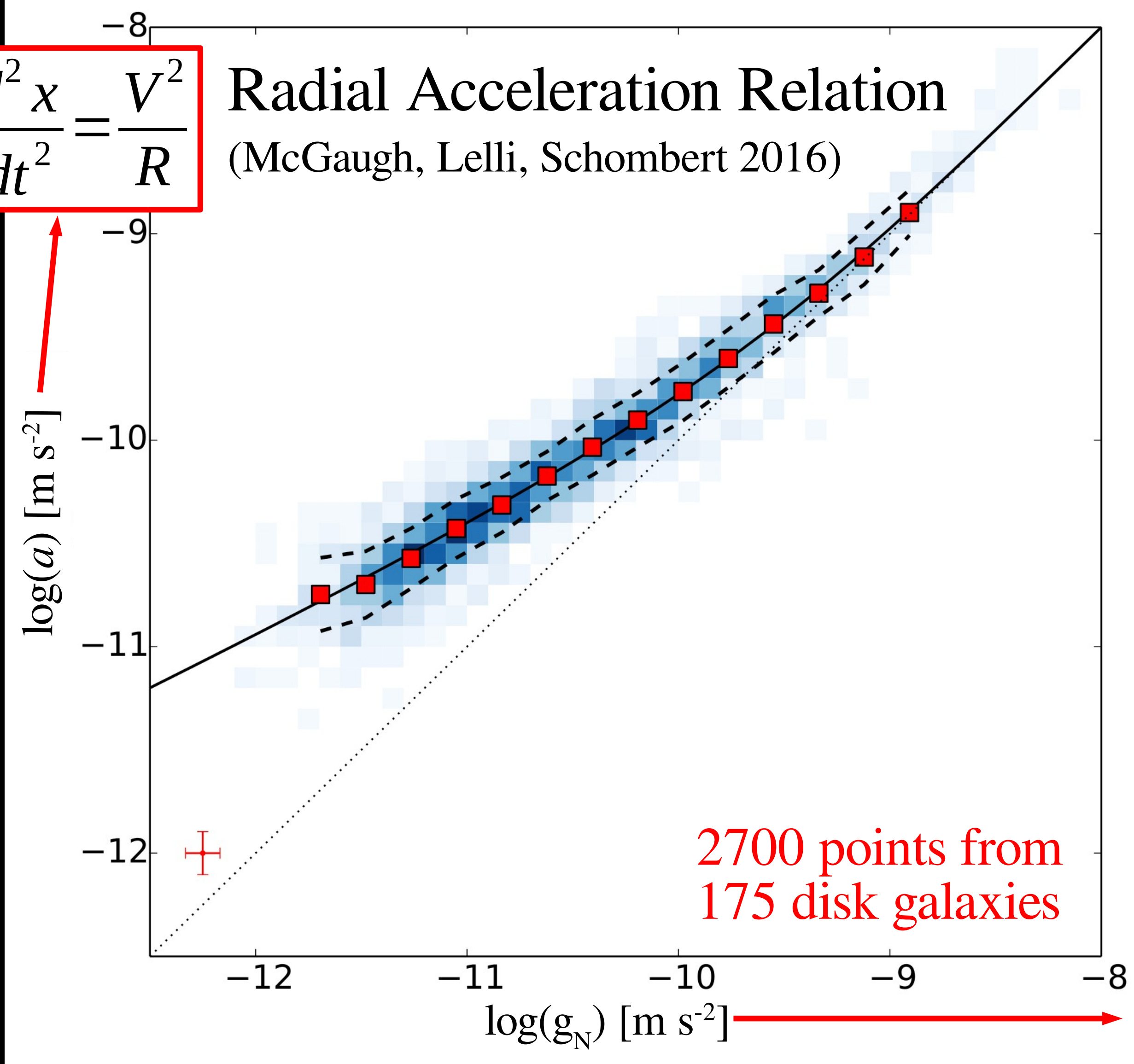

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- Asymptotic limits consistent with MOND
- Baryon distribution (g_N) ↔ rot. curve (a)
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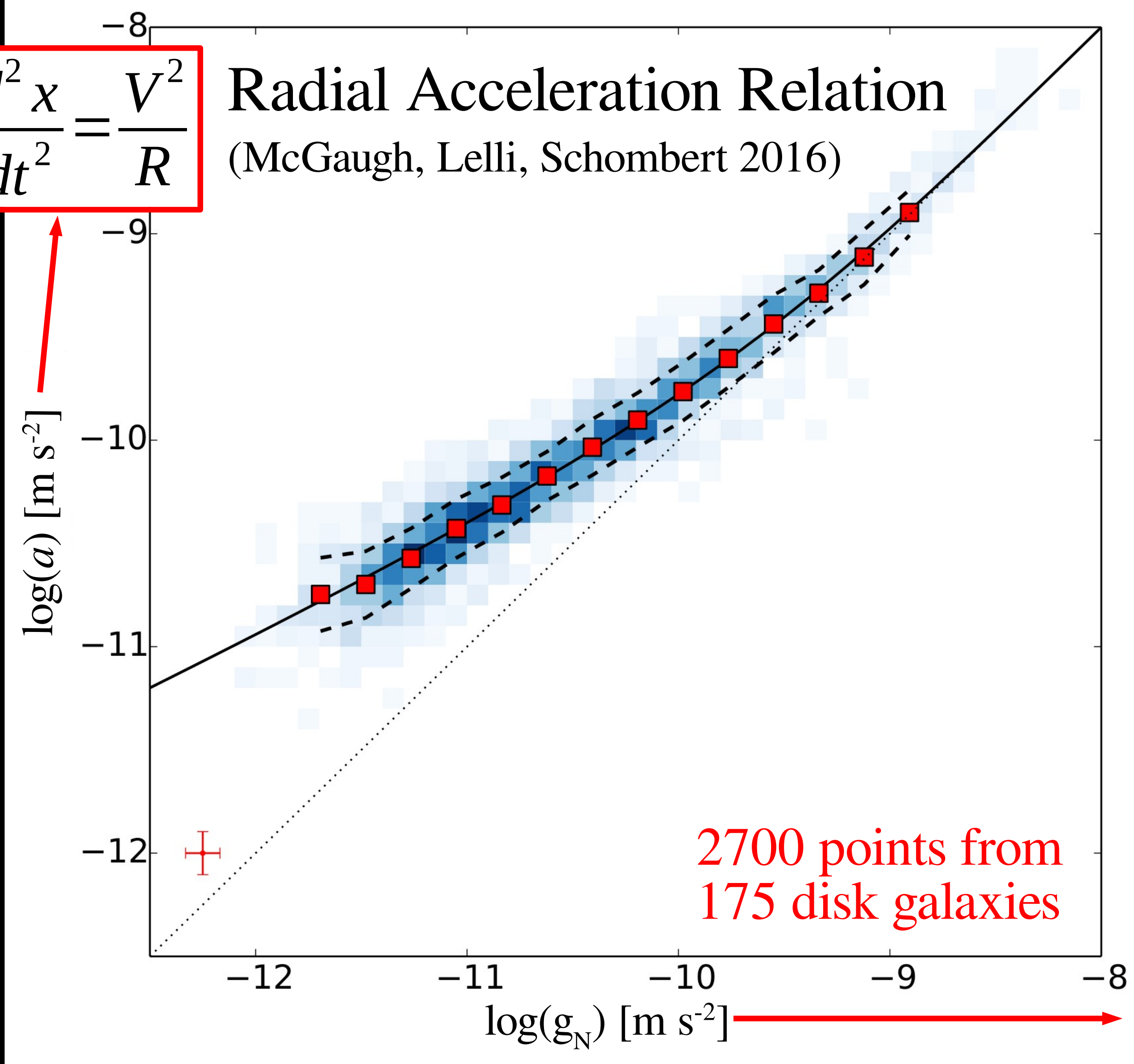

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- The RAR specifies the form of υ and μ

$$a = \nu\left(\frac{g_N}{a_0}\right) g_N \quad \nu = \mu^{-1}$$

We can now assume the RAR and predict rotation curves given ρ_b (within the errors).

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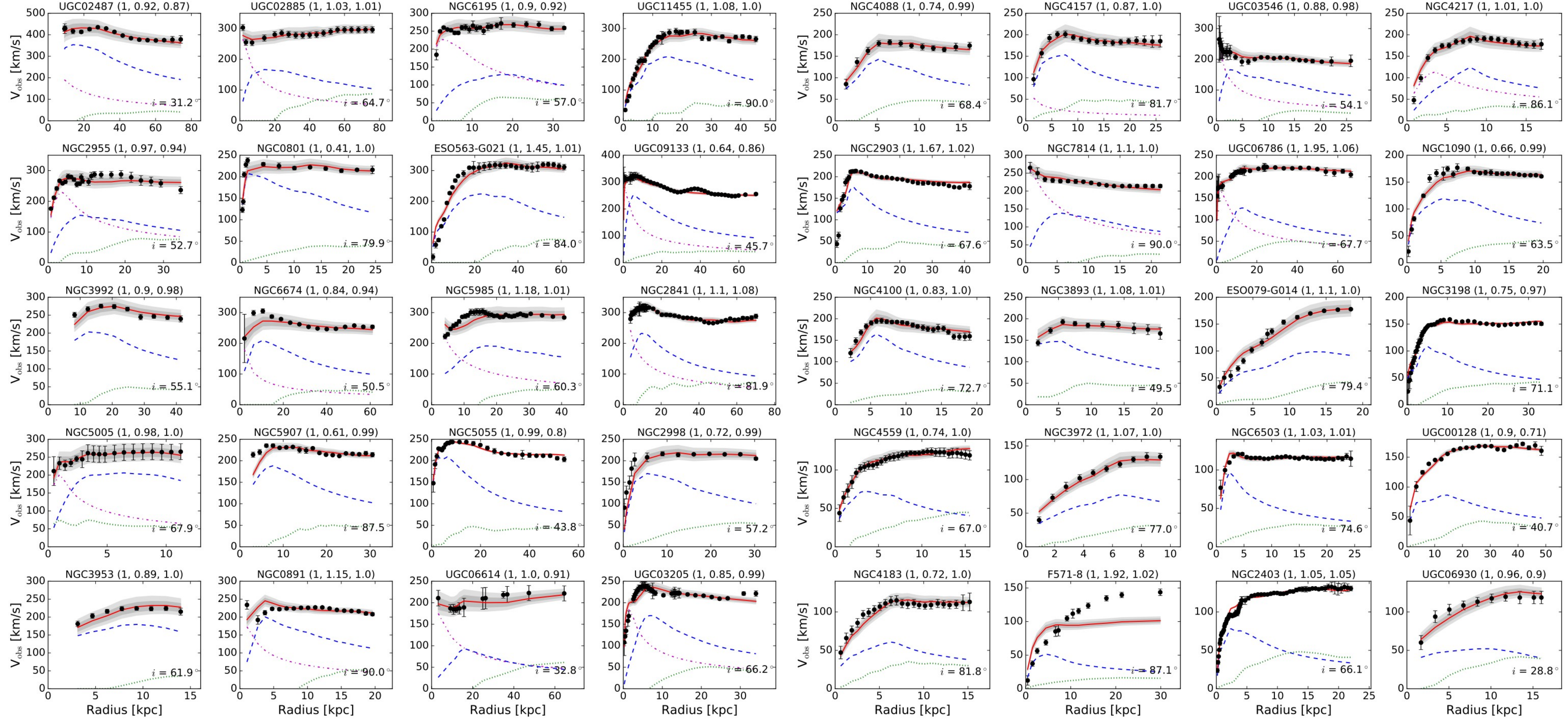

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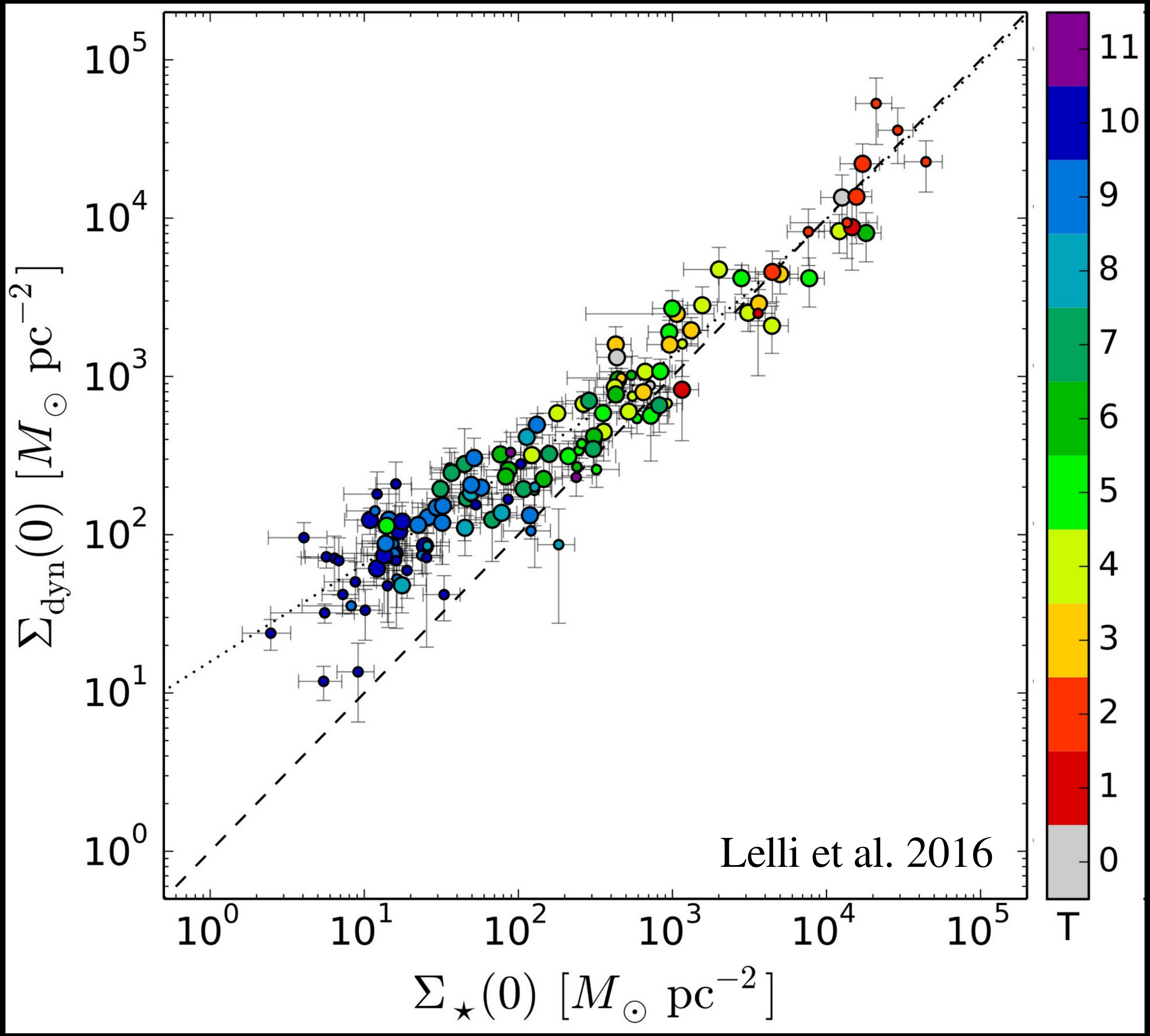

Central Dynamical Surface Density:

$$\Sigma_{dyn}(0) = \frac{1}{2\pi G}\int_0^\infty \frac{V^2}{R^2}\,dR$$

Newtonian formula from Toomre (1963).

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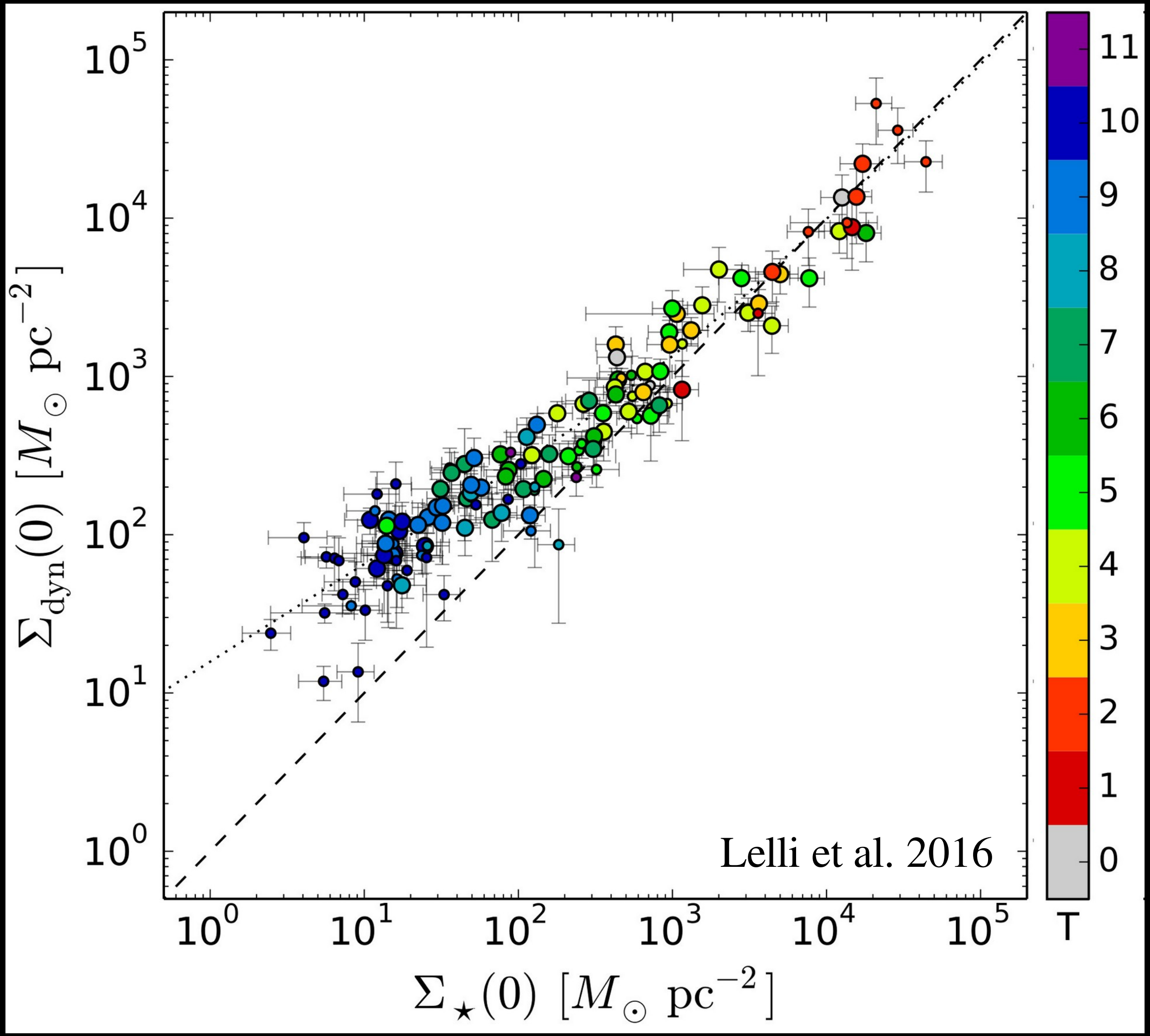

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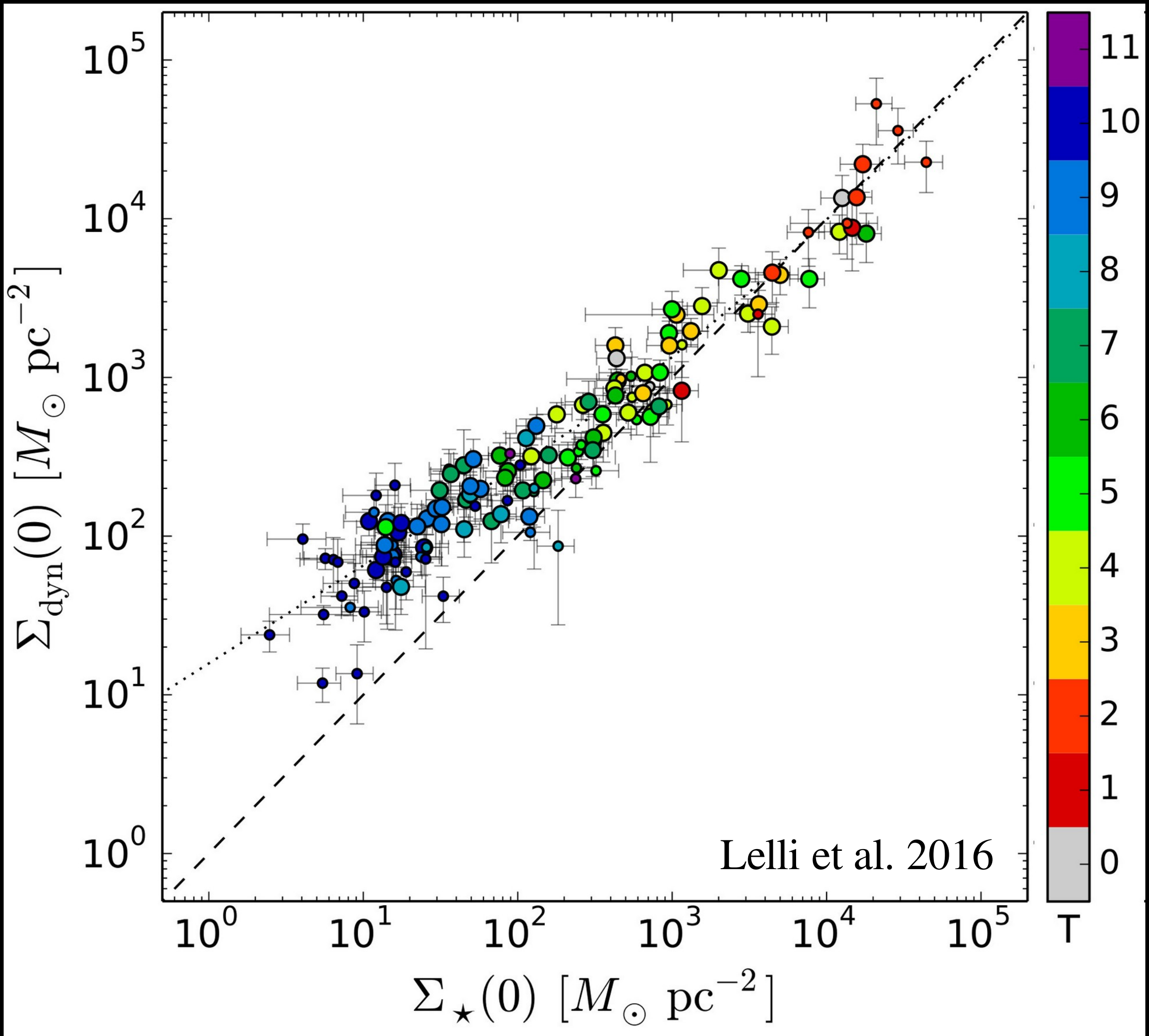

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In MOND, for R→0: $\Sigma_{dyn,0} = S(\Sigma_{b,0}/\Sigma_M)\,\Sigma_{b,0}$

$$S(y) = \int_0^y \nu(x)\,dx$$

Linked with the RAR interpolation function!

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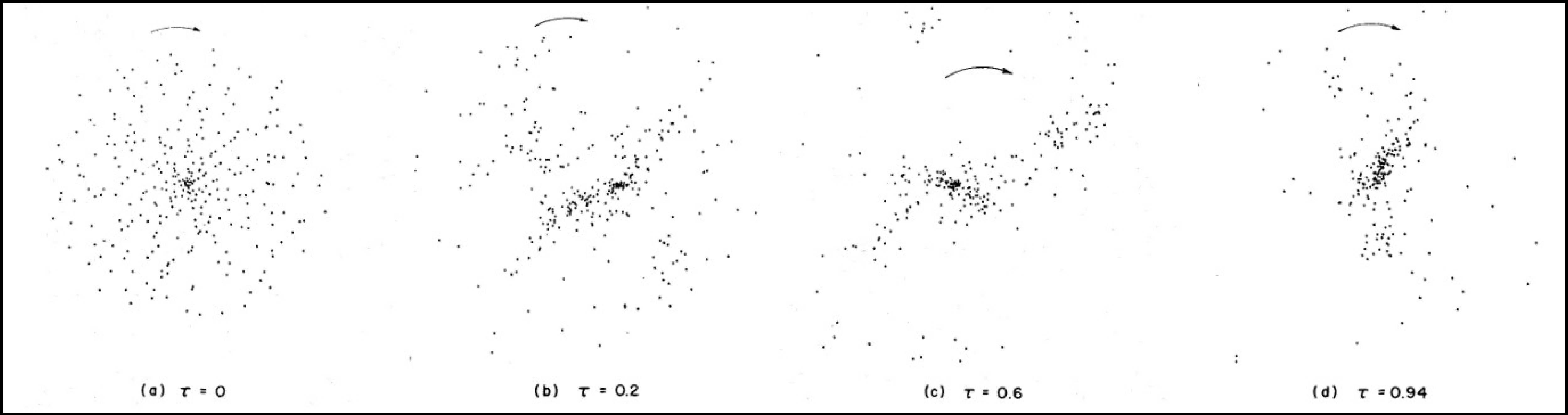
In Newtonian dynamics, self-gravitating stellar disks are unstable:

→ Ostriker & Peebles (1973): Bar instability develops and disk is destroyed

→ Historical reason to introduce spherical DM halos rather than DM disks

→ DM halo stabilizes the disk: bars/spirals cannot form when DM halo dominates



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In MOND, self-gravitating stellar disk are marginally stable:

In the Newtonian regime ($a \gg a_0$): $a \sim \rho R G \Rightarrow \delta a / a \sim \delta\rho/\rho$

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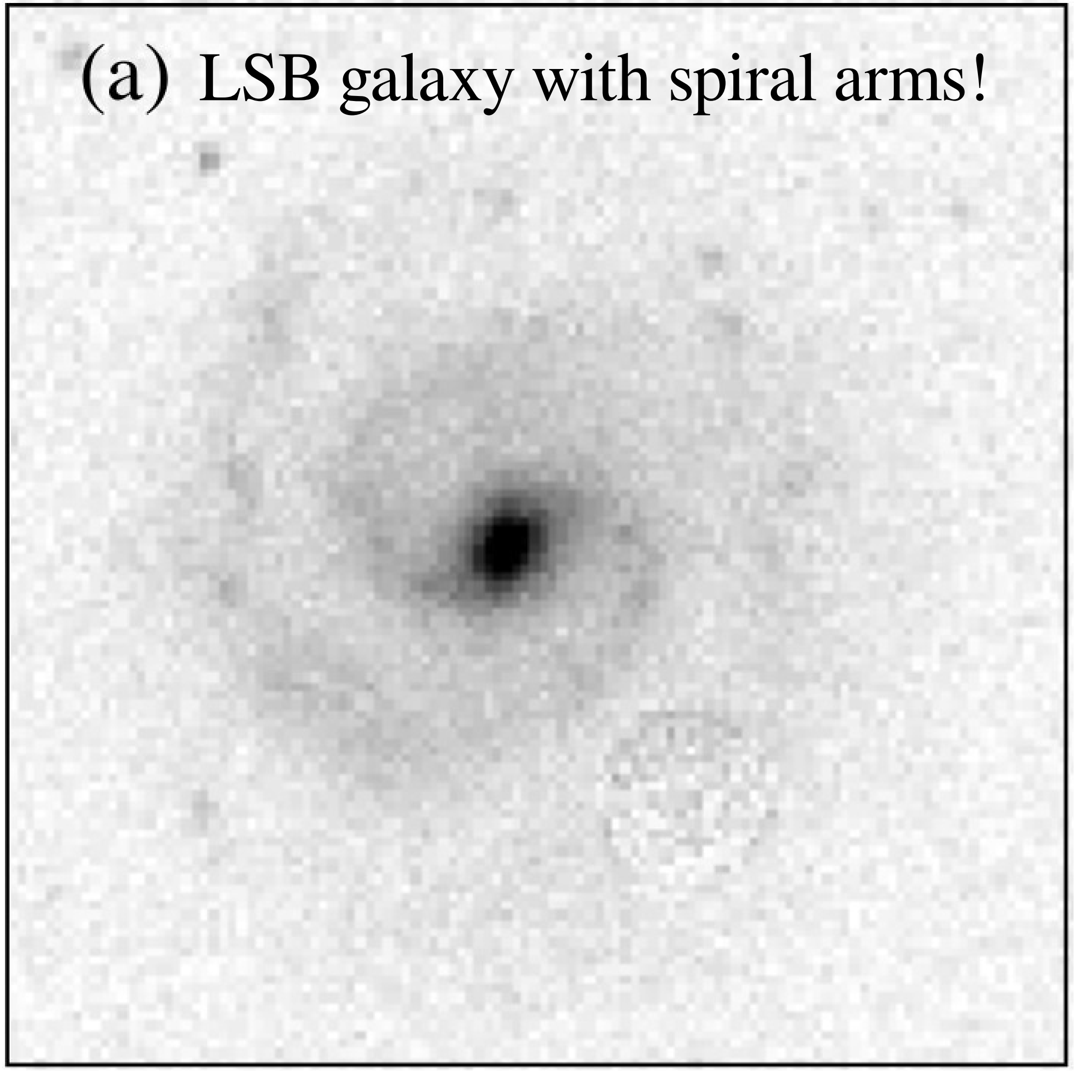
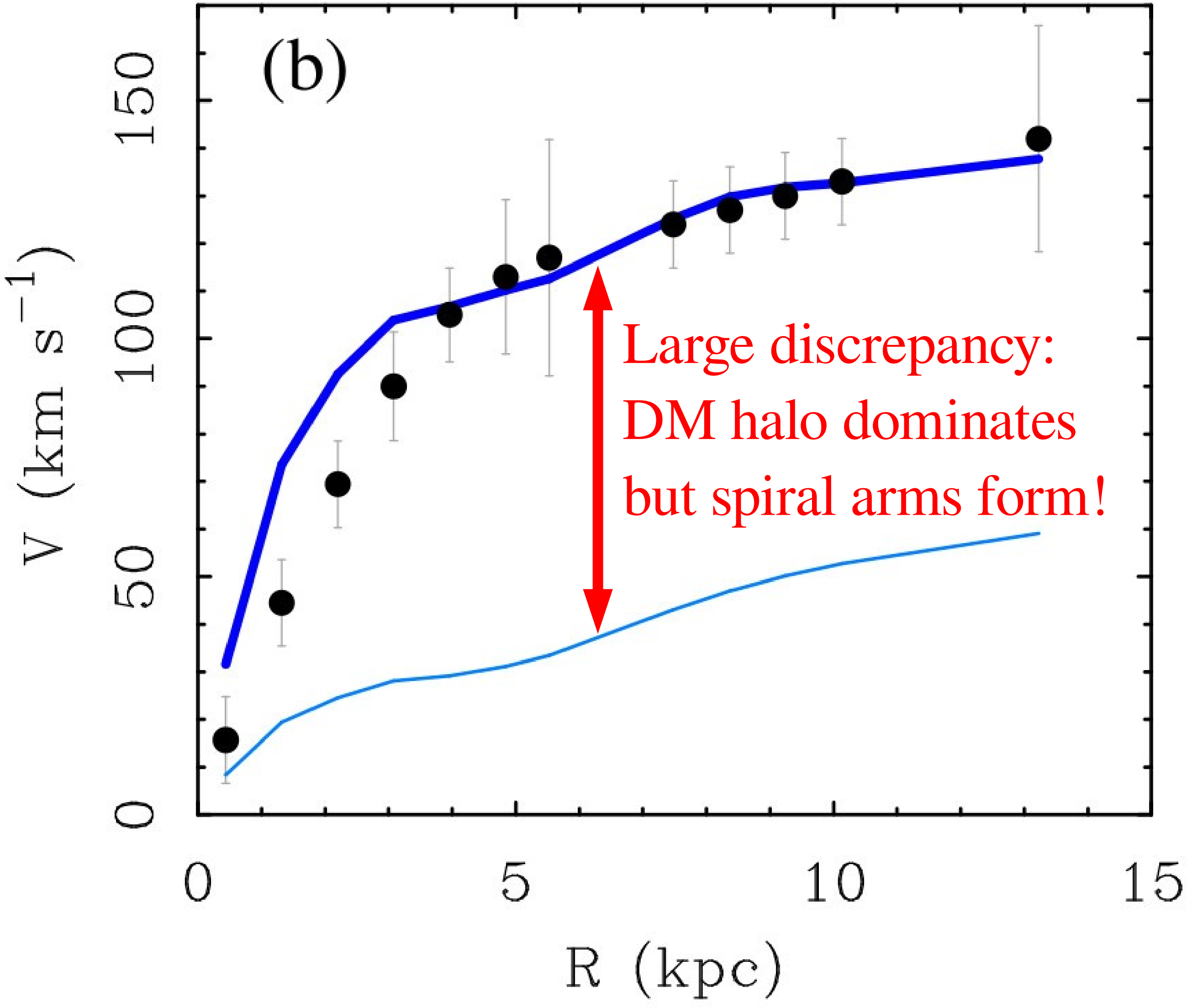
→ In the deep MOND limit, stability does NOT depend on the mass discrepancy.

→ Bars and spiral arms can form in any galaxy under appropriate conditions.

(6) Disk stability is increased and doesn't depend on mass discrepancy





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(6) Disk stability is larger and does NOT dependent on the mass-discrepancy ✓

(7) External field effect: strong equivalence principle is broken →

The external field in which a system is falling affects the internal dynamics

(7) External field effect: strong equivalence principle is violated

- Weak Equivalence Principle (WEP)

  Universality of free-fall: trajectory of an object in a gravitational field is independent of its composition and structure (center-of-mass motion)

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- **Einstein Equivalence Principle (EEP)**

  WEP + Lorentz invariance (same laws under spacetime rotations)
  + Local Position Invariance (LPI) for non-gravitational experiments
  (results of experiments do not depend on where/when they are performed)

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  WEP + Lorentz invariance (same laws under spacetime rotations)
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- **Strong Equivalence Principle (SEP)**

  WEP + Lorentz invariance + LPI for gravitational experiments too

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MOND is non-linear → consider both internal and external accelerations

For non-isolated systems, three possibilities:

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MOND is non-linear → consider both internal and external accelerations

For non-isolated systems, three possibilities:

(1) $g_{\rm N,\,ext} \ll g_{\rm N,\,int} \ll a_0 \rightarrow$ MOND regime

Example: nearly isolated galaxies

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Example: star clusters in the inner MW, low-acc experiments on the Earth

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(3) $g_{N,\,int} \ll g_{N,\,ext} \ll a_0$ → Newton with $G_{eff} \sim G\, a_0/g_{N,\,ext}$

Example: some dwarf satellites around the MW and Andromeda

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EFE is a general prediction but details depend on the specific MOND theory

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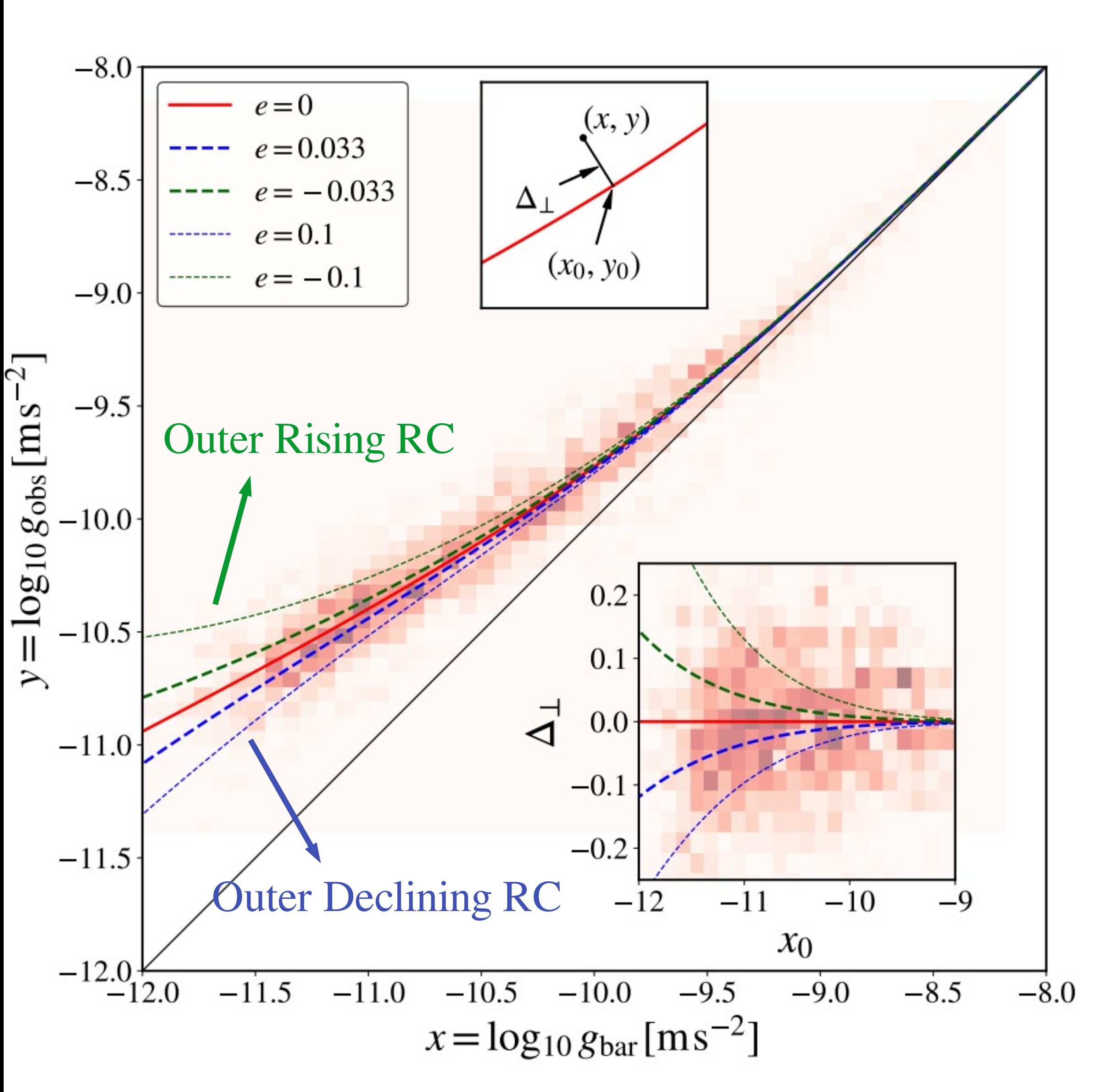

- For truly *isolated* galaxies:

$a = \nu(g_{N,int}/a_0)g_{N,int}$ → flat outer RCs

- For galaxies subjected to $e = g_{ext}/a_0$:

$a = \nu(g_{N,\,int}/a_0; e)g_{N,\,int}$ → declining outer RCs

Chae, Lelli, Desmond et al. 2020, ApJ

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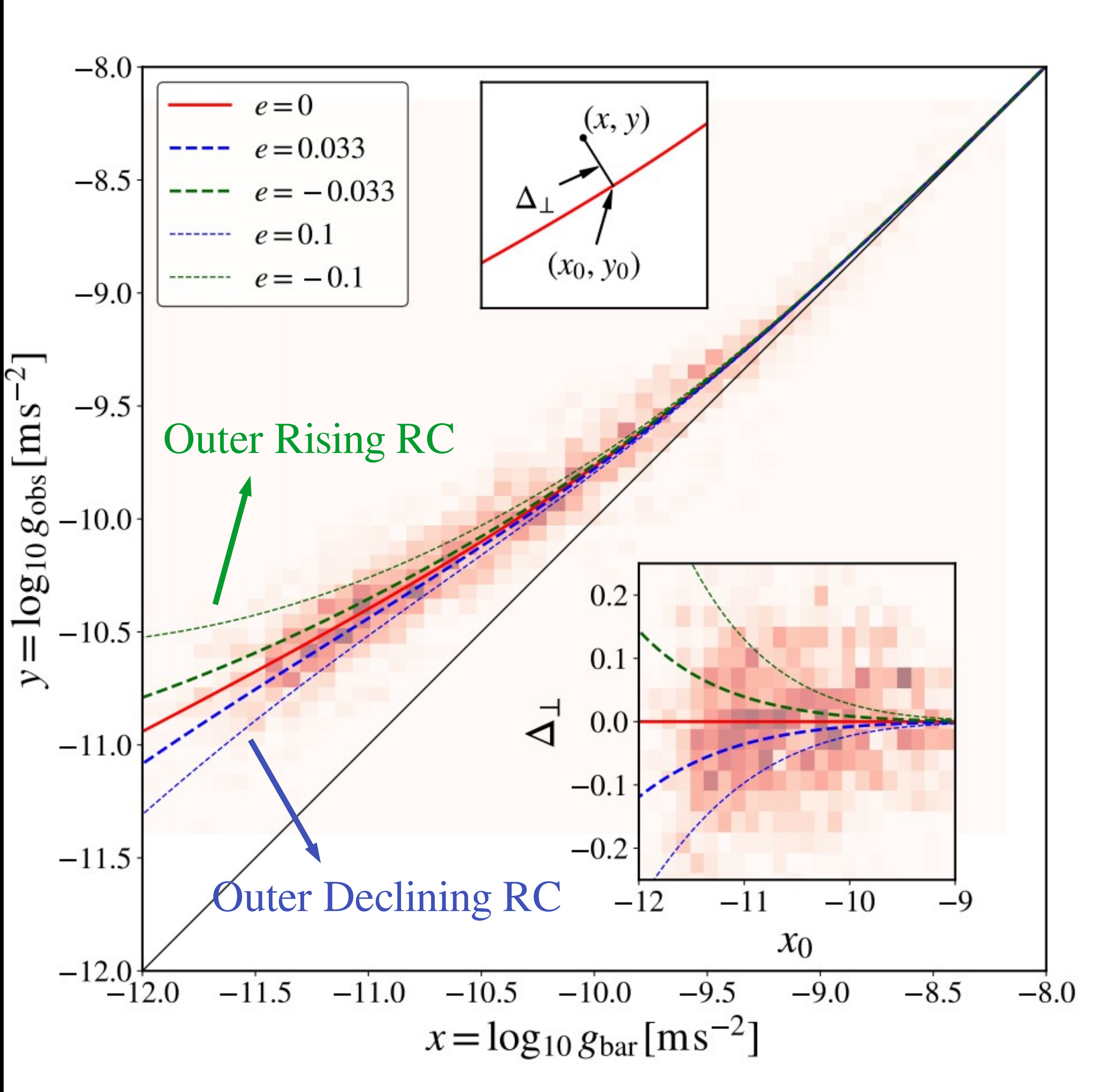

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- The RAR should be a ***family of curves*** depending on the galaxy environment
- We can fit RCs to infer the value of e and independently estimate e_{env} from the large-scale environment of galaxies

Chae, Lelli, Desmond et al. 2020, ApJ

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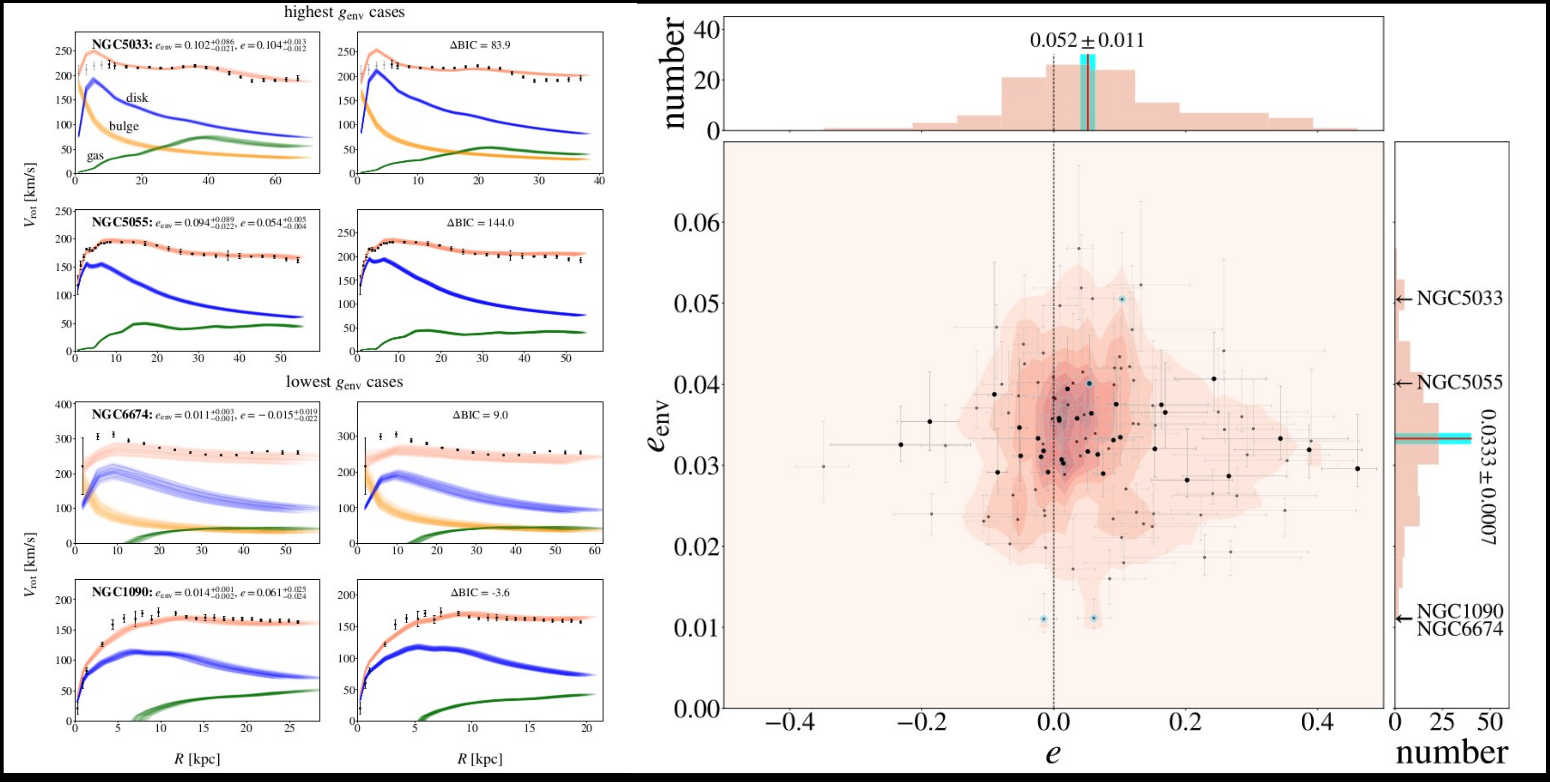

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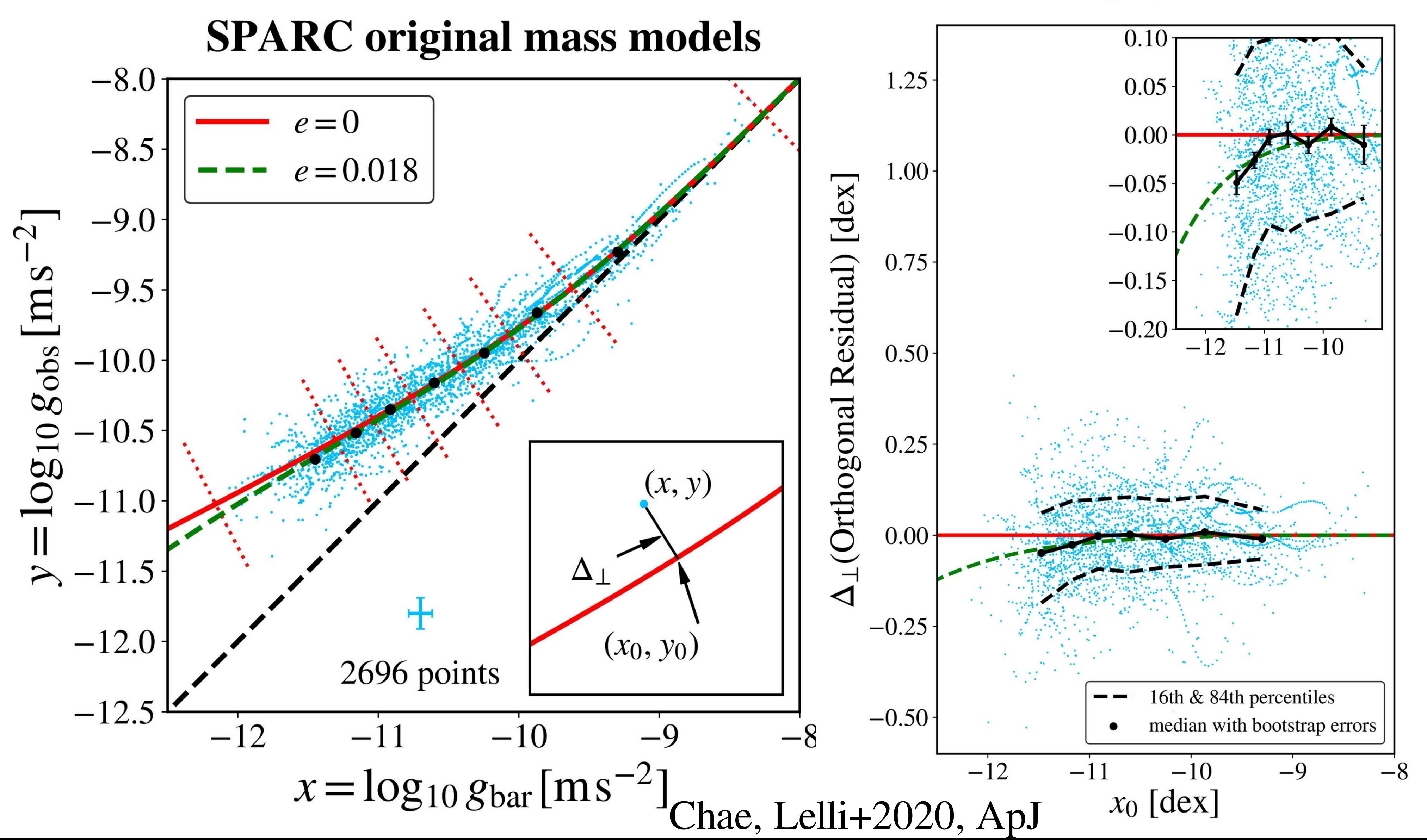

Chae, Lelli+2020, ApJ

Systematic deviation in the low acceleration part of the RAR → consistent with EFE from the average <g_{ext}> of the Local Universe (Chae, Lelli+2020)

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Roadmap of the Lecture

1. The general MOND paradigm

2. Non-relativistic MOND theories

3. Relativistic MOND theories

How do we interpret the MOND phenomenology?

Intuitive/naive way: multiply heuristic MOND relation by $m_i = m_g$ of a test particle

$$\vec{a}\,\mu\left(\frac{a}{a_0}\right) m_i = \vec{g}_N\, m_g$$

Modified inertia? → modify $\bar{F} = m_i \bar{a}$ (Newton's 2nd law)

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Modified gravity? → modify Newton's Gravitational law

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Modified gravity? → modify Newton's Gravitational law

BUT these equations cannot be generally valid. For m_1 & m_2 in the MOND regime:

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How do we interpret the MOND phenomenology?

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This is NOT symmetric in m_1 and m_2: It'd generally violate the Principle of Action & Reaction (Newton's 3rd law) → Linear momentum NOT conserved (we do NOT want this to happen...)

How do we interpret the MOND phenomenology?

The heuristic MOND law must emerge from a general theory in specific situations.

Let's consider the non-relativistic Newtonian Action:

$$S = \int dt\, L = \int dt \left(L_{matter} + L_{gravity} + L_{coupling} \right) = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8 \pi G} - \rho \Phi \right)$$

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Emmy Noether's Theorem:

Symmetry in $S \leftrightarrow$ conservation law

$t \to t + \Delta t$ Time translations: Total Energy

$\bar{x} \to \bar{x} + \Delta\bar{x}$ Space translations: Linear momentum

$\bar{x} \to \mathrm{R}\bar{x}$ Space rotations: Angular momentum

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Principle of Least Action.

(Ex. 4: derive the Eq. of motion):

$$\frac{\delta S}{\delta \Phi} = 0 \rightarrow \nabla^2 \Phi = 4\pi G \rho$$ Poisson's equation

$$\frac{\delta S}{\delta \vec{x}} = 0 \rightarrow \vec{a} = -\vec{\nabla}\Phi$$ Newton's 2nd Law

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Change this for modified inertia

Change this for modified gravity

Changing this modify both

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Three non-relativistic MOND theories:

(1) Modified Inertia (Milgrom 1994, 1999)

→ interesting but poorly developed: only a few general results

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(3) Quasi-linear Mod. Gravity: QUMOND (Milgrom 2010)

→ well developed & allows for easier numerical calculations

→ relativistic extension: BiMOND (Milgrom 2009, 2010)

(1) MOND as Modified Inertia

$$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla}\Phi_N$$

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$$\begin{cases} \gamma^3\left(\frac{v}{c}\right) a_x = \frac{F_x}{m} \\ \gamma\left(\frac{v}{c}\right) a_y = \frac{F_y}{m} \\ \gamma\left(\frac{v}{c}\right) a_z = \frac{F_z}{m} \end{cases} \Rightarrow \quad \vec{A}[\vec{x}(t); c] = F\left(\frac{d^i \vec{x}}{dt^i} \text{ for } i = 1, 2; c\right) = F(v, a; c)$$

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In MOND no full theory yet setting A from varying S but two general results (Milgrom 1994):

(A) IF we impose the Newtonian and MOND limits at high and low accelerations + Galilei Invariance → Eq. of motions are the same in all inertial frames: $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

Theory is time non-local: $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i\vec{x}}{dt^i}; i=1,2,\ldots N\right)$

Accelerations at $(\bar{x}, t)$ depend on the full orbital history!

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Accelerations at ($\overline{x}$, t) depend on the full orbital history!

(B) For purely circular orbits: $\vec{a}\,\mu\left(\frac{a}{a_0}\right) = \vec{g}_N$ holds exactly (e.g. RAR for disk galaxies)

The interpolation function is a derived concept valid for circular orbits.

Hints for building a modified inertia theory

Two remarkable numerical coincidences (Milgrom 1983a, Milgrom 1999):

$$a_0 \sim \frac{H_0 \cdot c}{2\pi}$$

H_0 = Hubble constant → maybe $a_0(t) \sim H(t)$???

$$a_0 \sim \frac{c^2 \sqrt{\Lambda/3}}{2\pi}$$

Λ = Cosmological constant → relation to Dark Energy???

IF this numerology has some deeper, fundamental meaning:
either the state of the Universe at large enters in local dynamics,
or the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

(2) MOND as Non-Linear Modified Gravity

$$S = \int dt\, L = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Lagrangian is quadratic in ∇Φ → standard Poisson's equation

$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla}\Phi|^2}{a_0^2} \right)$$

AQUAL (AQUAdratic Lagrangian)
Bekenstein & Milgrom (1984)

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AQUAL (AQUAdratic Lagrangian) Bekenstein & Milgrom (1984)

Principle of least Action: $\nabla \cdot \left[\mu\left(\frac{|\vec{\nabla}\Phi|}{a_0} \right) \vec{\nabla}\Phi \right] = 4\pi G \rho$ Modified Poisson's Equation

$$\mu(\sqrt{x}) = \frac{dF(x)}{dx} \qquad x = \frac{|\vec{\nabla}\Phi|^2}{a_0^2}$$

F is a free function (new degree of freedom) in *L* that is linked to the interpolation function μ or υ.

(2) MOND as Non-Linear Modified Gravity

$$\nabla \cdot \left[\mu\left(\frac{|\vec{\nabla}\Phi|}{a_0} \right) \vec{\nabla}\Phi \right] = 4\pi G \rho \quad \Rightarrow \quad \vec{a} = \nu\left(\frac{g_N}{a_0}\right)\vec{g_N}$$

in spherical symmetry only!

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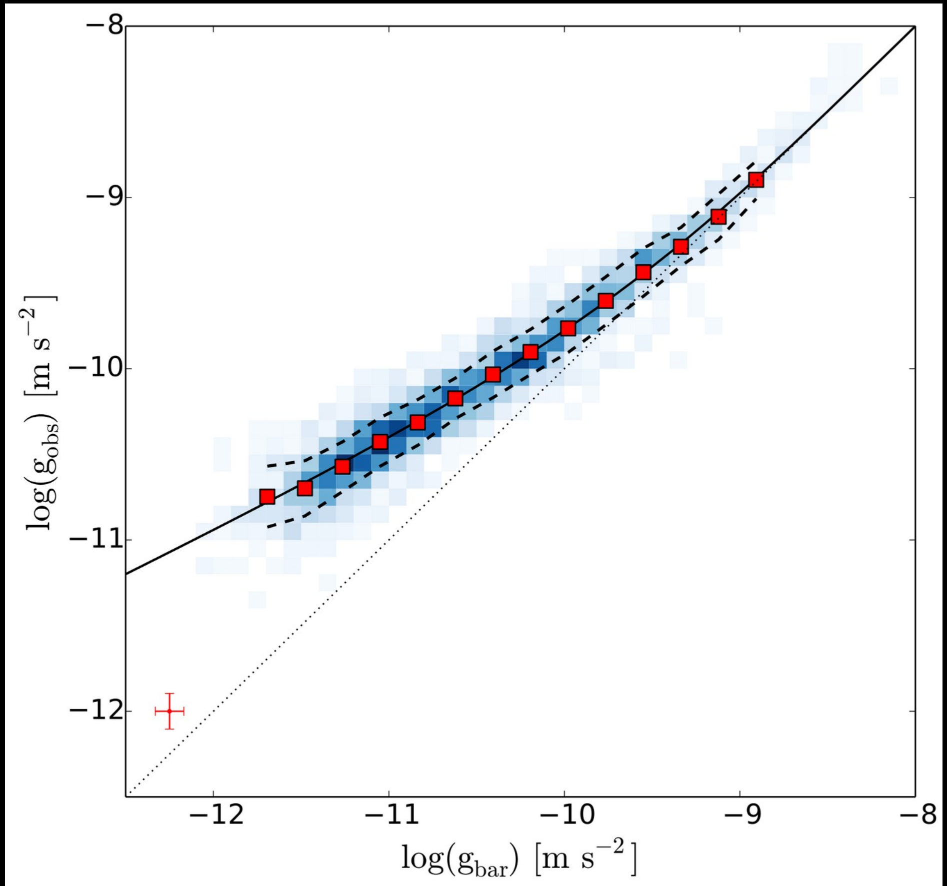
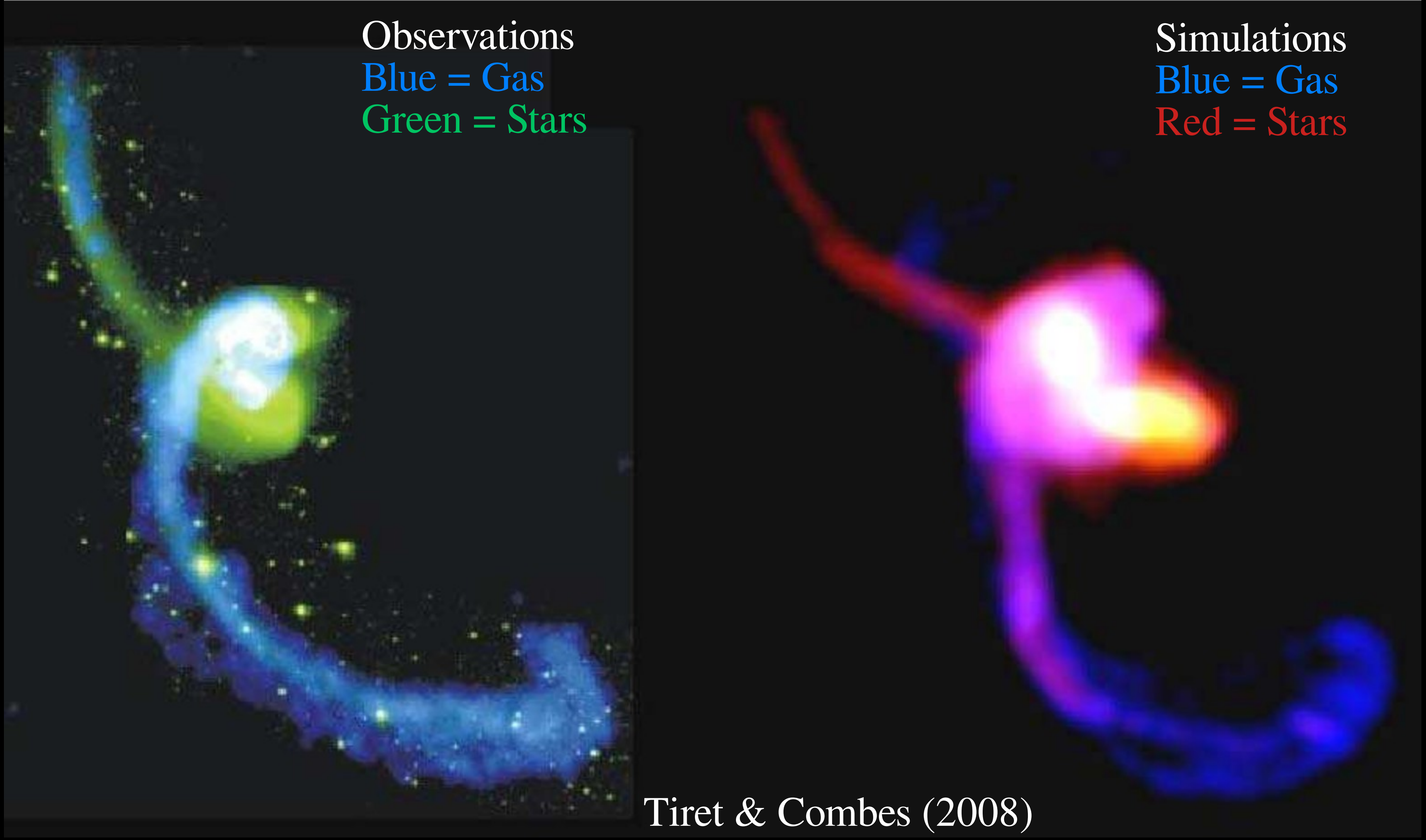
Important observational implications:

If MOND is due to modified gravity, the RAR of disk galaxies (which aren't spherical) must be an approximate relation with intrinsic scatter.

If MOND is due to modified inertia, the RAR of disk galaxies holds exactly (circular orbits).



Application of AQUAL: The Antennae Merger



(3) MOND as Quasi-Linear Modified Gravity

$$S = \int dt\, L = \int dt\, d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla}\Phi|^2}{8\pi G} - \rho\Phi \right)$$

Single gravitational potential Φ

$$\downarrow$$

$$\frac{-1}{8\pi G}\left[2\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi_N - a_0^2 Q\left(\frac{|\vec{\nabla}\Phi_N|^2}{a_0^2} \right) \right]$$

Two potentials: Φ and Φ_N!

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Two potentials: Φ and Φ_N!

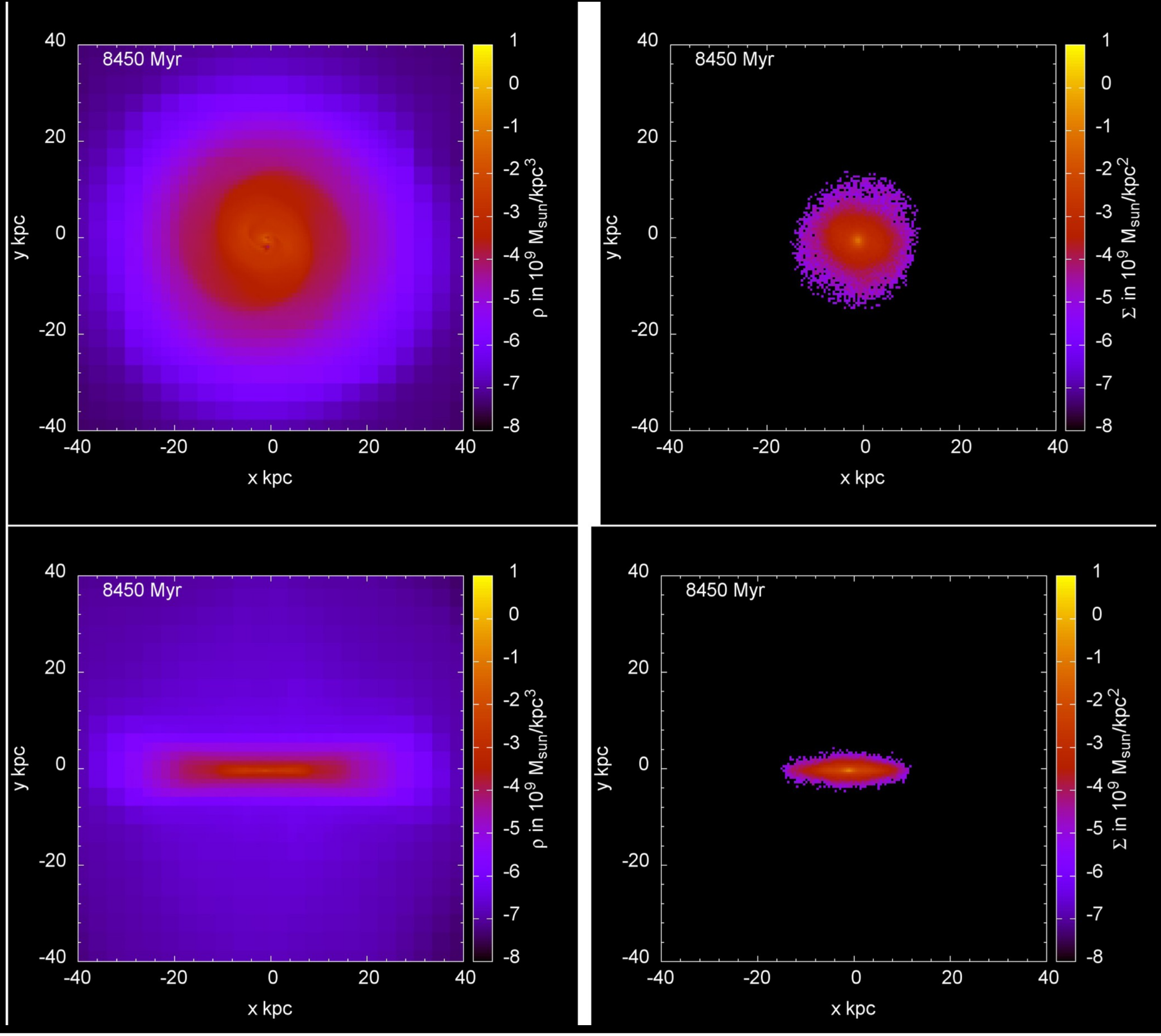
Principle of least Action varying Φ, Φ_N and $\bar{x}$ → set of 3 equations (Milgrom 2010)

$\nabla^2 \Phi_N = 4\pi G \rho$ ⟶ Standard, linear Poisson's equation for Φ_N

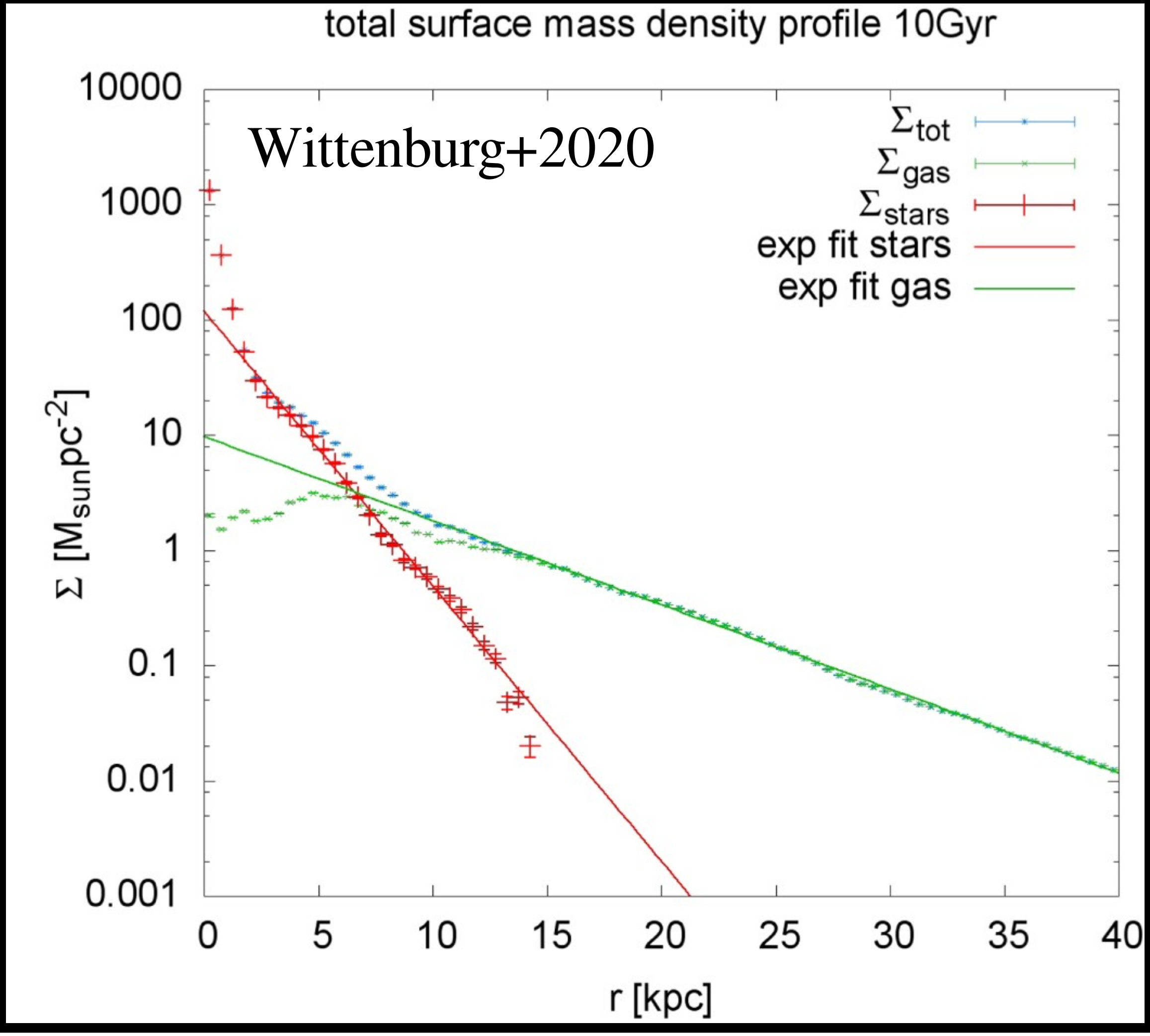
$\nabla^2 \Phi = \vec{\nabla} \cdot \left[\nu\left(|\vec{\nabla}\Phi_N| / a_0 \right) \vec{\nabla}\Phi_N \right]$ ⟶ Non-linear step: get Φ from Φ_N $\quad \nu(\sqrt{x}) = \frac{dQ(x)}{x}$

$\vec{a} = -\vec{\nabla}\Phi$ ⟶ Acceleration/force set by second potential Φ

Application of QUMOND: Formation of Galaxy Disks



Gas collapse → Exponential disk



Summary on non-relativistic MOND theories:

(1) Modified Inertia (Milgrom 1994, 1999)

→ $a = \upsilon(g_N/a_0)g_N$ holds for circular orbits only (for any geometry)

→ No calculations possible beyond circular orbits (so far)

(2) Non-linear Mod. Gravity: AQUAL (Bekenstein & Milgrom 1984)

→ $a = \upsilon(g_N/a_0)g_N$ applies in spherical symmetry (for any orbit)

→ Numerical simulations on binary galaxies → interactions & mergers

(3) Quasi-linear Mod. Gravity: QUMOND (Milgrom 2010)

→ $a = \upsilon(g_N/a_0)g_N$ applies in spherical symmetry (for any orbit)

→ Full hydrodynamical simulations of galaxy formation!

Roadmap of the Lecture

1. The general MOND paradigm
2. Non-relativistic MOND theories
3. Relativistic MOND theories

Lovelock-Grigore Theorem:

GR (+Λ) is the only theory that satisfy these assumptions:

1- Geometry is Reimannian

2- The Action depends only on $g_{\mu\upsilon}$

3- It is diffeomorphism invariant

4- It is local

5- It leads to 2^{nd} order field equations

Many ways to build a relativistic version of MOND

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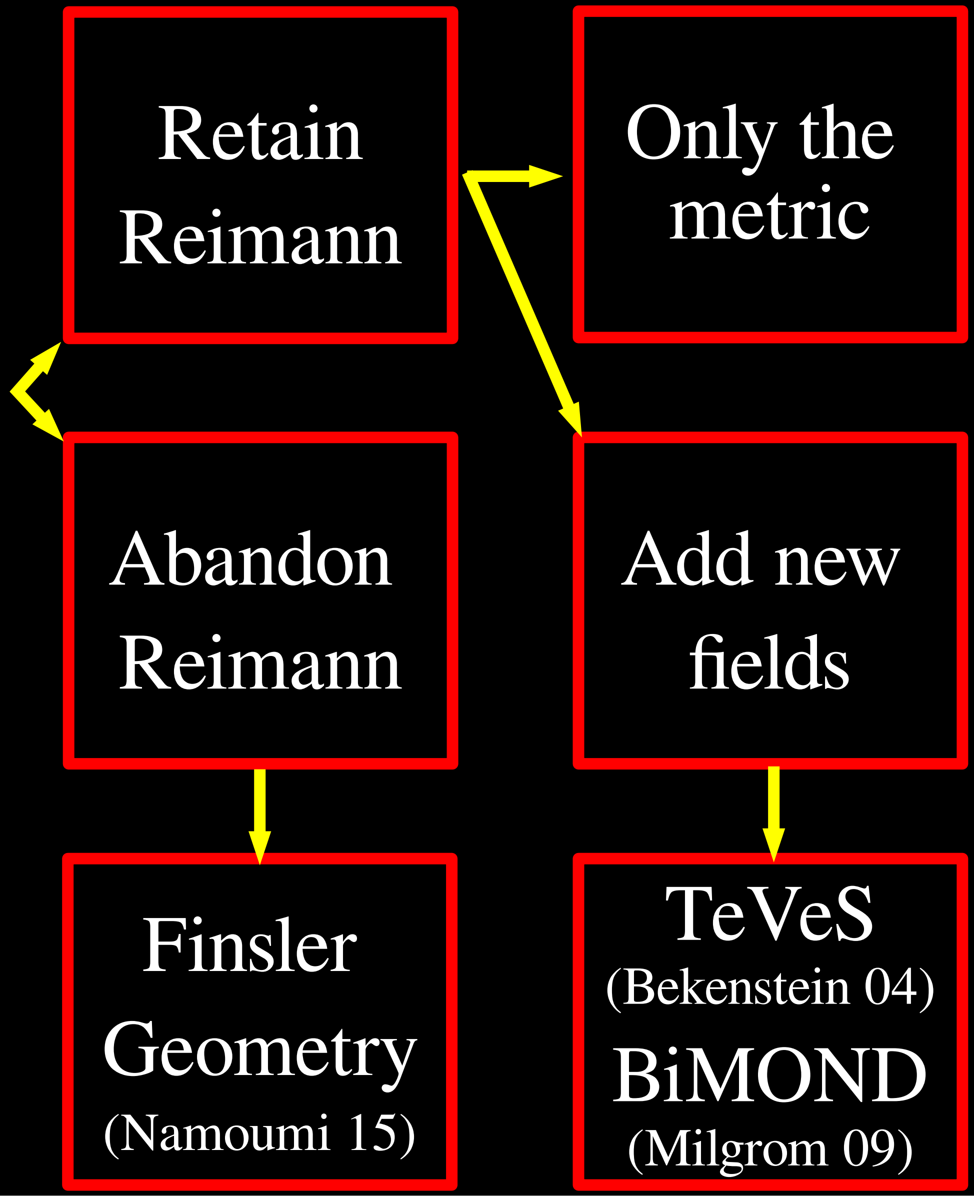

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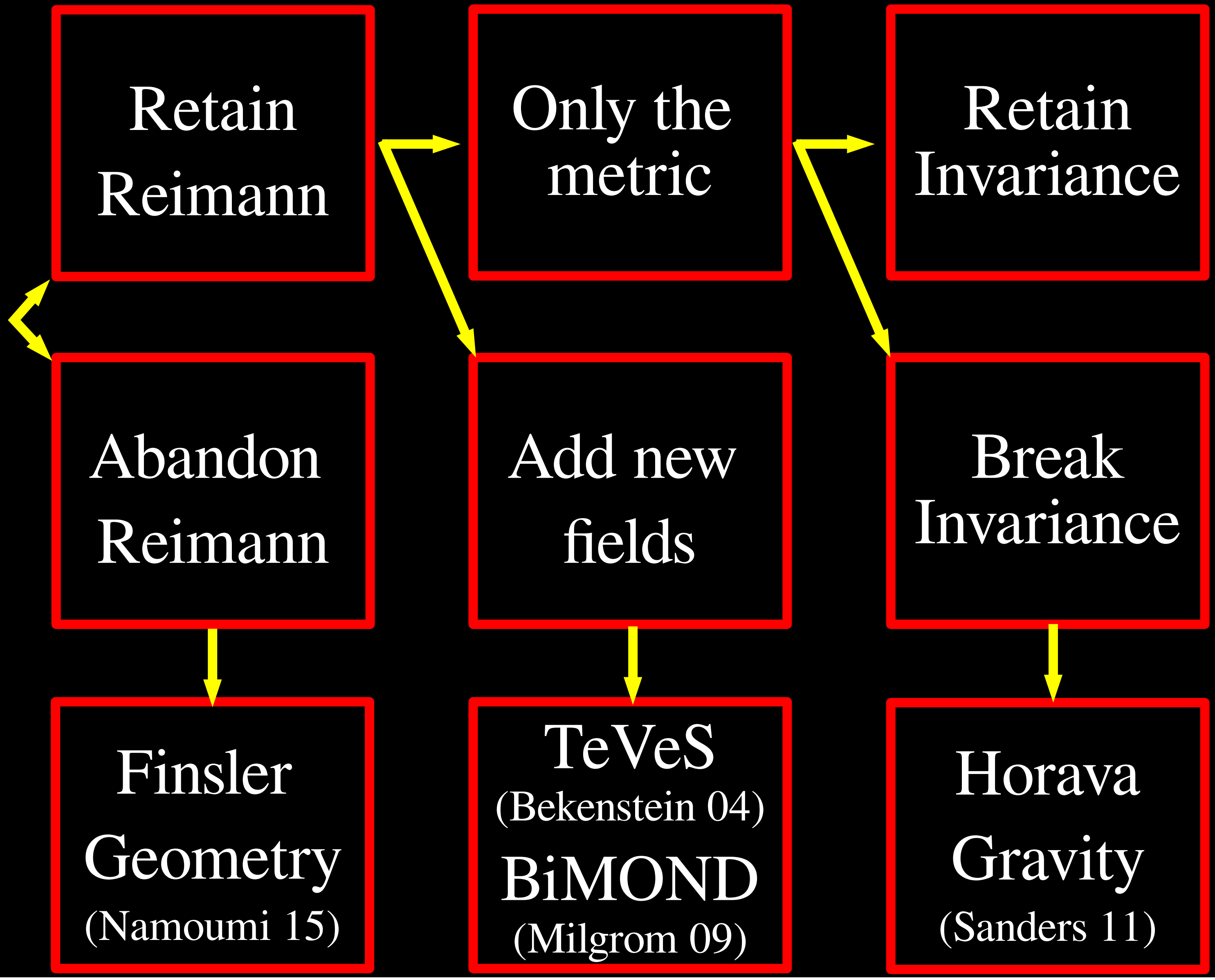

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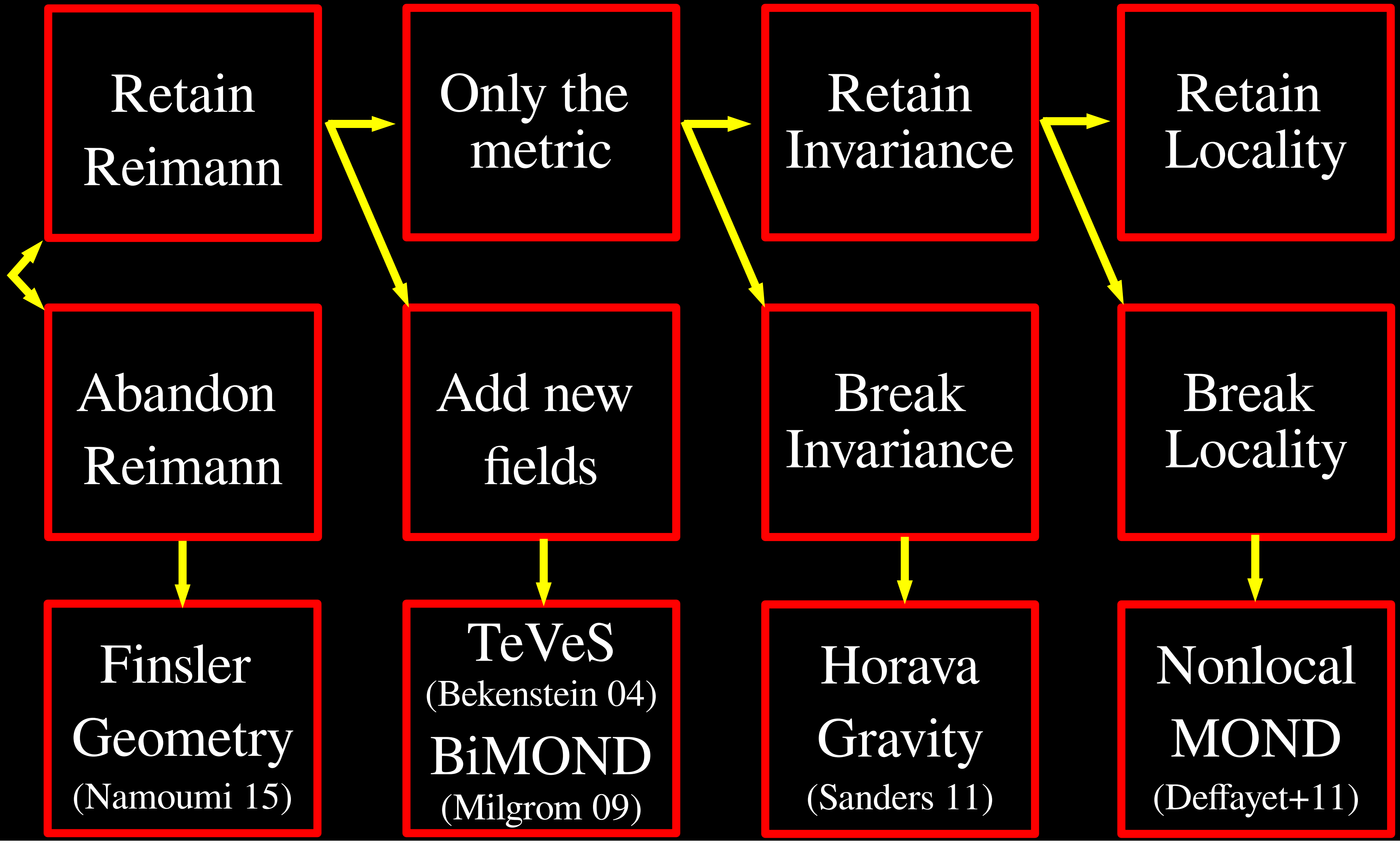

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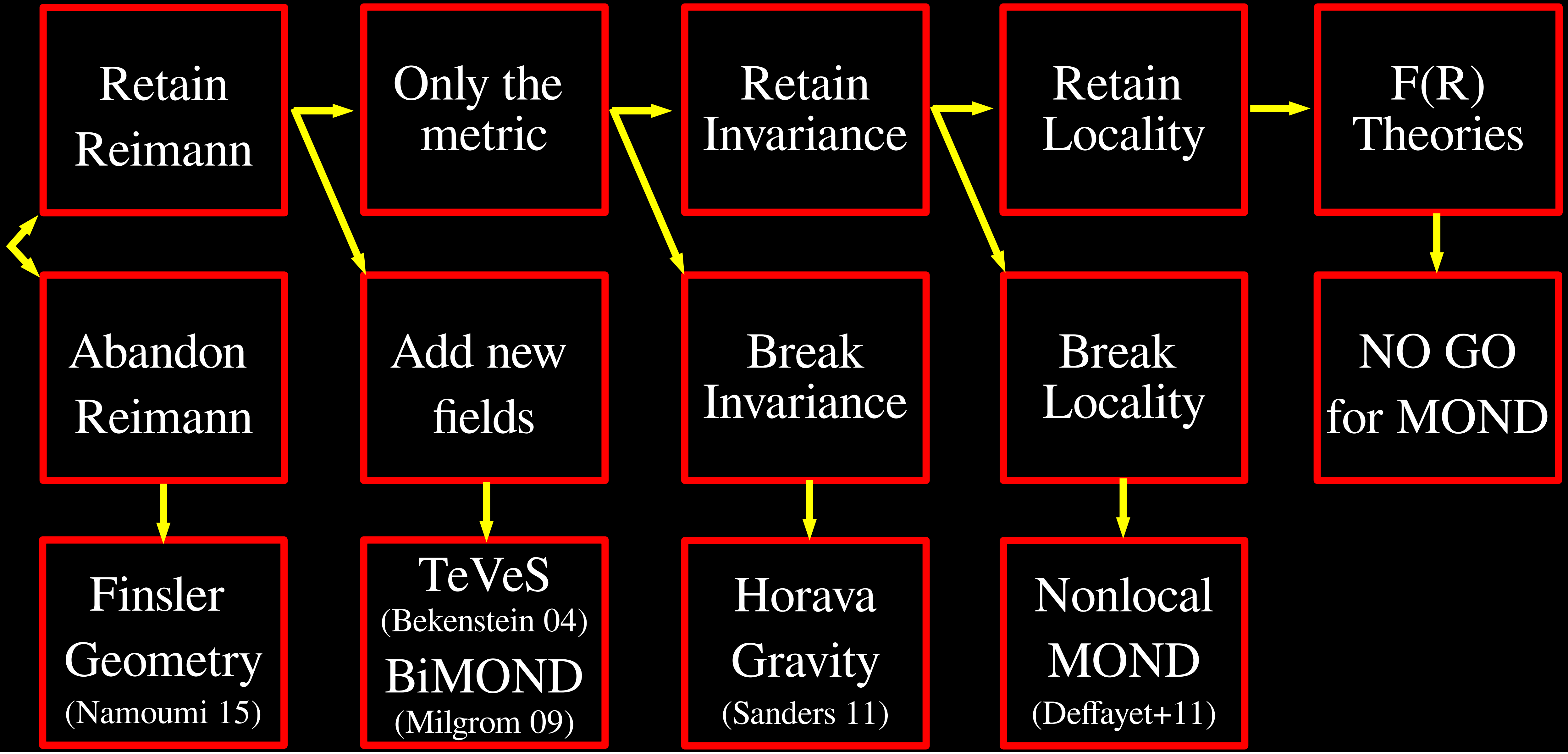

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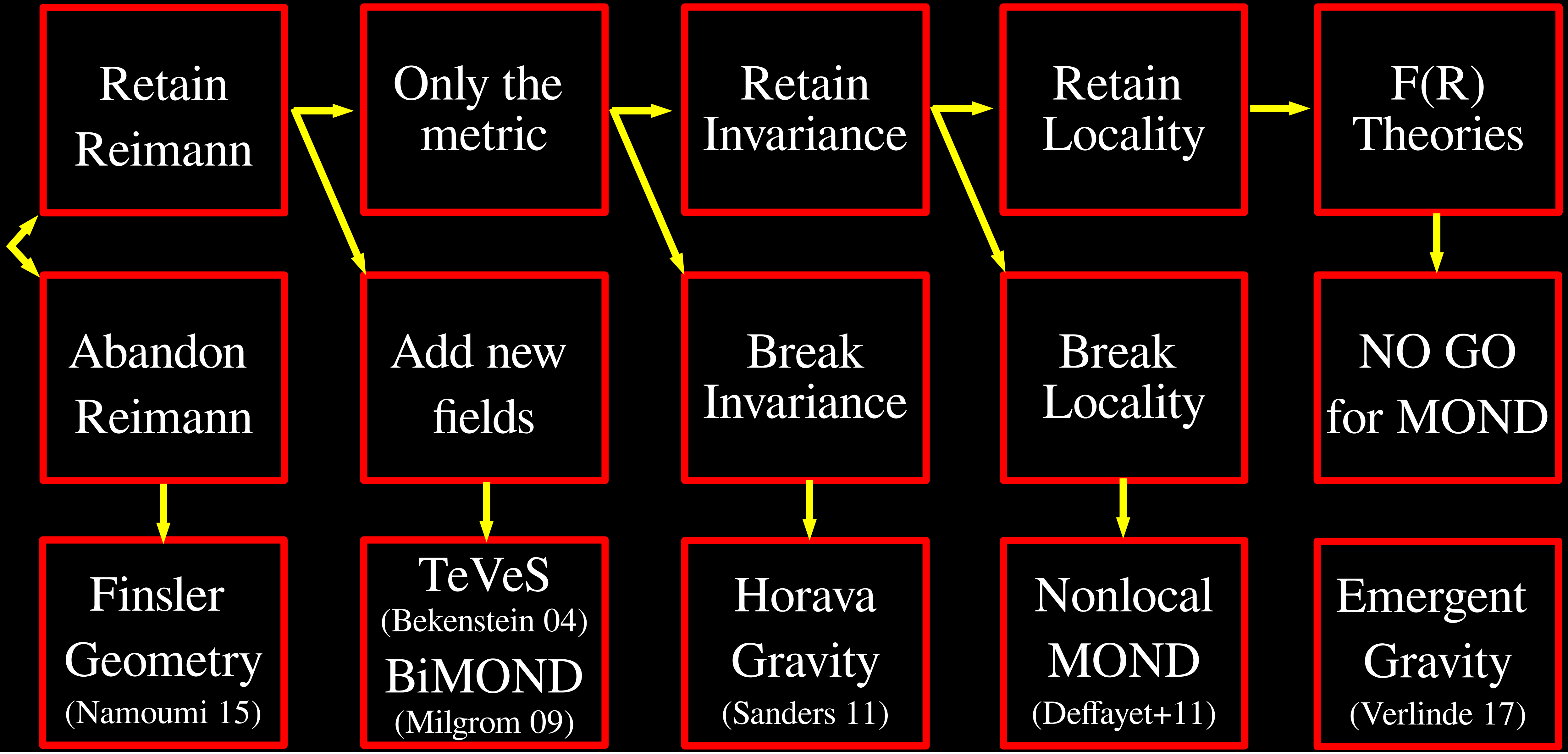

Bekenstein's TeVeS (Tensor-Vector-Scalar):

- Tensor $g_{\mu\nu}$ → Einstein's metric
- Vector A^{μ} → to get the "right" gravitational lensing (Sanders 1997)
- Scalar Φ → to get the DM effect for matter (Bekenstein & Milgrom 1984)
- Free Function → interpolation function (similar to AQUAL, QUMOND)

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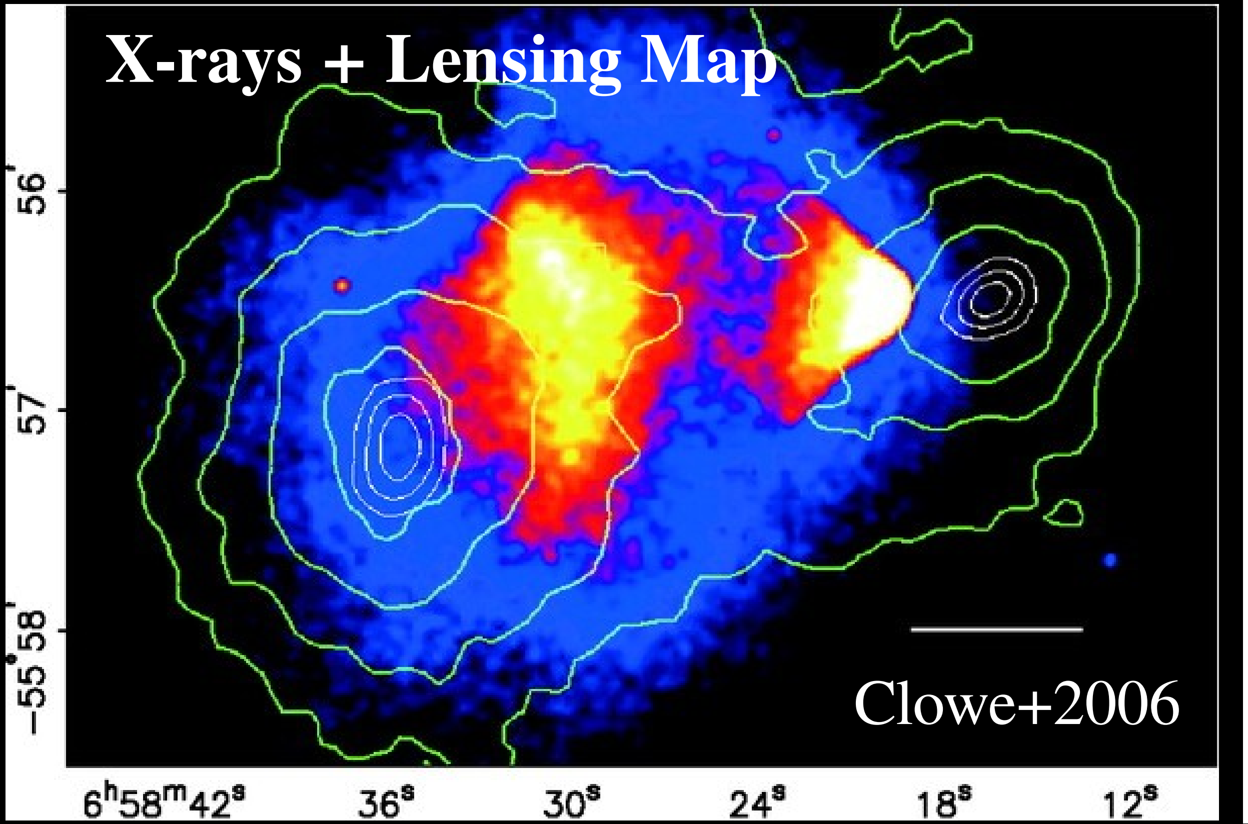
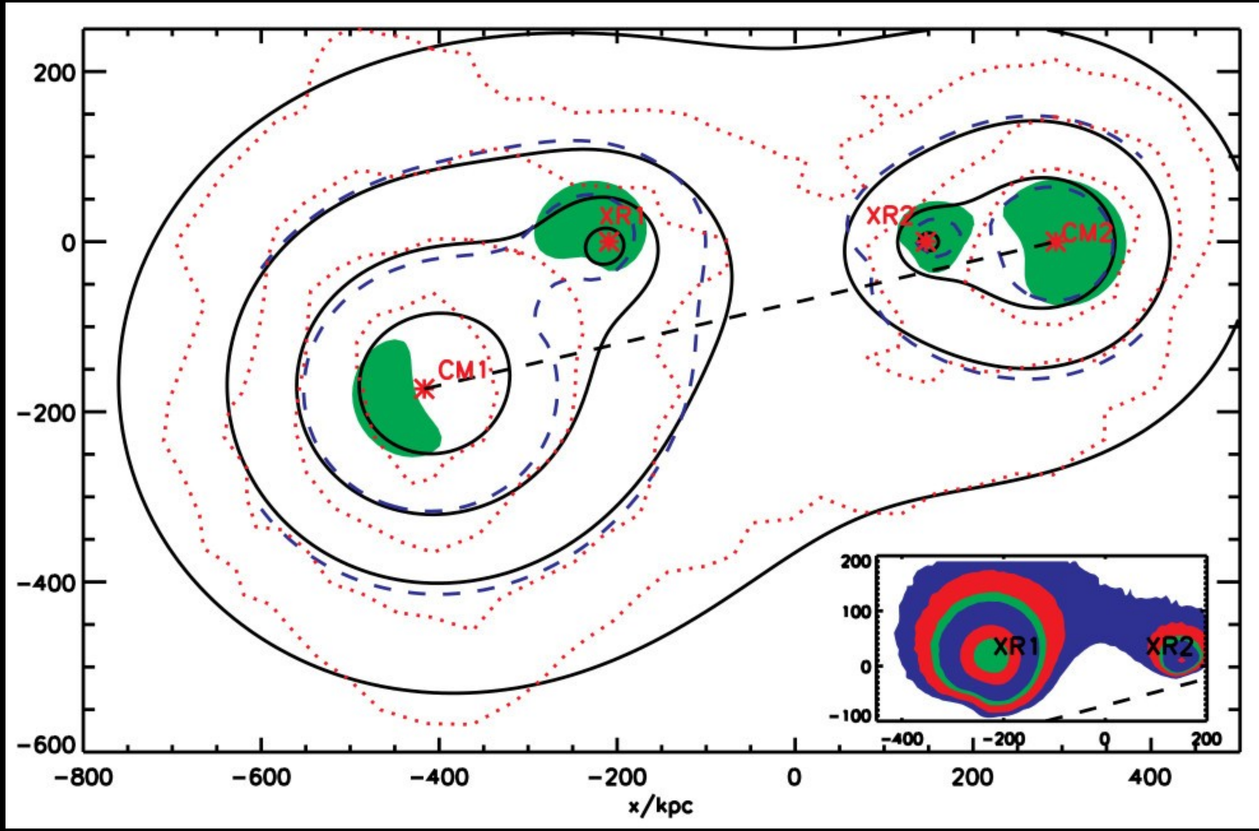
Matter follows a "physical metric" given by a disformal transformation:

$$\tilde{g}_{\mu,\nu} = g_{\mu,\nu} e^{-2\phi} + A_{\mu} A_{\nu} e^{-2\phi} - A_{\mu} A_{\nu} e^{2\phi} = e^{-2\phi} g_{\mu,\nu} - 2 A_{\mu} A_{\nu} \sinh(2\phi)$$

Application: Bullet Cluster in Bekenstein's TeVeS

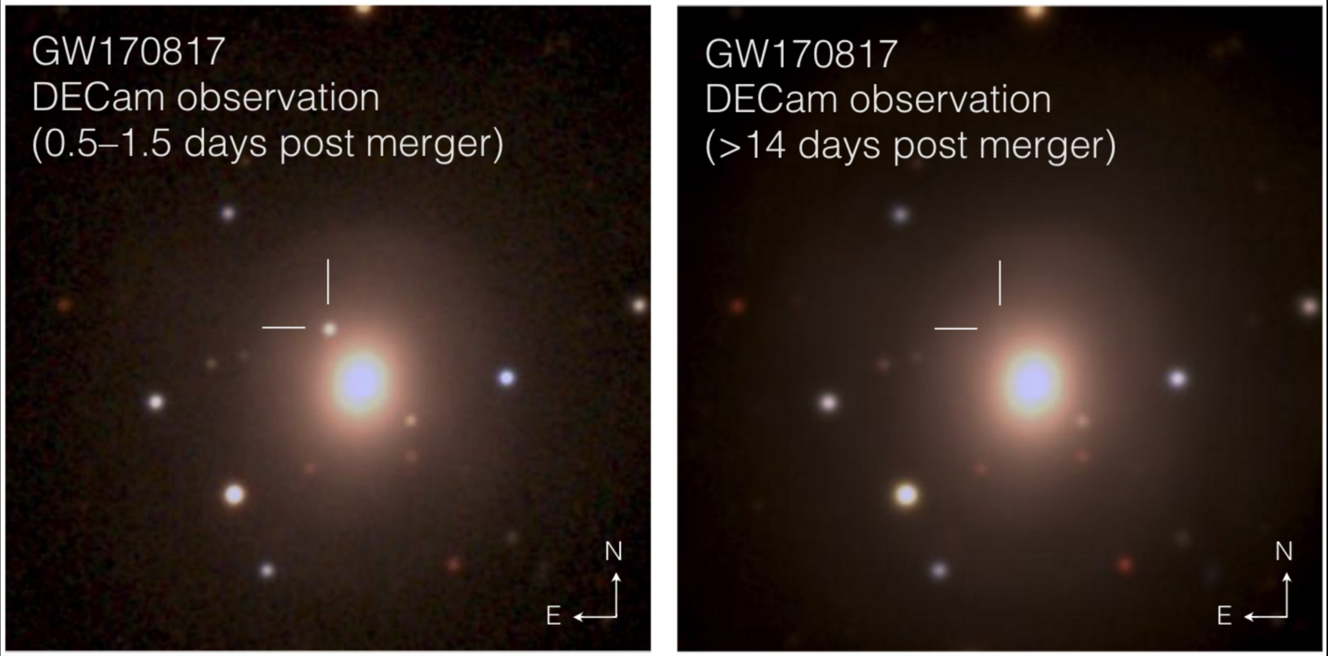


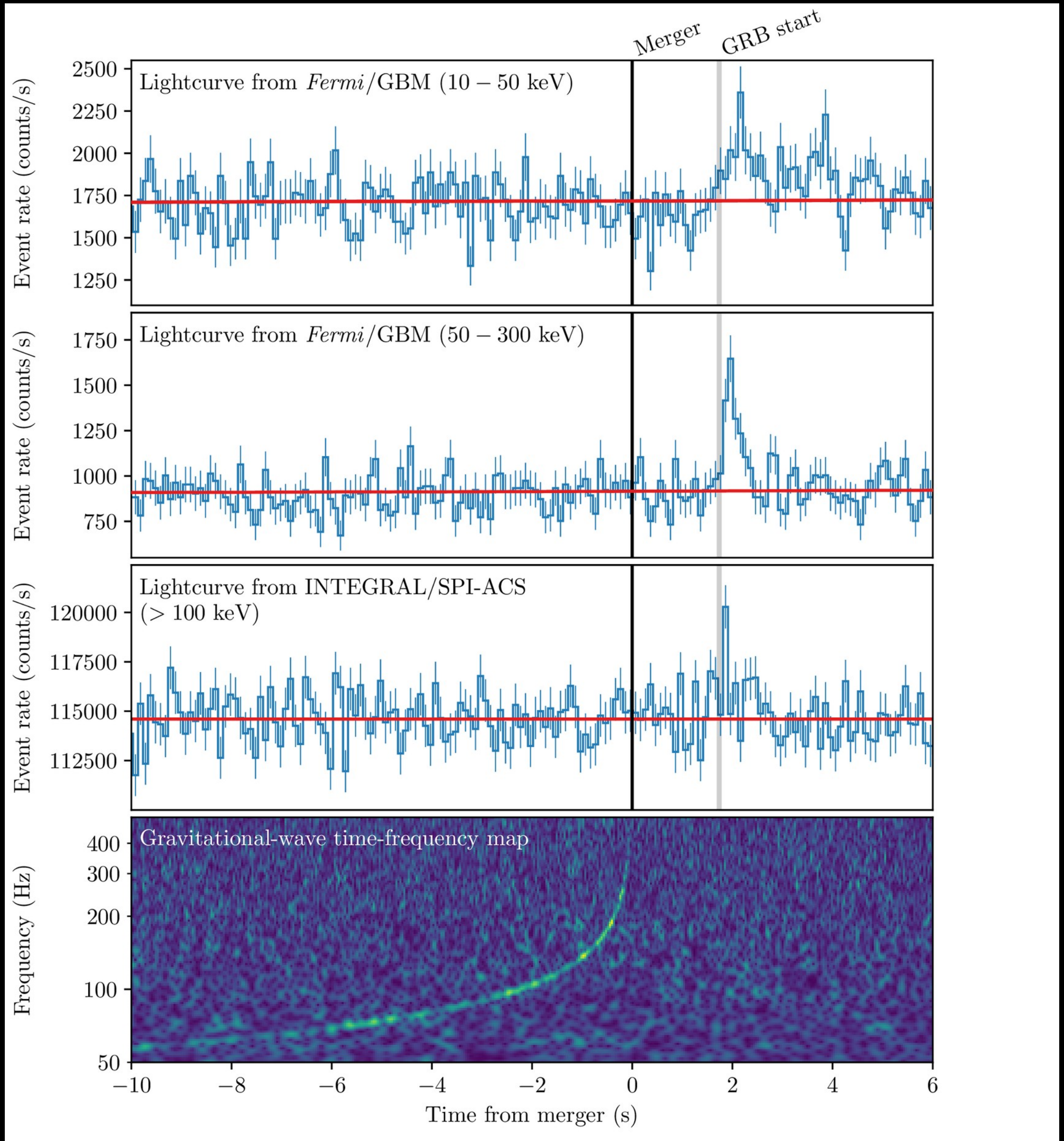


High collision speed (~4500 km/s) is rare in ΛCDM but natural in MOND.
(Hayashi & White 2006; Farrar & Rosen 2006; Angus+2007; Angus & McGaugh 2008)

MOND model with 2eV υ (Angus+2007)**:**
Red: Observed lensing convergence map
Black: best-fit MOND+υ convergence map
Blue: total surface densities (baryons+υ).

TeVeS is ruled out by kilonova discovery (GW170817)





Gravitational Wave signal immediately followed by gamma-ray signal:

$|c_{GW} - c_{EM}| < 10^{-15}\, c_{EM}$

But TeVeS predicted $c_{GW} \neq c_{EM}$!

New TeVeS-like theory (Skordis & Zlosnik 2020):

Combine scalar & vector in new time-like vector:

$B^{\mu} = e^{-2\phi} A^{\mu}$ such that $B^2 = g^{\mu\nu} B_{\mu} B_{\nu} = -e^{-2\phi}$

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The Action has free terms that are fixed requiring 5 conditions:

(1) General Relativity when $\nabla\Phi \gg a_0$ in *quasi-static situations*

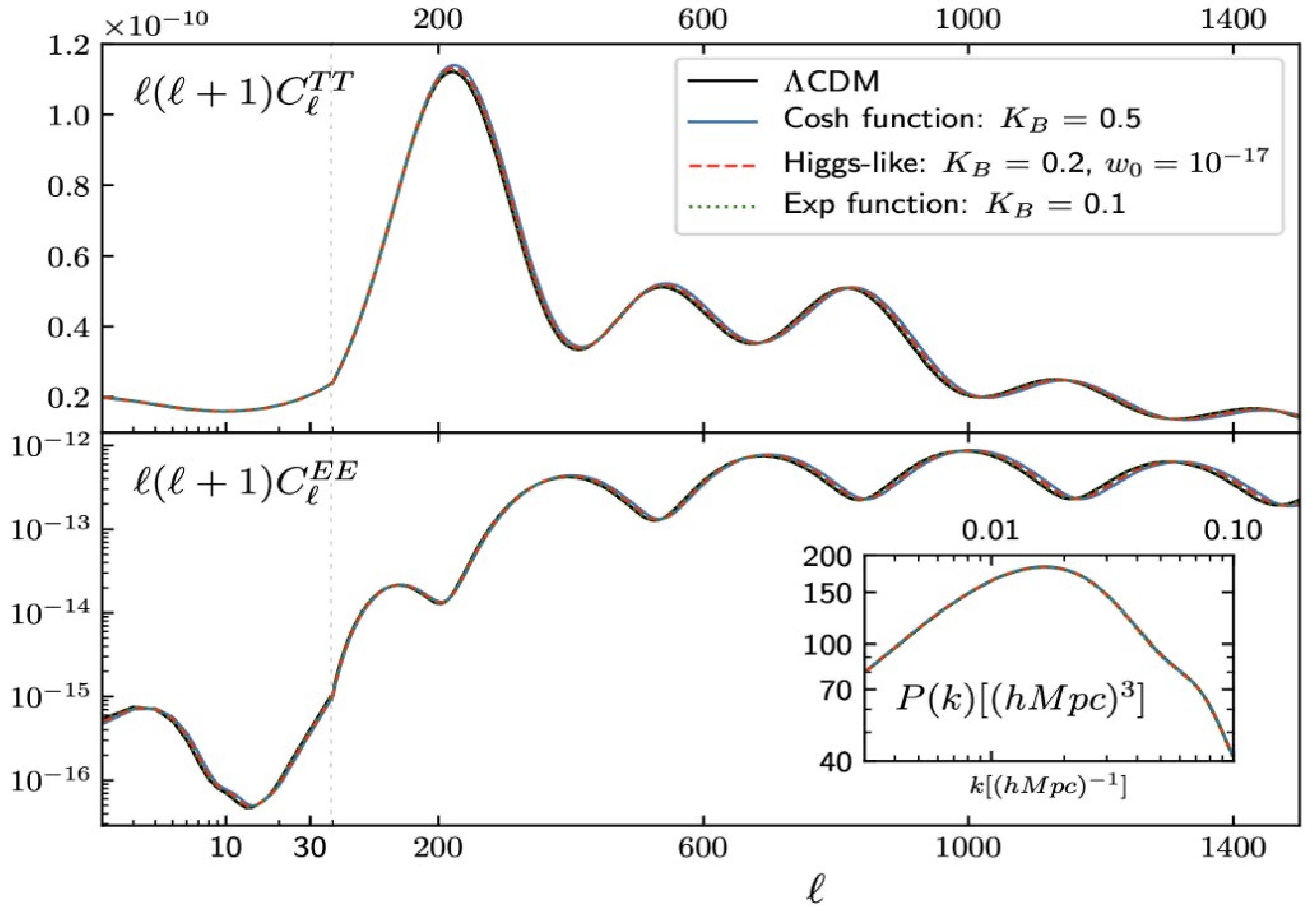
(2) MOND/AQUAL when $\nabla\Phi \ll a_0$ in *quasi-static situations*

(3) Gravitational lensing without dark matter

(4) Tensor mode of GW propagates at the speed of light

(5) FLRW background with the same expansion history as LCDM

New TeVeS-like theory (Skordis & Zlosnik 2020):



CMB power spectrum (both temperature and polarization) and matter power spectrum *P(k)* similar to LCDM.

Gravitational lensing and c_{GW}=c_{EM} are fine.

Lots of work left to do: non-linear formation of LSS, galaxy formation...

Success of MOND at different scales

Small Scales (~1-100 kpc) | **Intermediate Scales (~1-5 Mpc)** | **Large Scales (>100 Mpc)**