

Small Scales Problems for Λ CDM Cosmology

Federico Lelli

Arcetri Astrophysical Observatory

Largely based on:

Bullock & Boylan-Kolchin (2017)

Ann. Rev. Astron. & Astrophys, 55:343.



Testing Λ CDM at Different Scales

Galaxy Scales (~1-100 kpc)

Andromeda (spiral galaxy)



Messier 87 (elliptical galaxy)

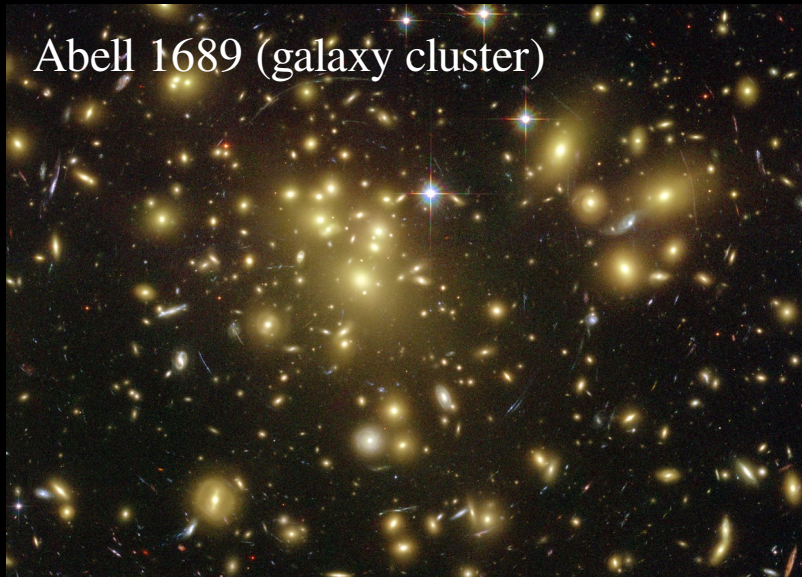


Group/Cluster Scales (~1-5 Mpc)

Stephan's Quintet (galaxy group)

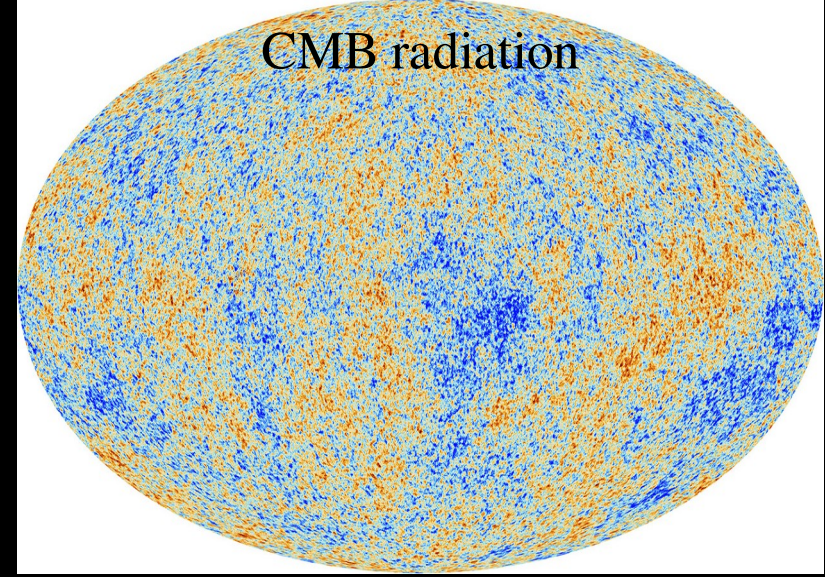


Abell 1689 (galaxy cluster)

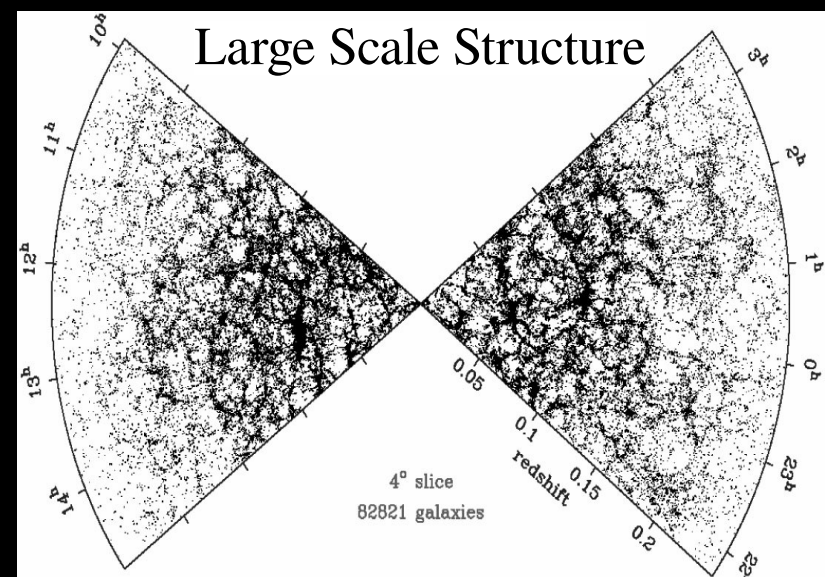


Cosmological Scales (>100 Mpc)

CMB radiation



Large Scale Structure



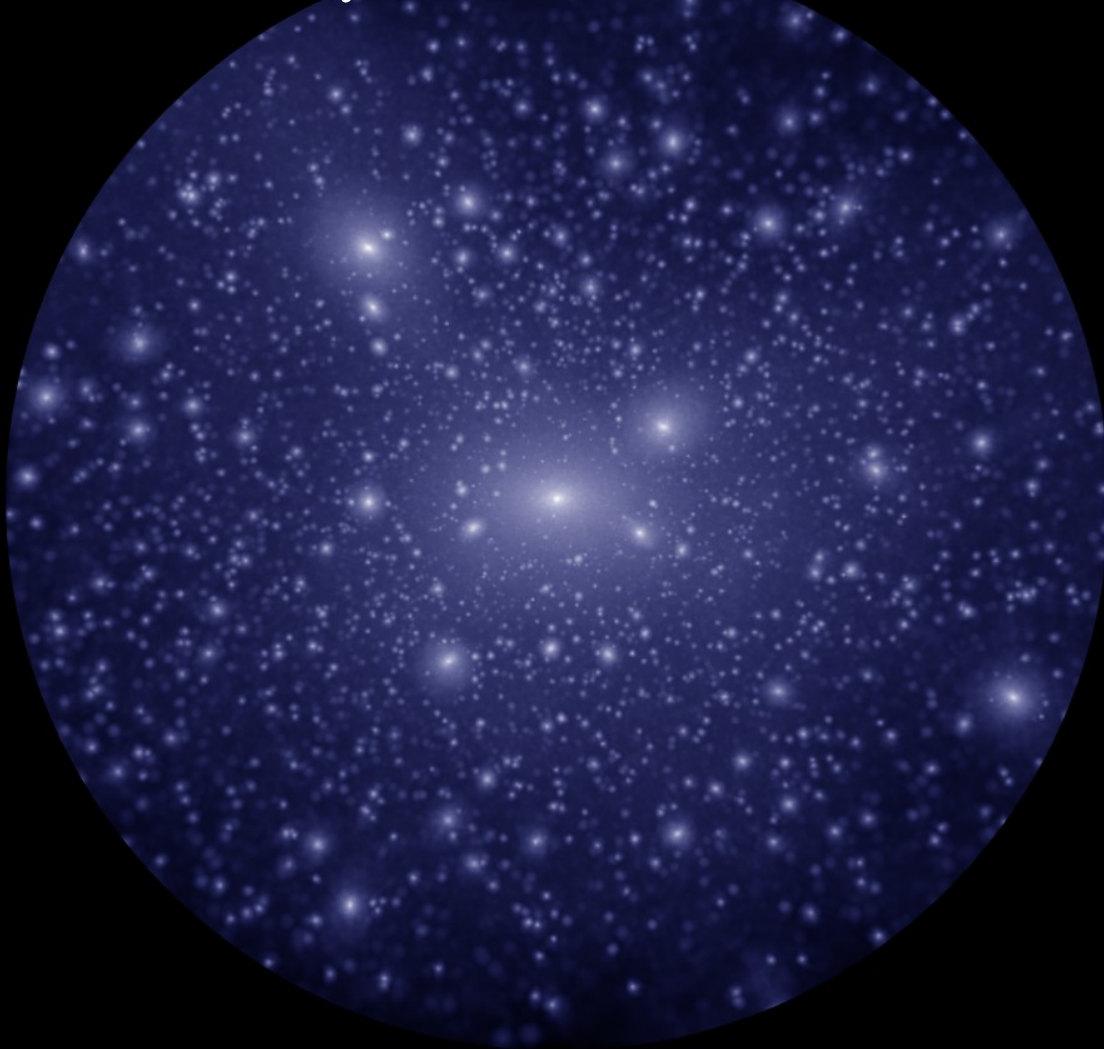
Small Scale Λ CDM Problems:

1. Missing Satellites
2. Cusp vs Core
3. Too-Big-To-Fail
4. Regularity vs Diversity
5. Planes of Satellites

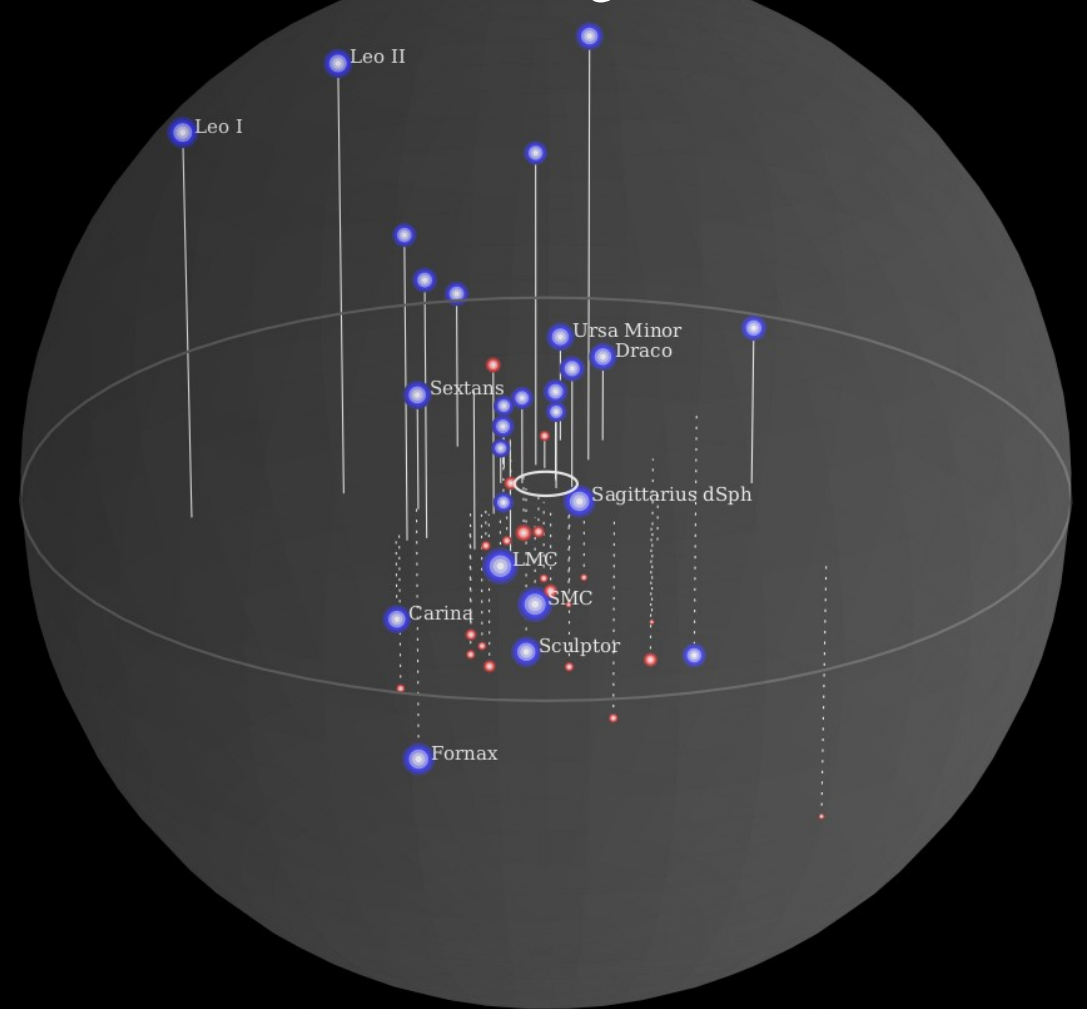
1. Missing Satellites Problem (Mass Function Problem)

The Missing Satellites Problem

Λ CDM N-body Simulation: 1000s sub-halos



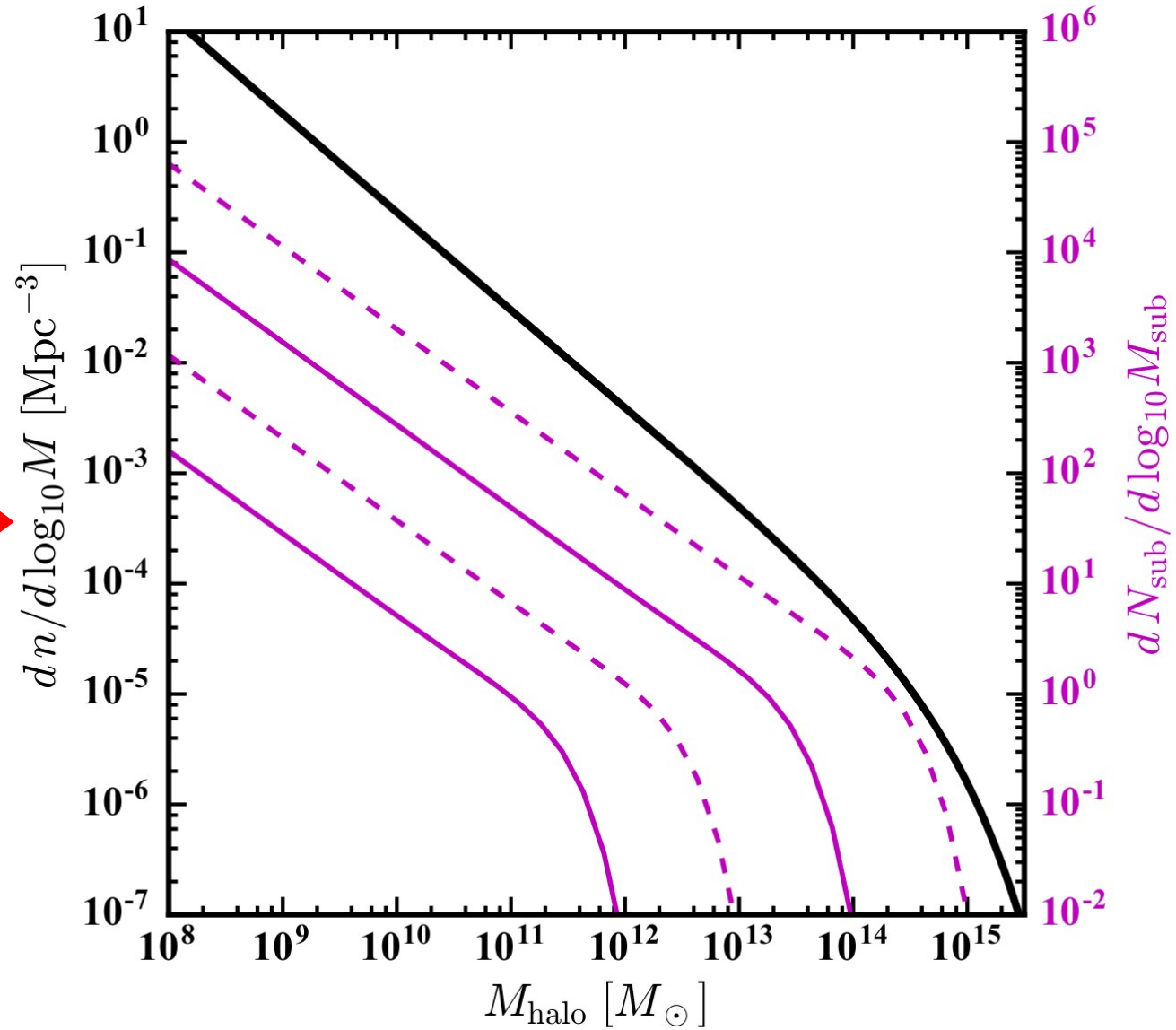
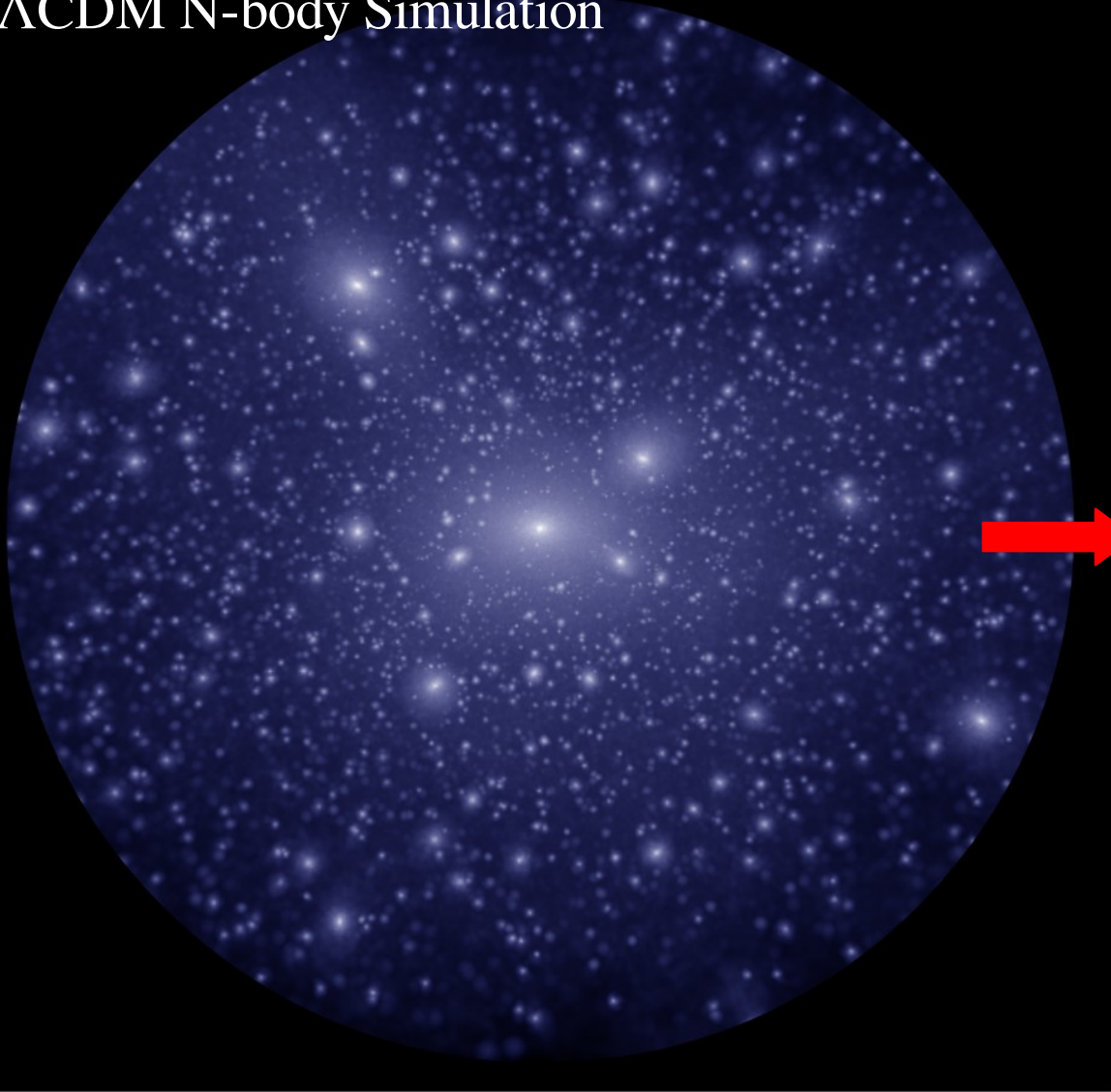
Observations: ~50 satellite galaxies around MW



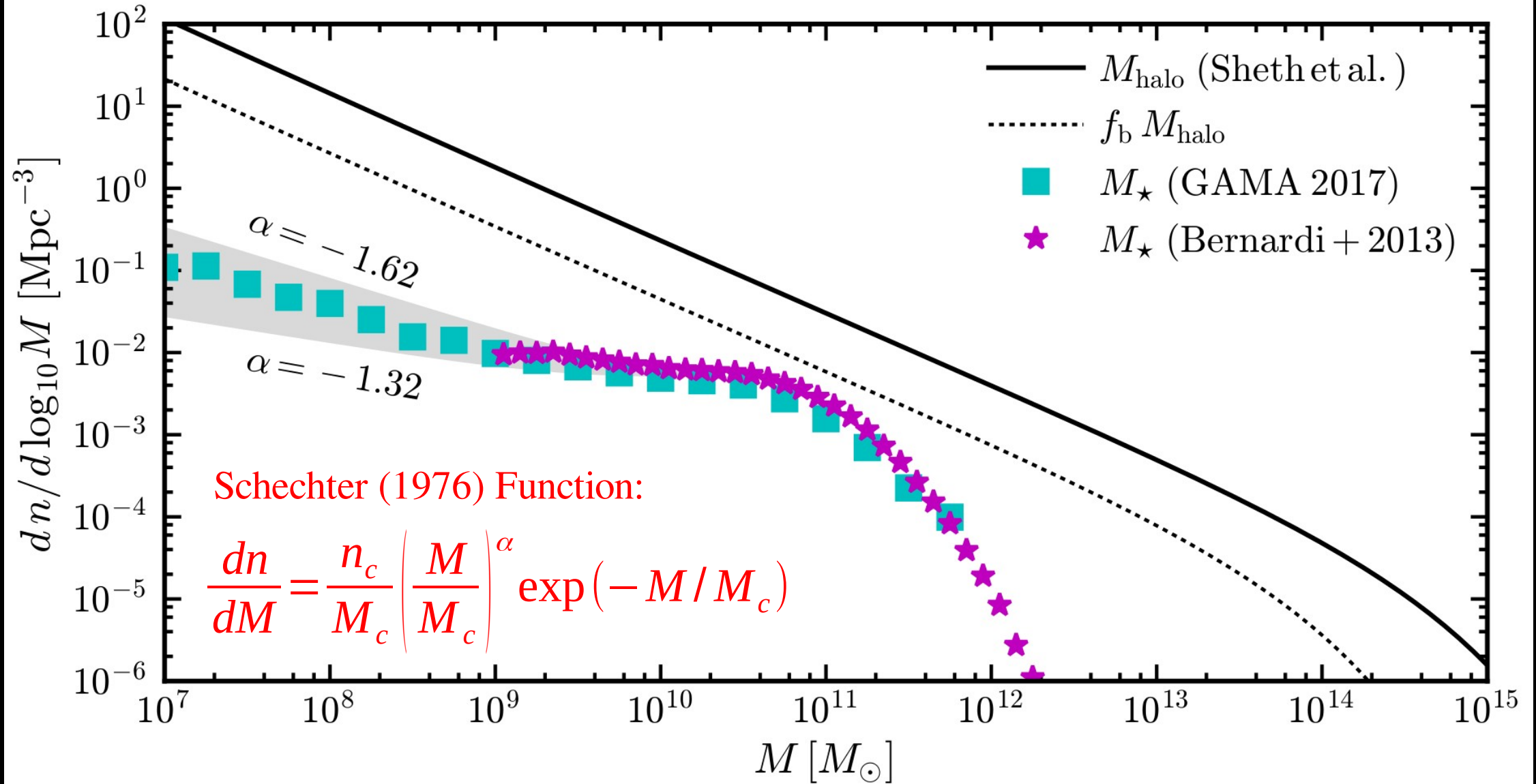
Pawlowski/Bullock/Boylan-Kolchin

The Mass Function Problem

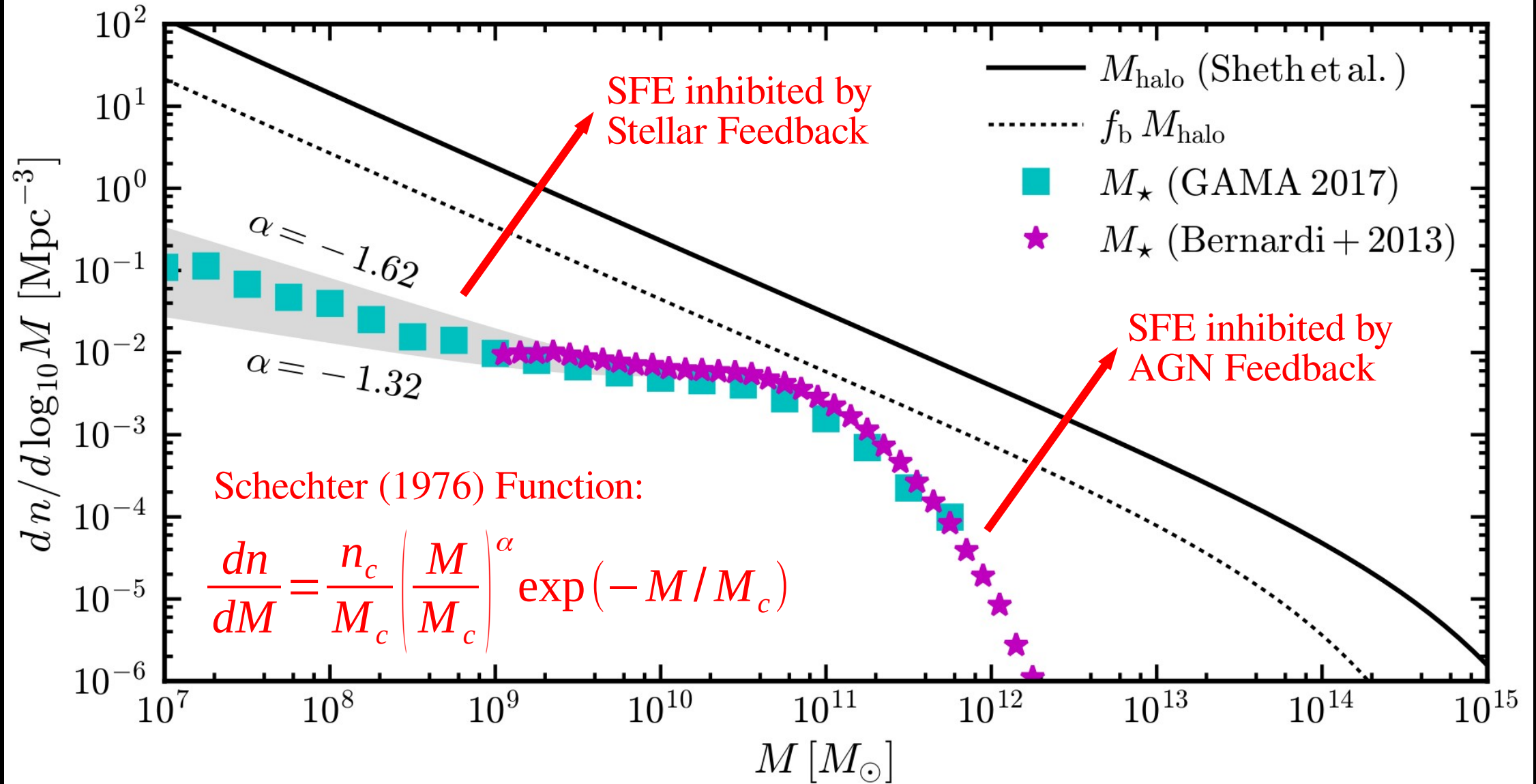
Λ CDM N-body Simulation



Stellar vs Halo Mass Function

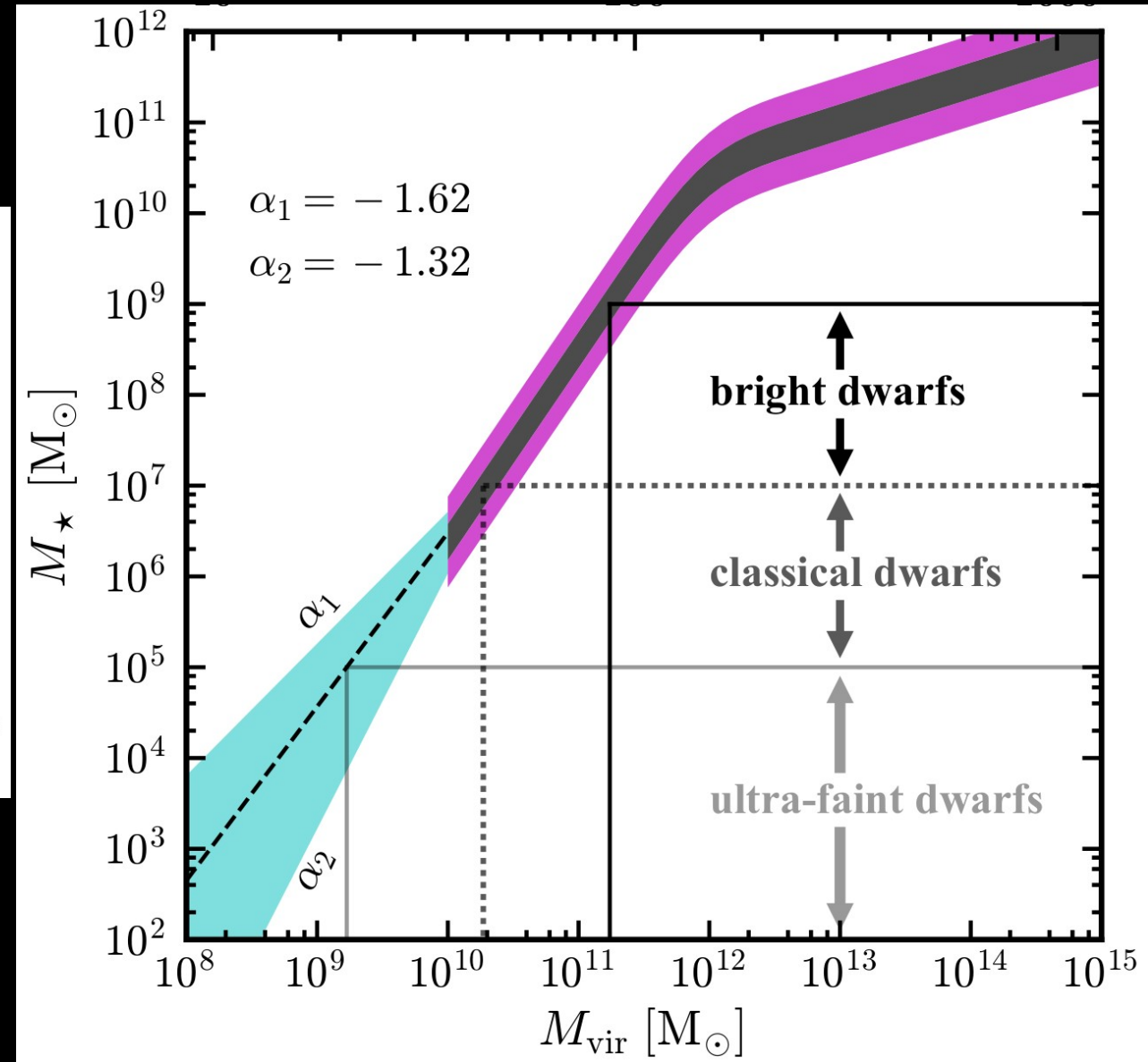
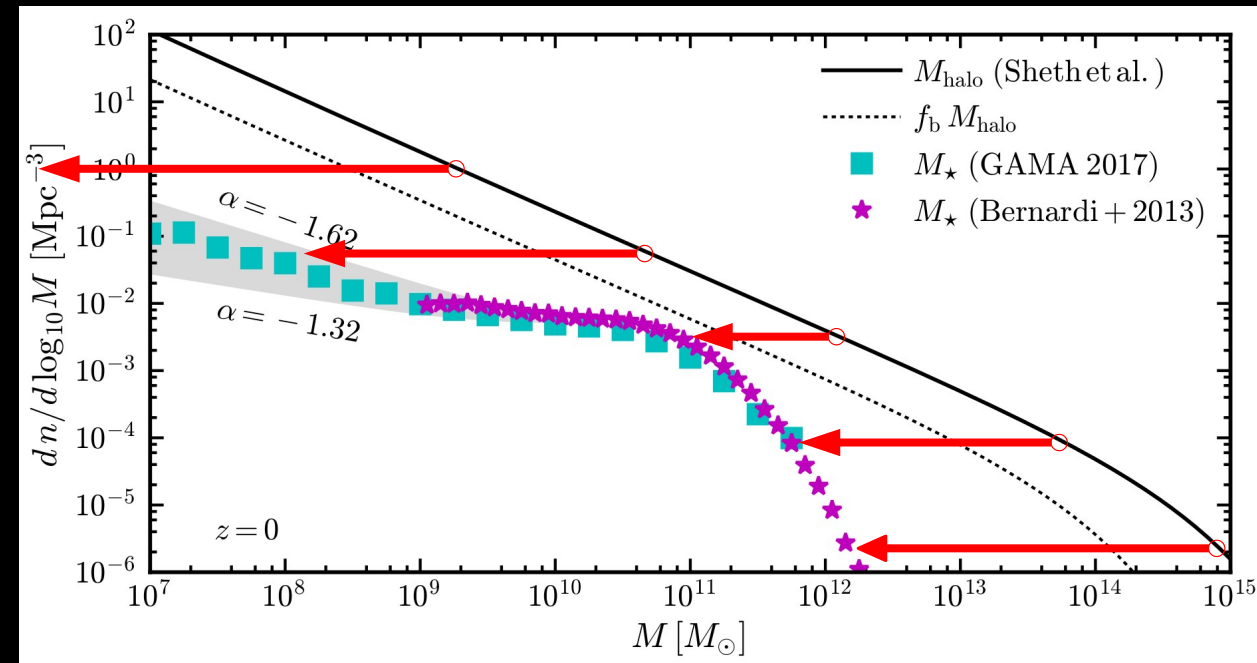


Stellar vs Halo Mass Function

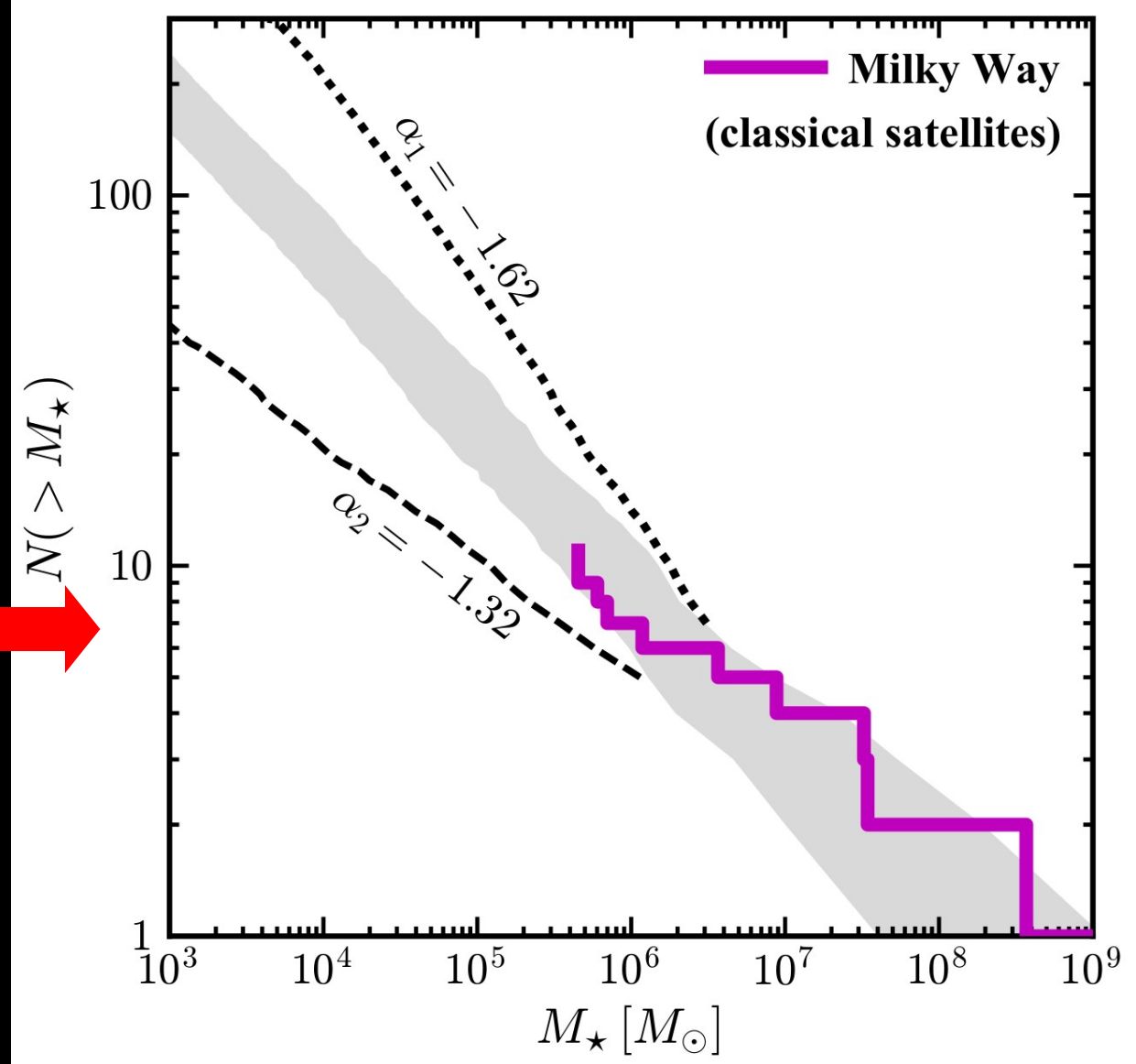
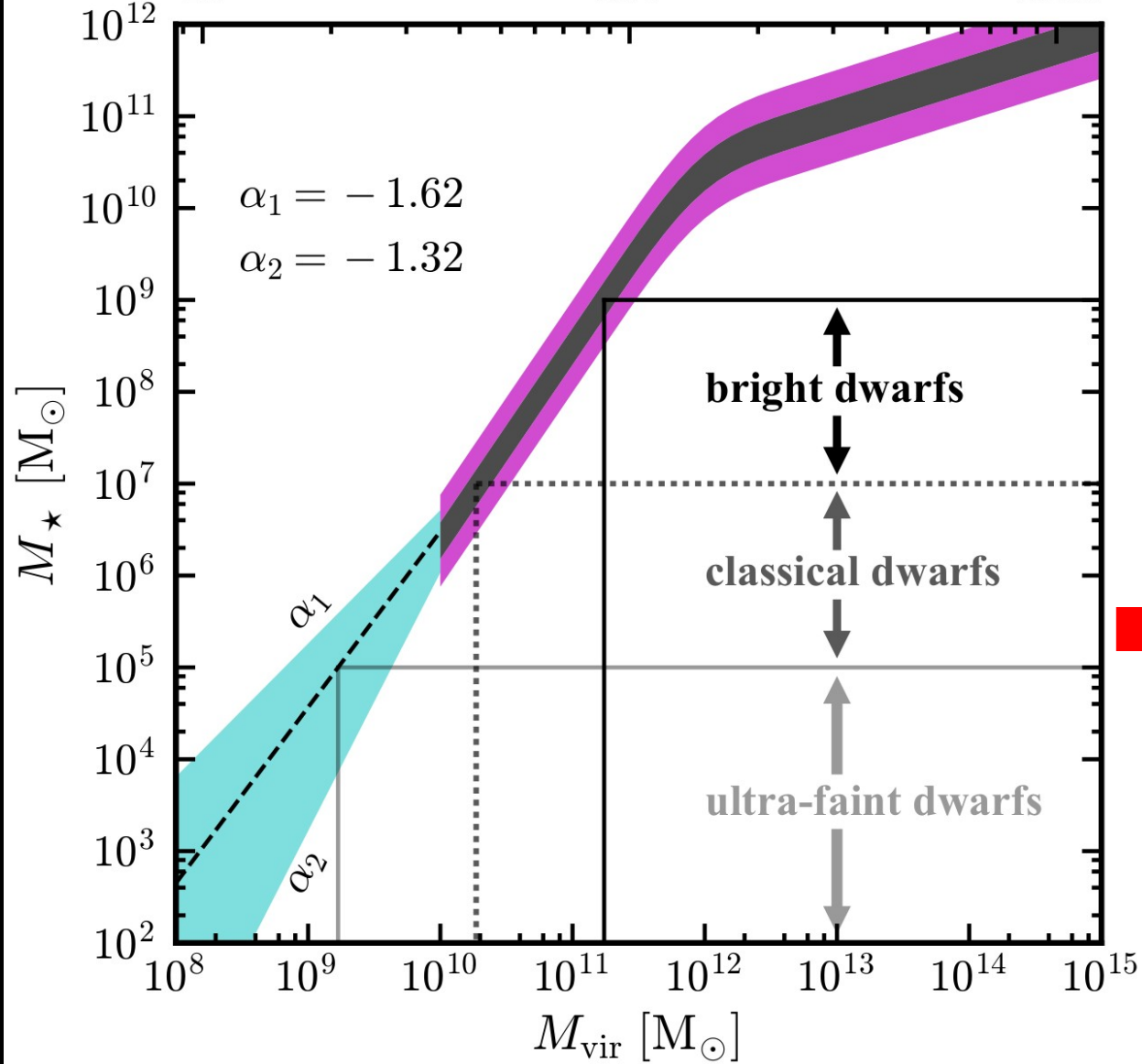


Stellar Mass – Halo Mass Relation

Abundance Matching:
Most massive galaxy \rightarrow most massive halo



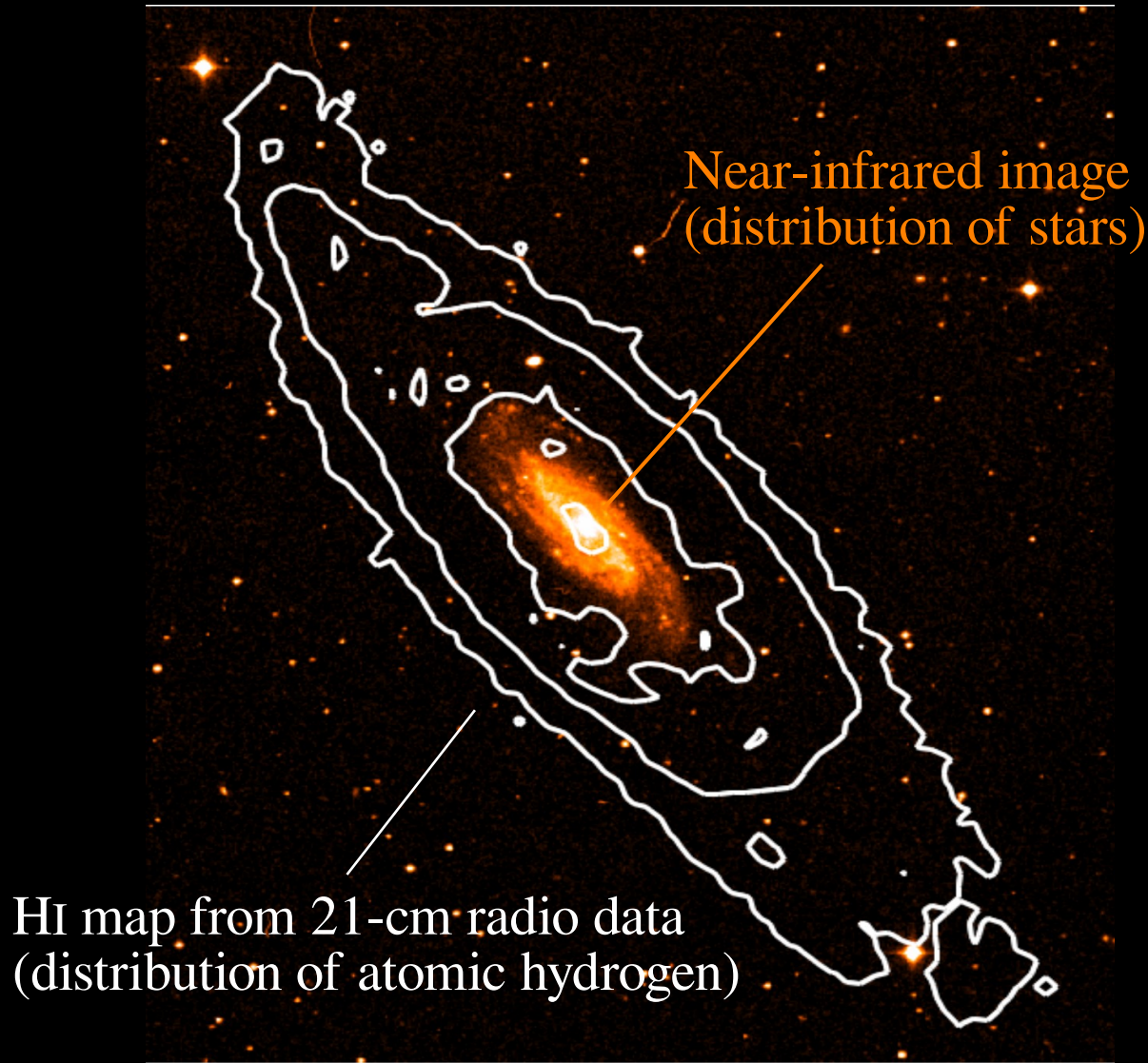
Abundance Matching \leftrightarrow Missing Satellites



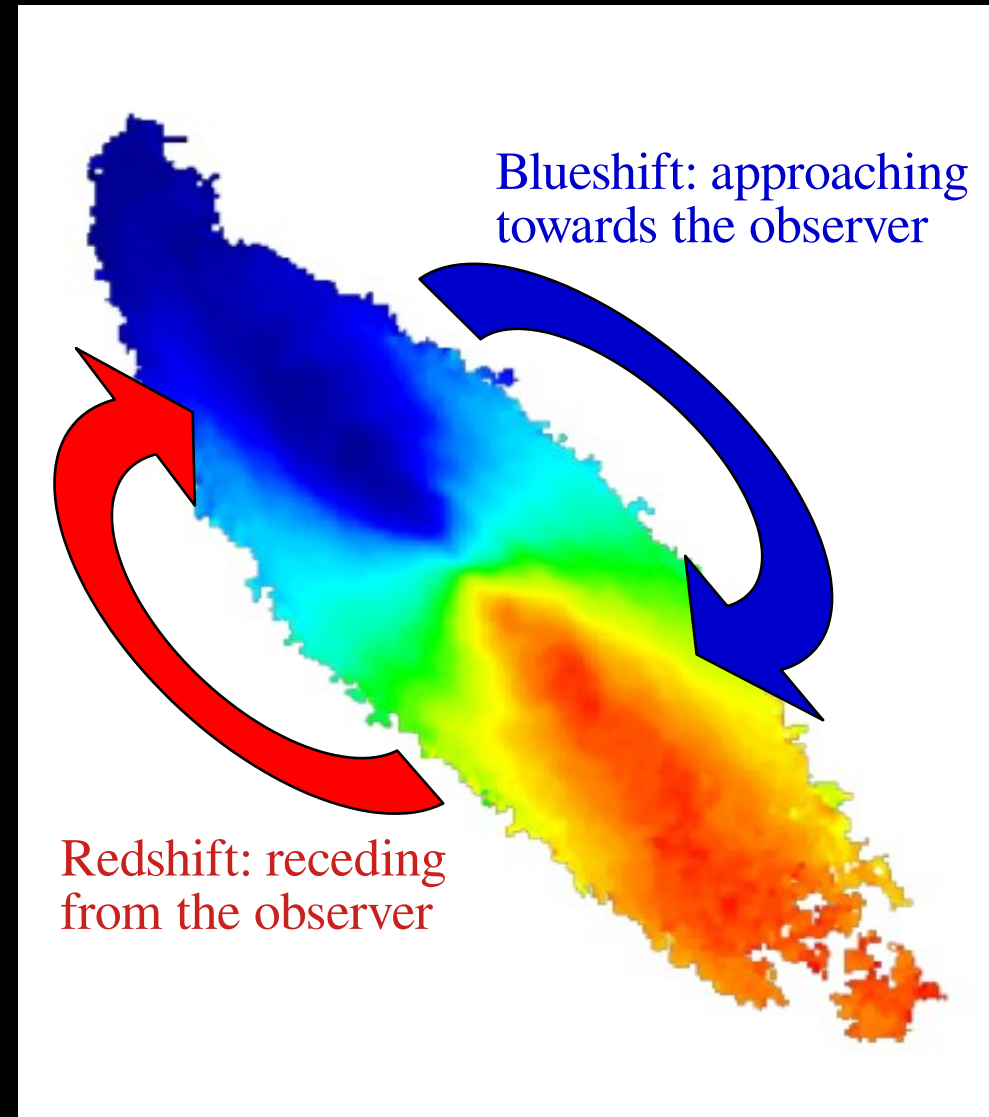
2. Cusp vs Core Problem (Rotation Curves Problem)

Rotation Curves of Disk Galaxies

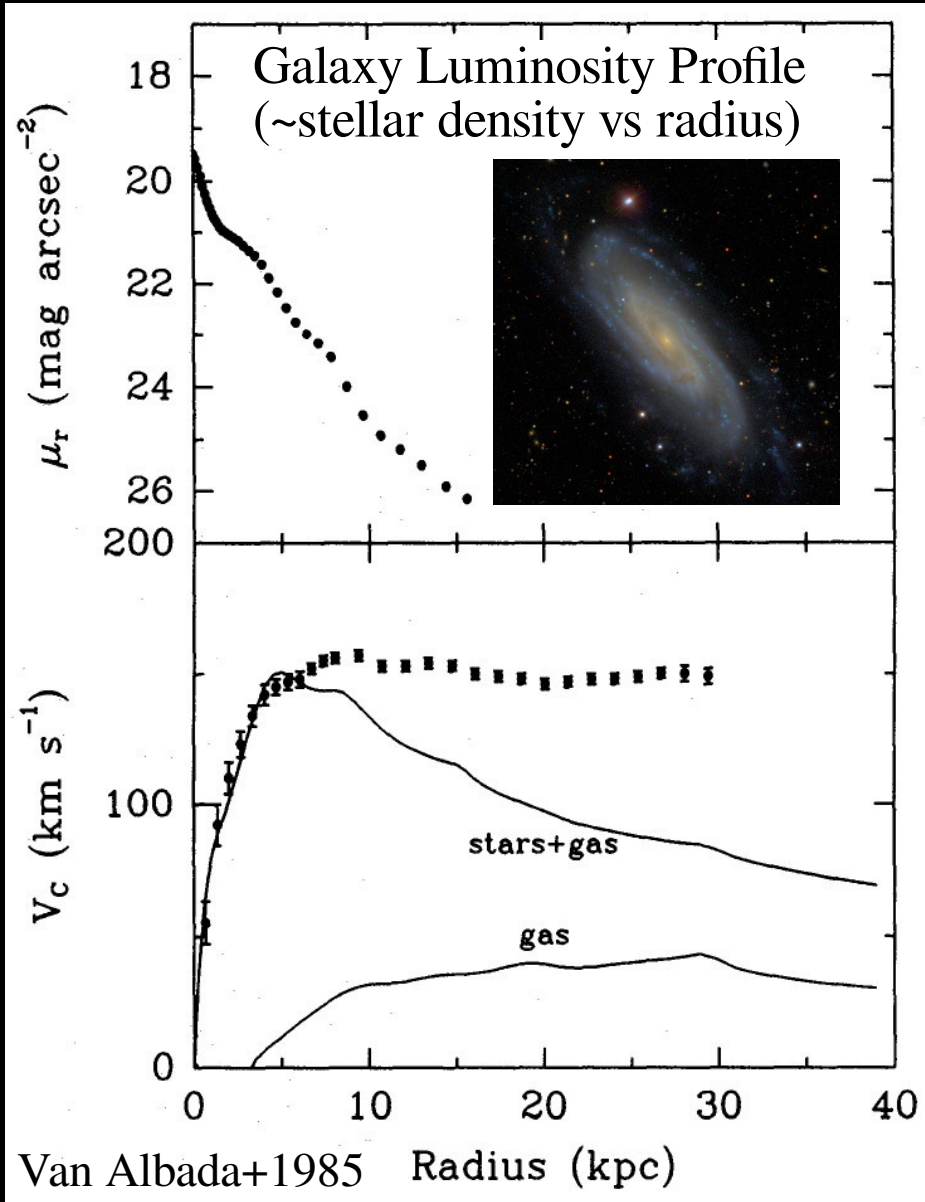
Distribution of baryons (gas & stars)



Gas Velocity along the Line of Sight



Building a Newtonian Mass Model



- Solve (numerically) Poisson's equation in cylindrical coordinates for each component ($i = \text{stars, gas}$):

$$\nabla^2 \Phi_i(R, z) = 4\pi G \rho_i(R, z)$$

- Find expected circular velocity in disk mid-plane:

$$\frac{V_i^2(R, z=0)}{R} = - \frac{\partial \Phi_i(R, z=0)}{\partial R}$$

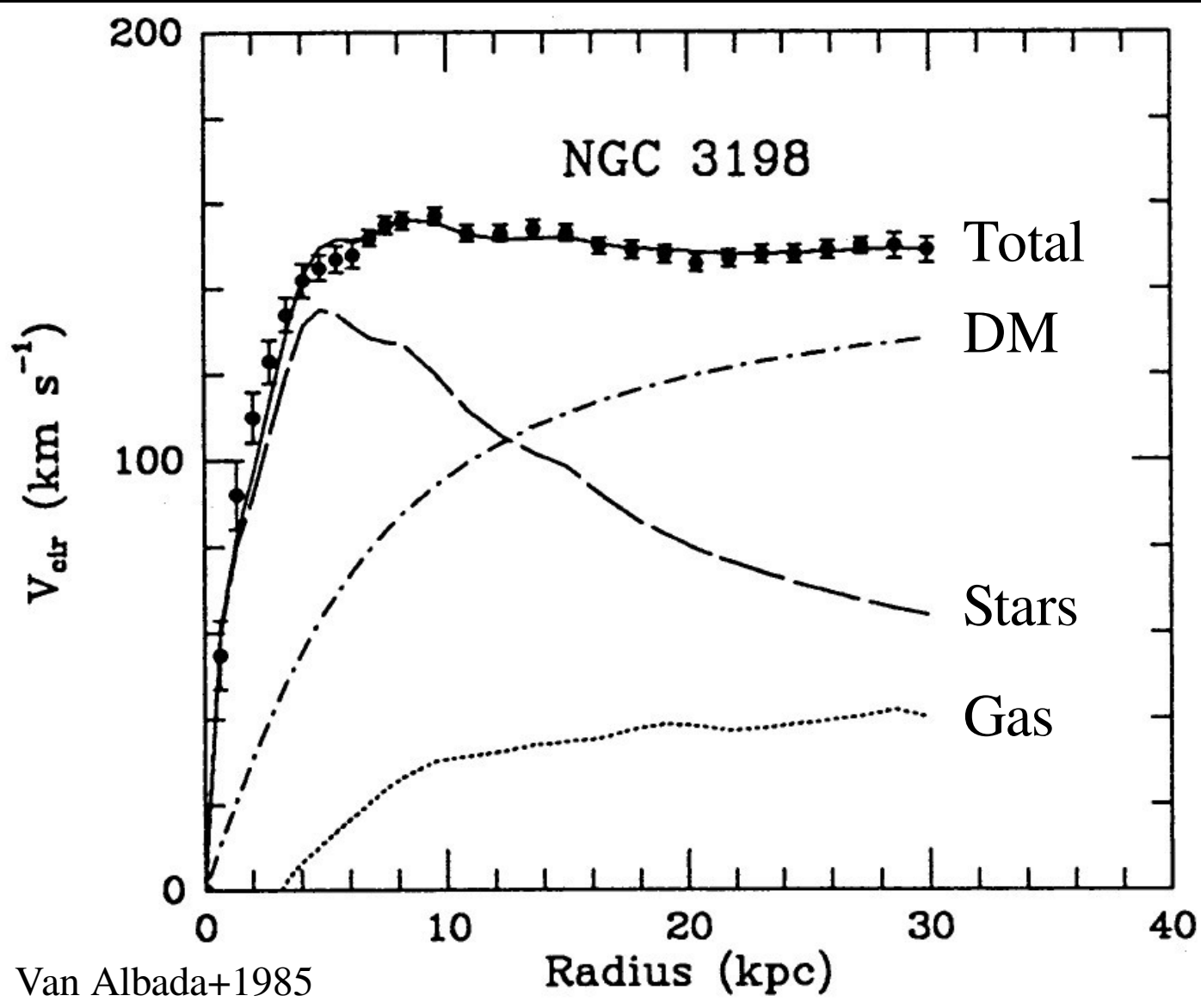
- Sum over gravitational fields ($g_i = V_i^2/R$):

$$V_b^2(R) = Y_s V_s^2(R) + Y_g V_g^2(R)$$

$Y_s = M_s/L$ estimated from stellar population models

$Y_g =$ known for HI from atomic physics (spin-flip) + small corrections for H₂, He, heavier elements

Mass Model with a Dark Matter Halo



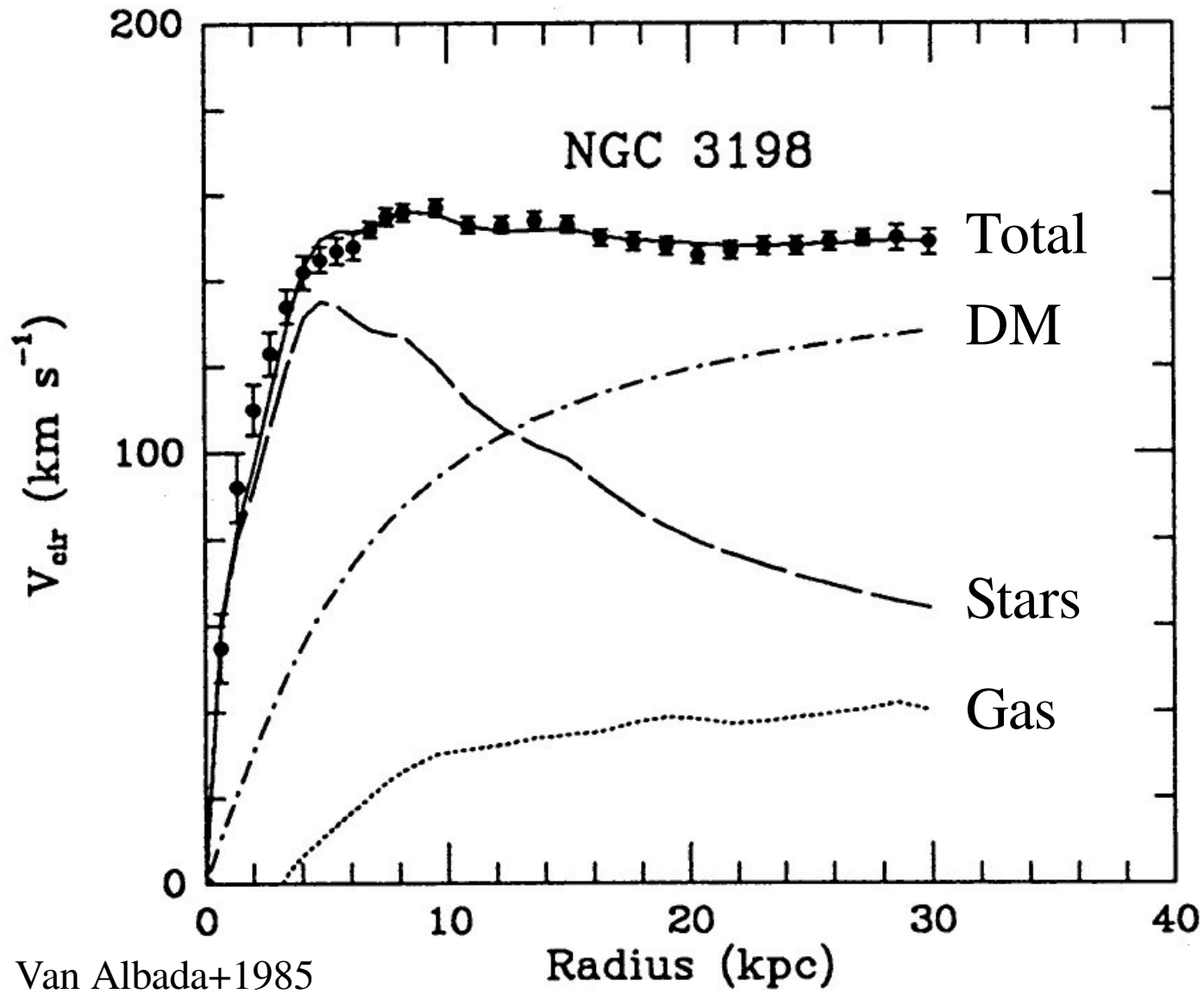
- Assume spherical DM halo profile:

$$\rho_{DM} = \rho(r; \rho_c, r_c)$$

- Add it together with the baryons:

$$V_c^2 = Y_s V_s^2 + Y_g V_g^2 + V_{DM}^2(\rho_c, r_c)$$

Mass Model with a Dark Matter Halo



- Assume spherical DM halo profile:

$$\rho_{DM} = \rho(r; \rho_c, r_c)$$

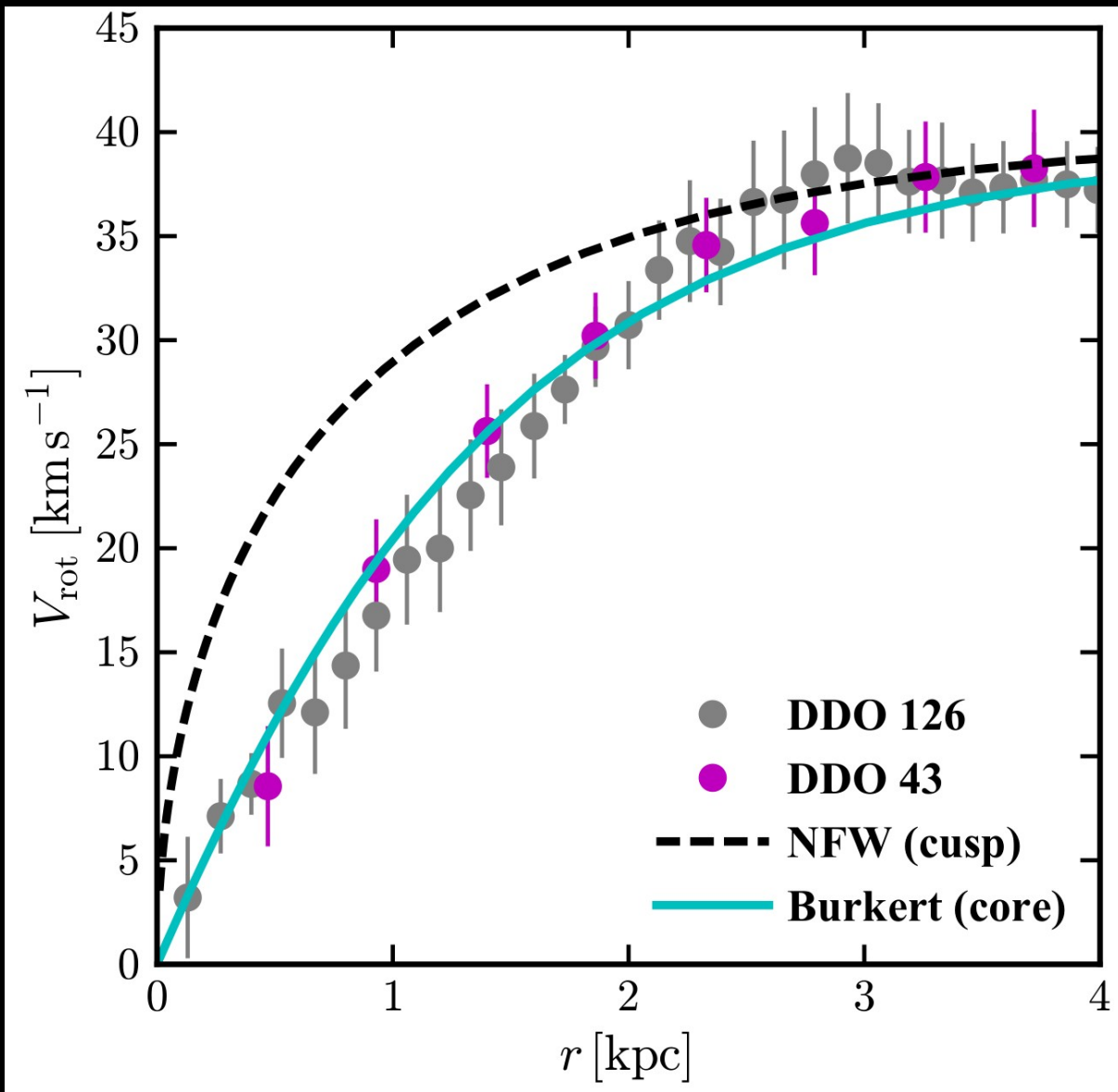
- Add it together with the baryons:

$$V_c^2 = Y_s V_s^2 + Y_g V_g^2 + V_{DM}^2(\rho_c, r_c)$$

For spiral galaxies like the Milky Way, baryons dominate in the inner parts while DM is needed in the outer regions → **the sum of the two gives the flat part!**

Why are rotation curves flat? Unclear!
This is called “disk-halo conspiracy”
(van Albada & Sancisi 1986)

The Cusp vs Core Problem



NFW profile (from N-body sims):

$$\rho_{DM}(r) = \frac{4\rho_c}{(r/r_c)(1+r/r_c)^2}$$

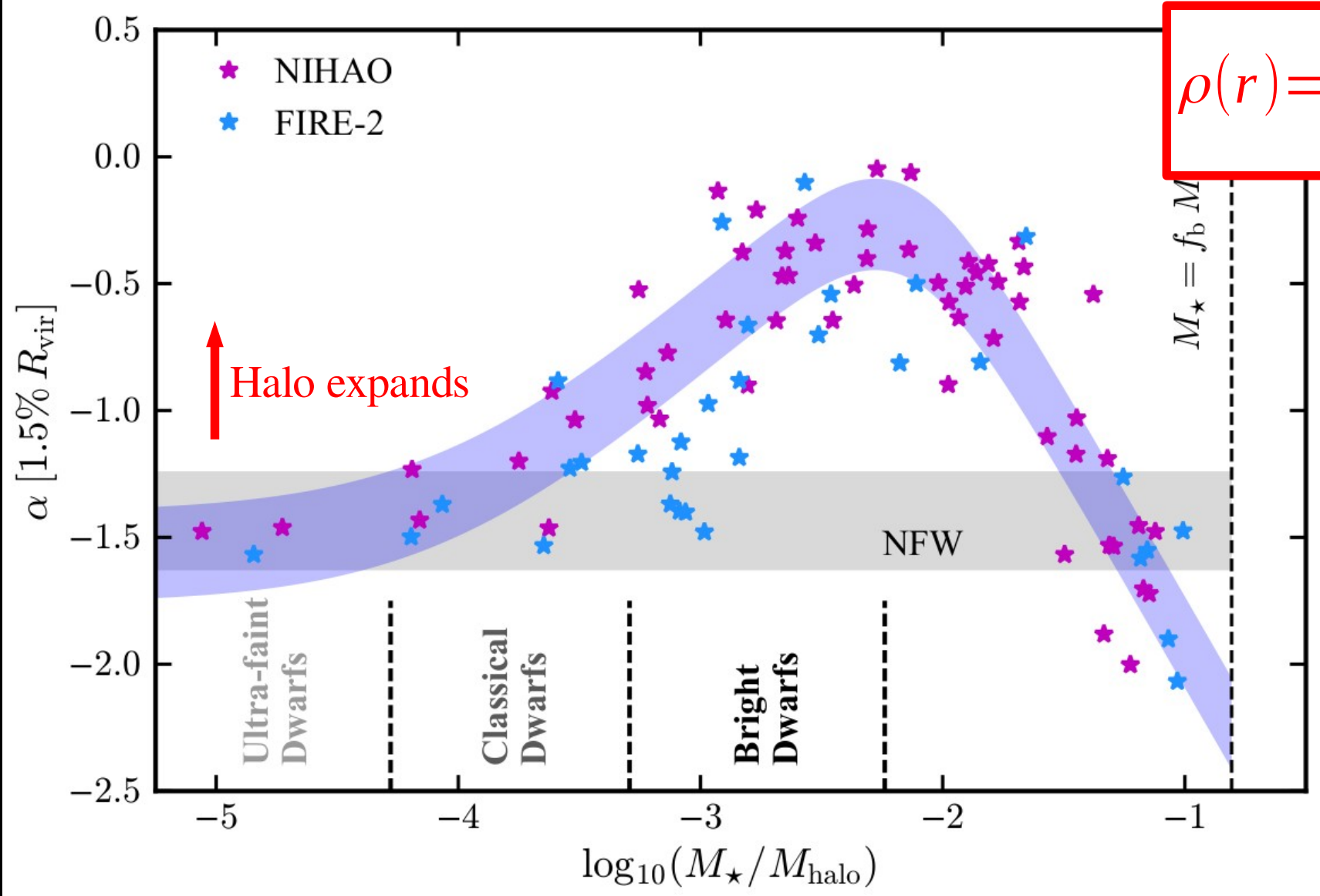
Burkert profile (empirical):

$$\rho_{DM}(r) = \frac{\rho_c}{(1+r/r_c)[1+(r/r_c)^2]}$$

Pseudo-isothermal profile (empirical):

$$\rho_{DM}(r) = \frac{\rho_c}{1+(r/r_c)^2}$$

Core Formation from Stellar Feedback



$$\rho(r) = \frac{4\rho_c}{(r/r_c)^\alpha (1+(r/r_c)^\gamma)^{(\beta-\alpha)/\gamma}}$$

$\alpha\beta\gamma$ profile fitted to DM halos from hydro sims of galaxy formation with stellar feedback

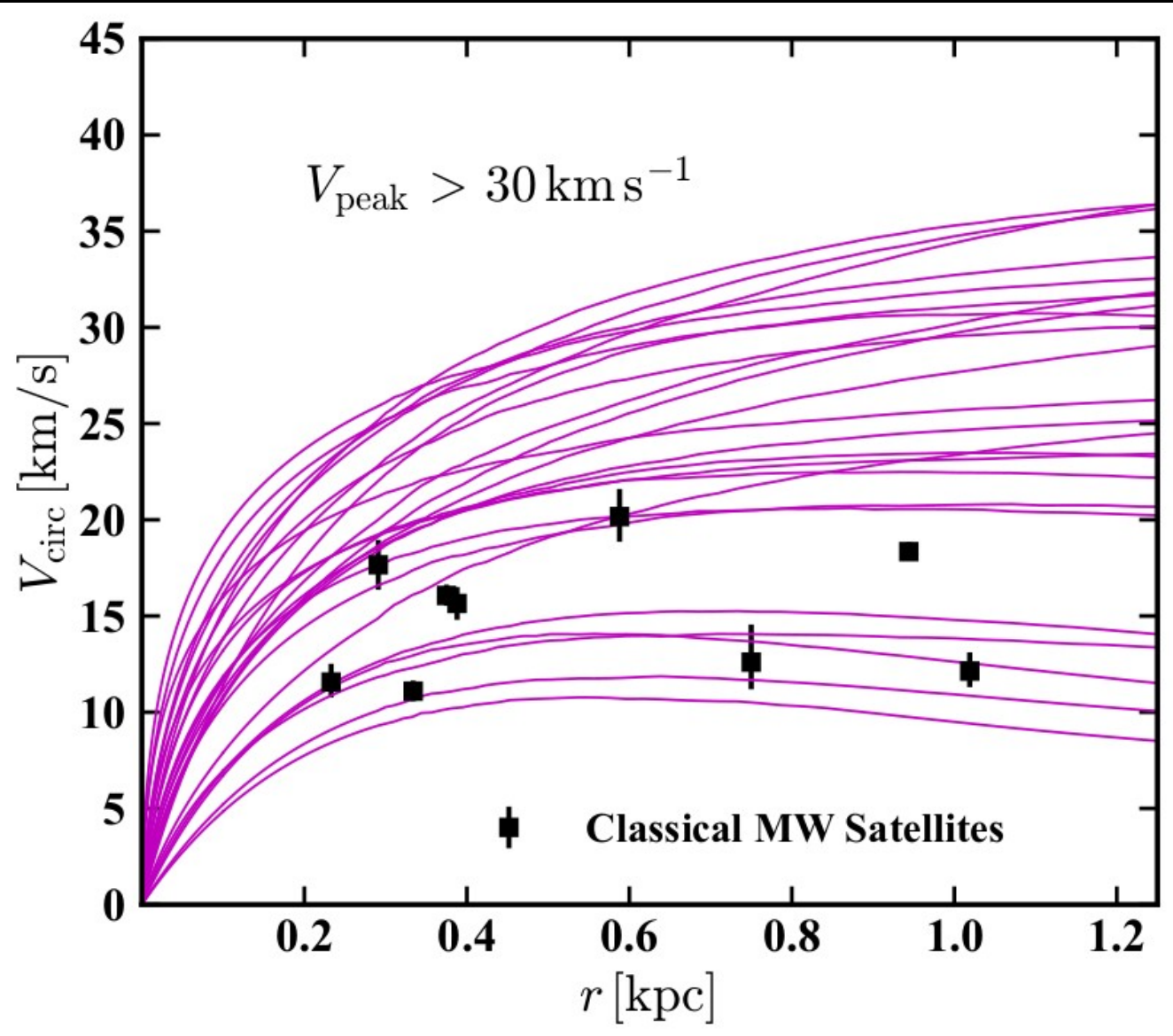
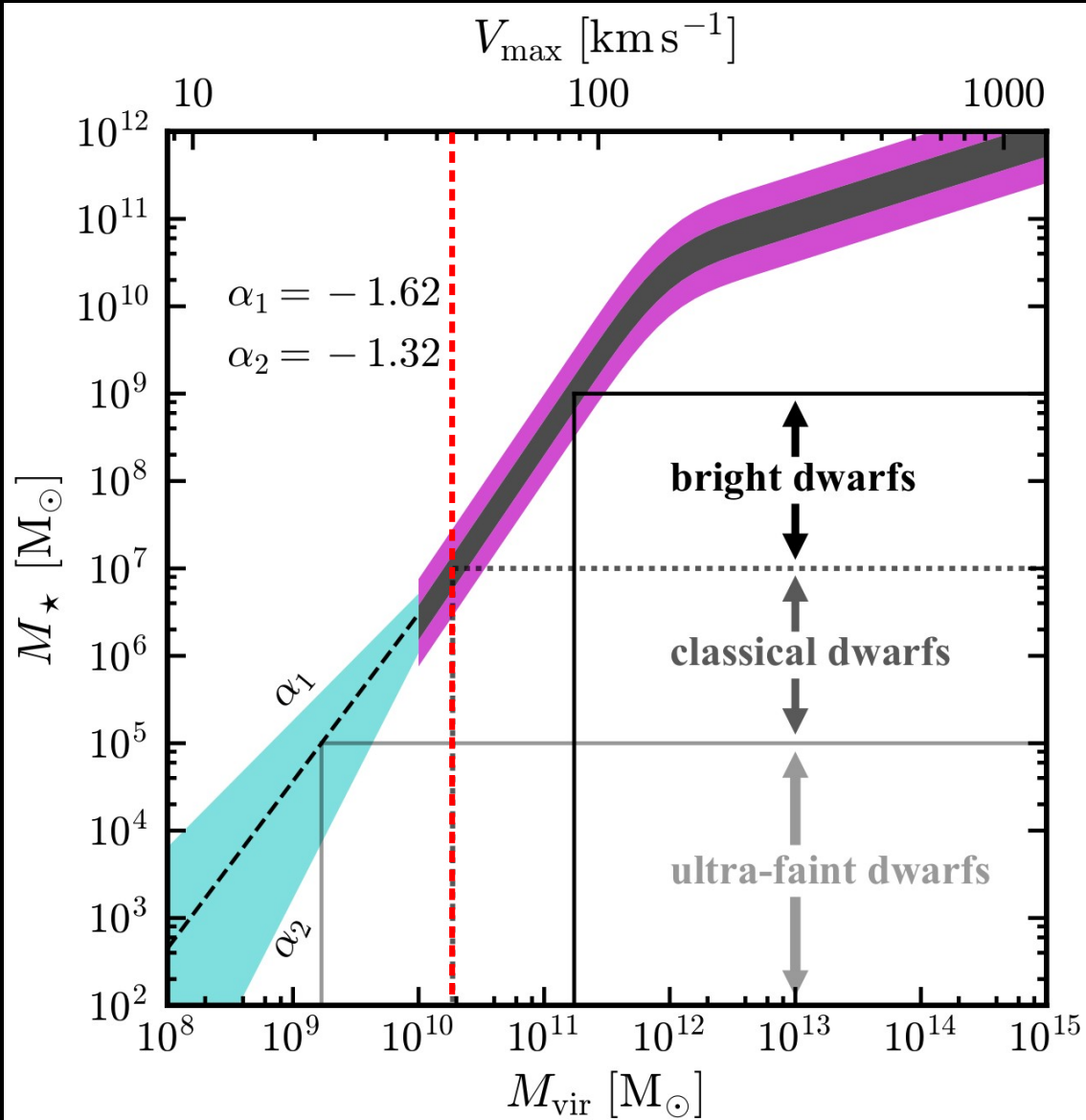
$$\alpha = f_1(M_{\star}/M_{\text{halo}})$$

$$\beta = f_2(M_{\star}/M_{\text{halo}})$$

$$\gamma = f_3(M_{\star}/M_{\text{halo}})$$

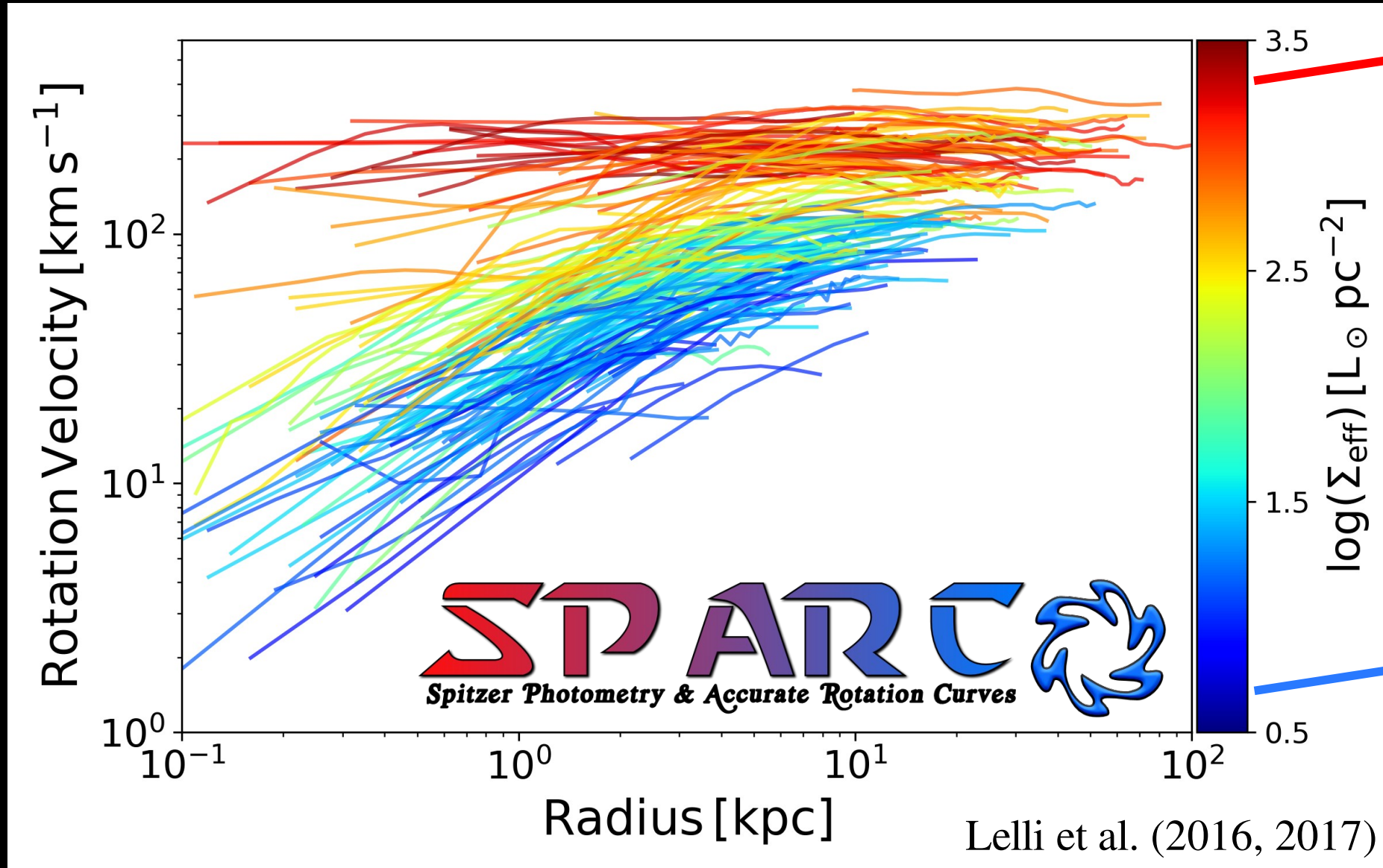
3. Too-Big-To-Fail Problem (Problems 1+2 Reloaded)

Too-Big-To-Fail Problem



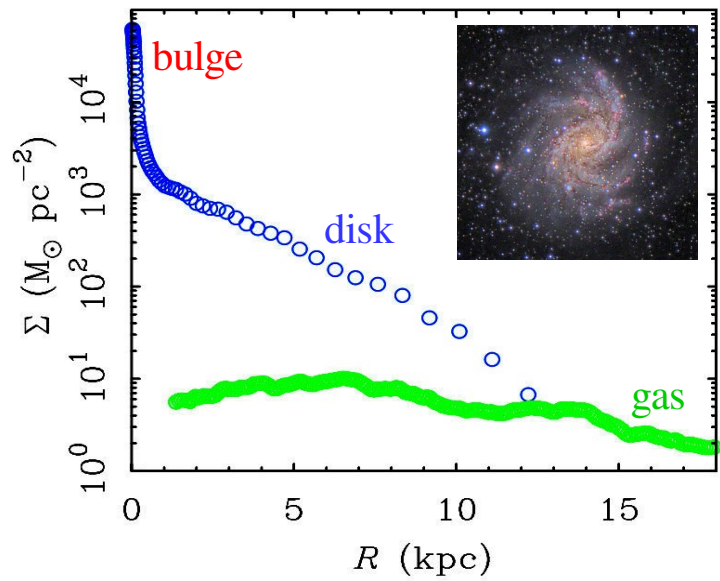
4. Regularity vs Diversity (Baryon-DM Coupling)

Rotation Curves \leftrightarrow Baryon Distribution

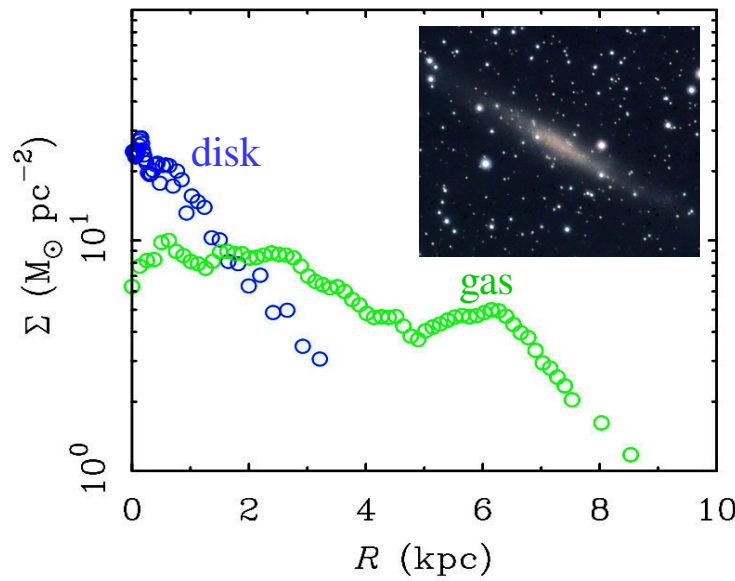


High surface brightness
(very dense) galaxies
→ Steeply Rising RCs
→ Baryon dominated
in the inner regions

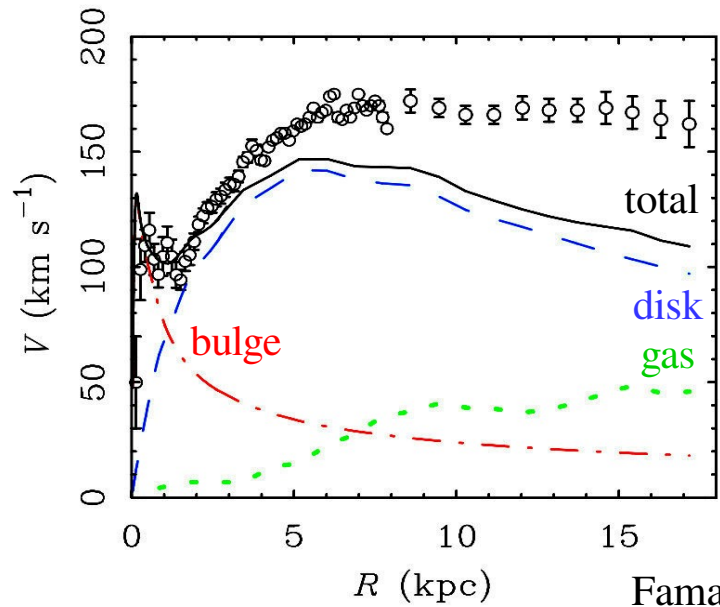
Low surface brightness
(very diffuse) galaxies
→ Slowly Rising Rcs
→ DM dominated down
to the inner regions



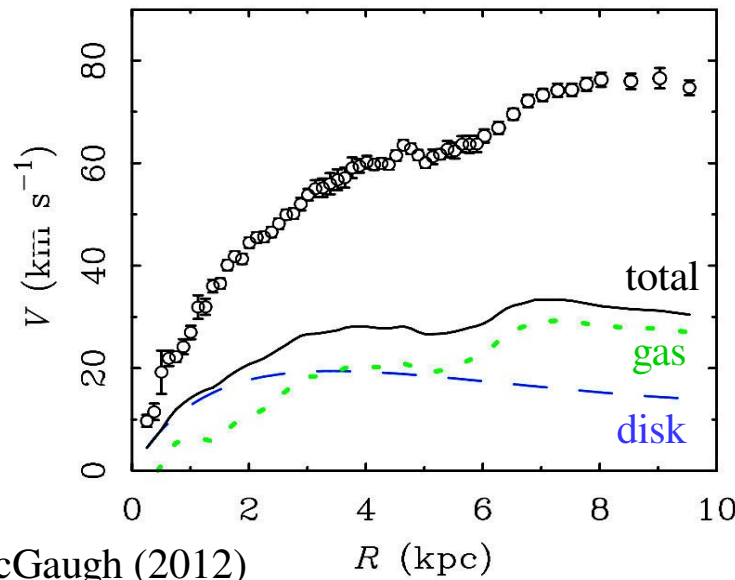
Baryon-dominated Spiral



DM-dominated Dwarf

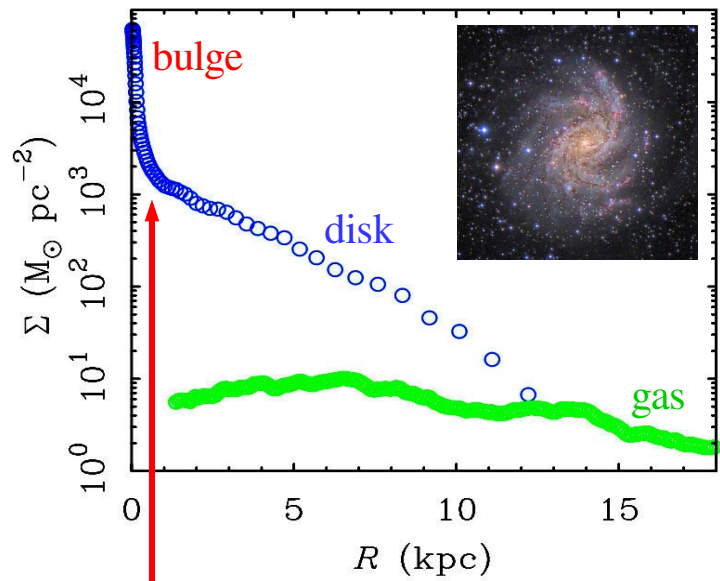


Famaey & McGaugh (2012)

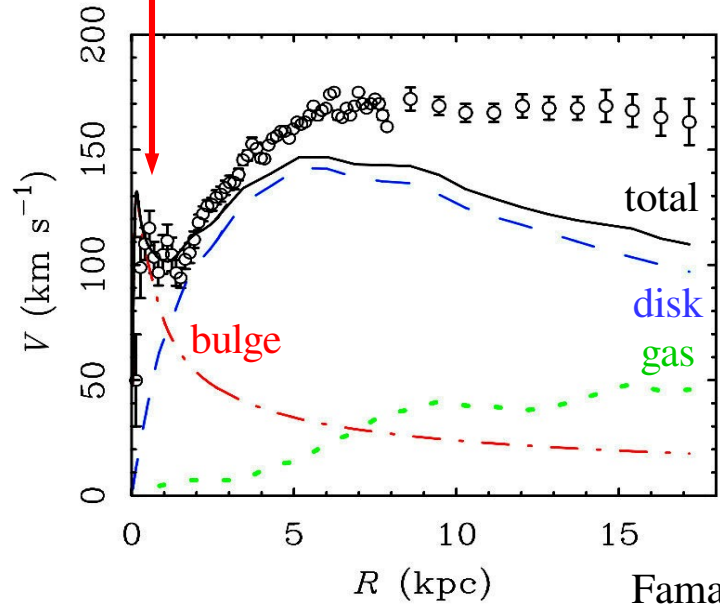


Baryons ↔ Dynamics

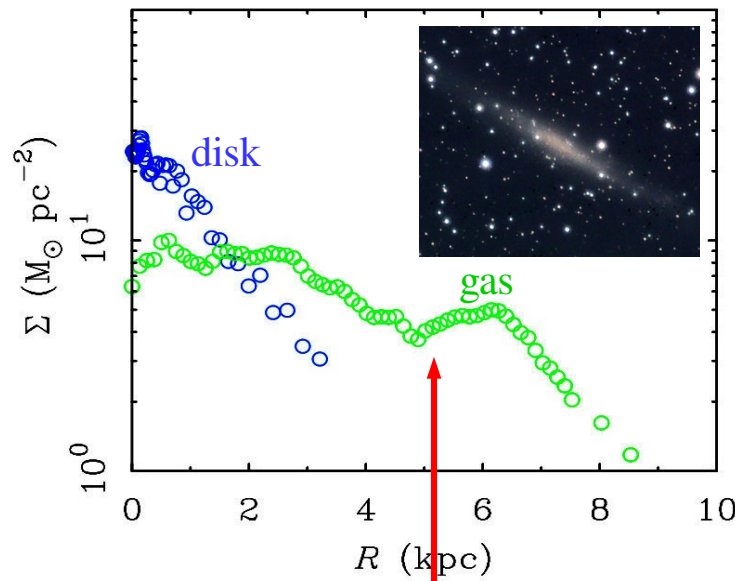
Rotation curves are flat at very large radii, but can display structures/features in the inner regions.



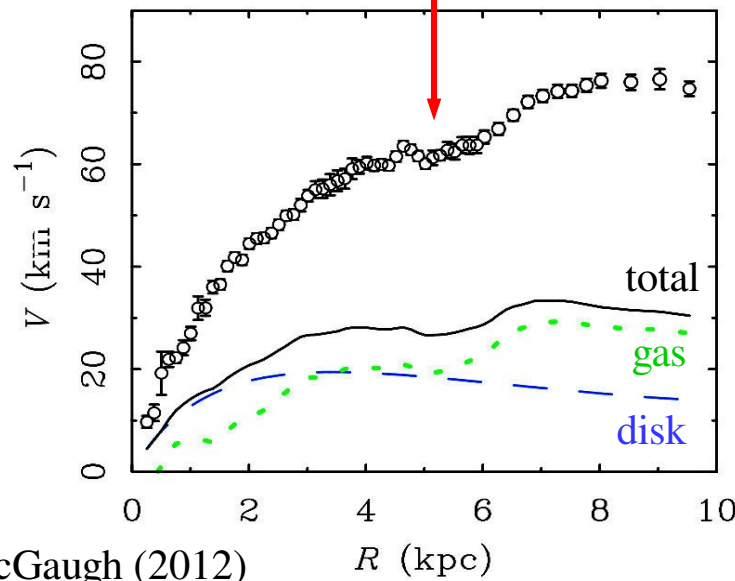
Baryon-dominated Spiral



Famaey & McGaugh (2012)



DM-dominated Dwarf



Baryons ↔ Dynamics

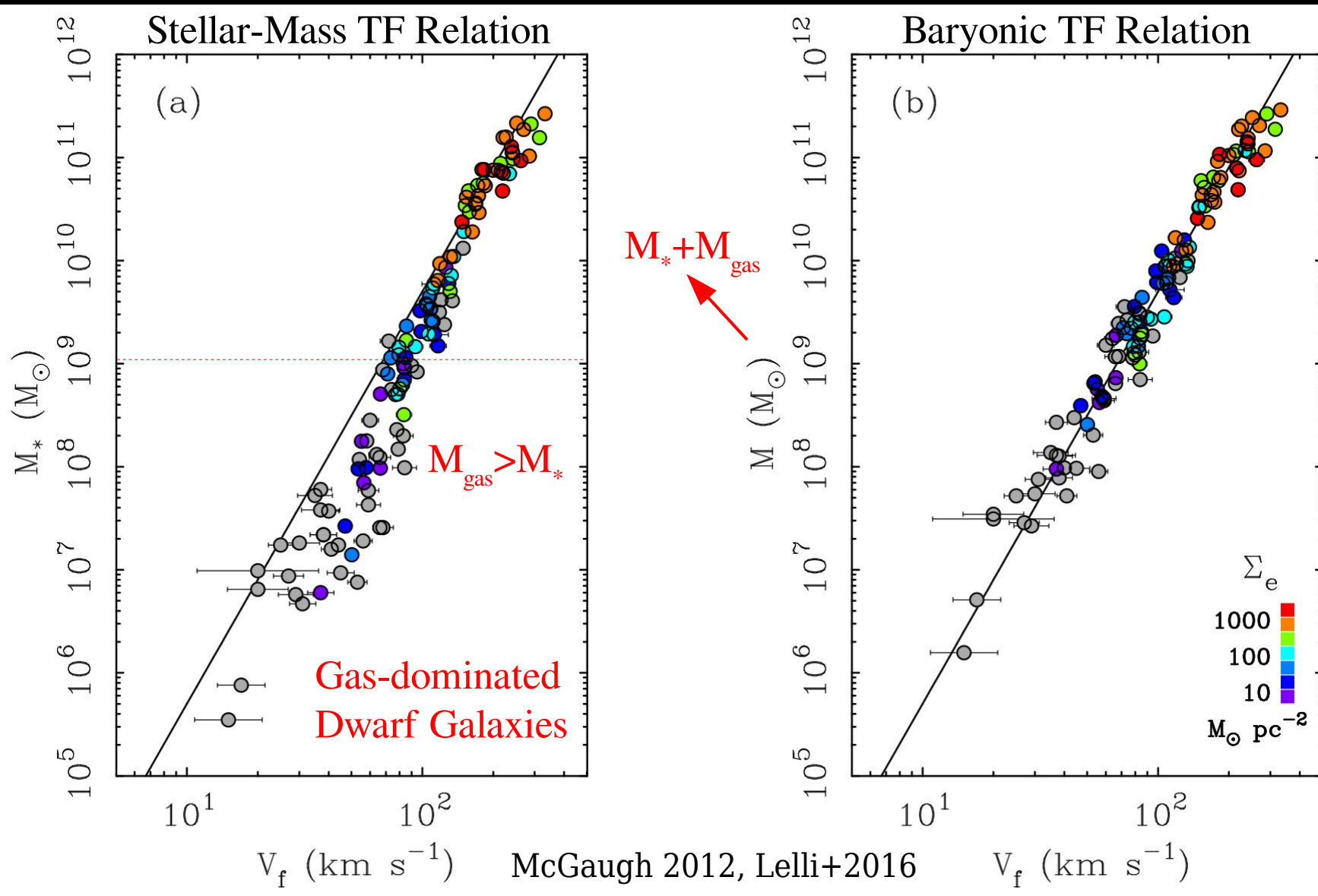
Rotation curves are flat at very large radii, but can display structures/features in the inner regions.



Renzo's Rule (Sancisi 2004):

“For any feature in the luminosity profile of a galaxy, there is a corresponding feature in the rotation curve, and vice versa”

Tully Fisher Relation: Mass vs Velocity



Newton's Law gives:

$$V^4 = \frac{\pi^2 G^2}{f_b^2} \Sigma_b M_b$$

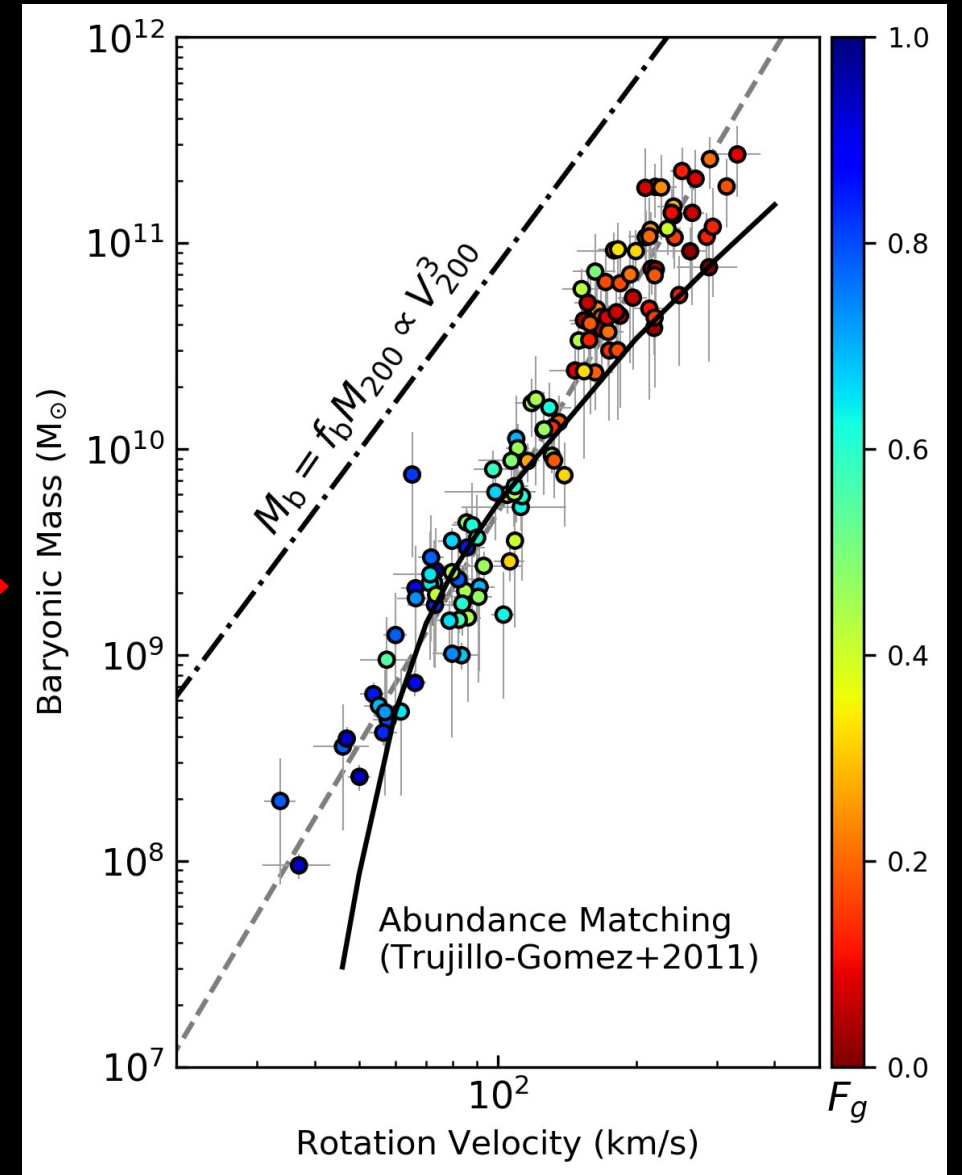
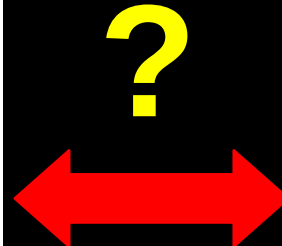
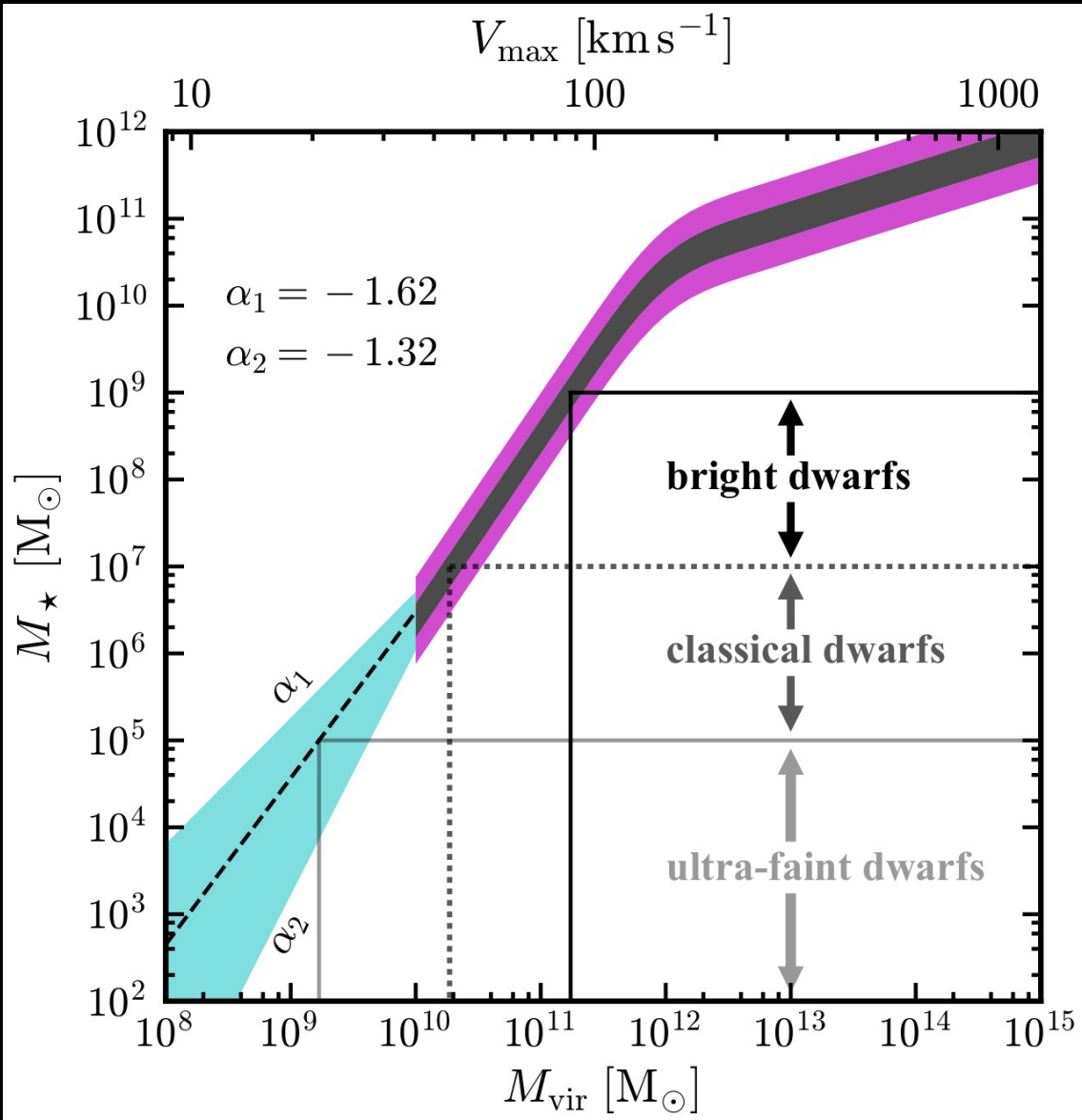
$$f_b = M_b / M_{\text{tot}}$$

But no dependence on Σ_b is observed.

$$\rightarrow \Sigma_b / f_b^2 \sim \text{const}$$

Fine-tuning problem

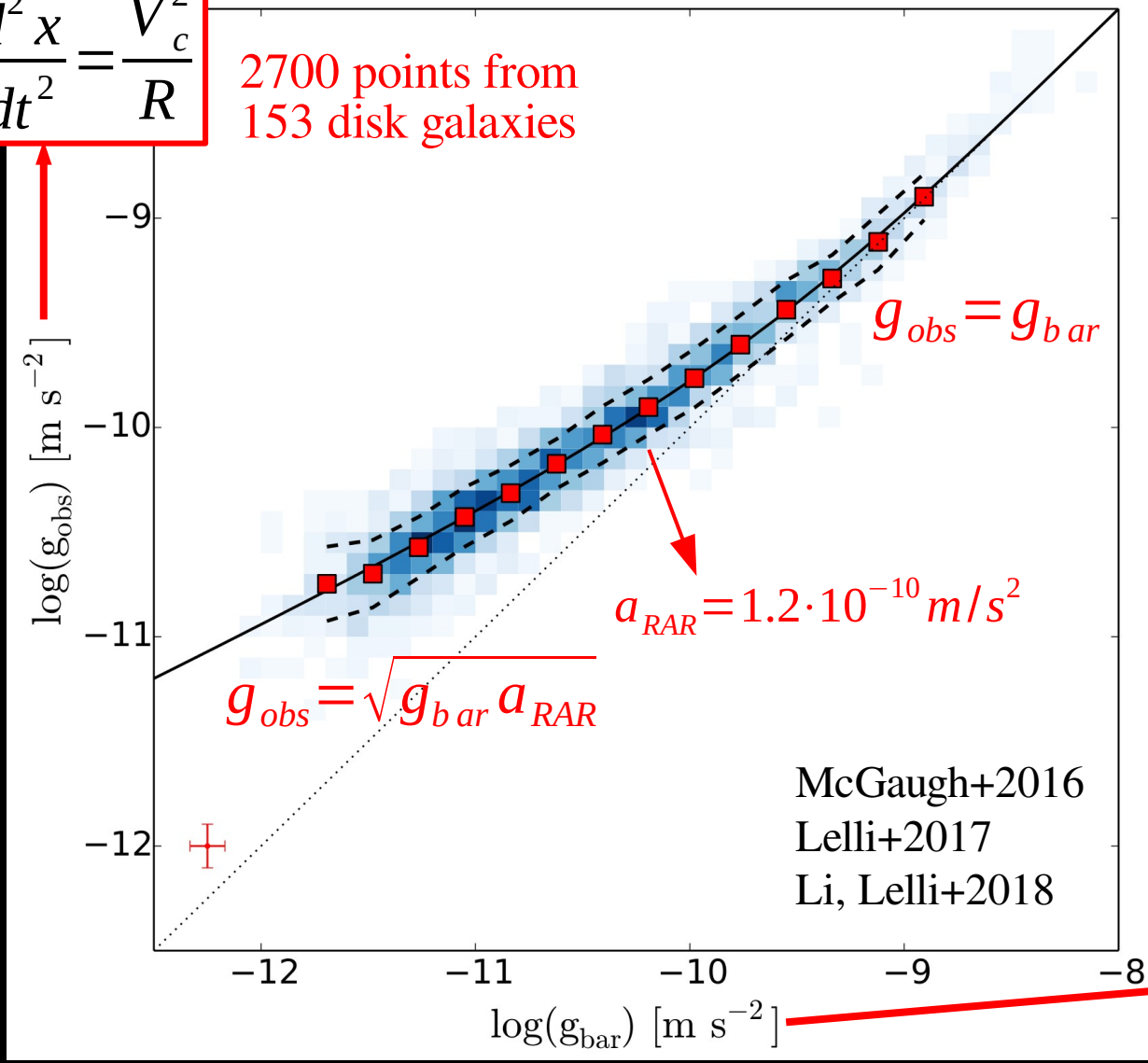
Abundance Matching → Curvated BTFR



Radial Acceleration Relation (RAR)

$$\frac{d^2 x}{dt^2} = \frac{V_c^2}{R}$$

2700 points from
153 disk galaxies



Observables:

g_{obs} → centripetal acceleration from RCs

g_{bar} → gravitational field from baryons

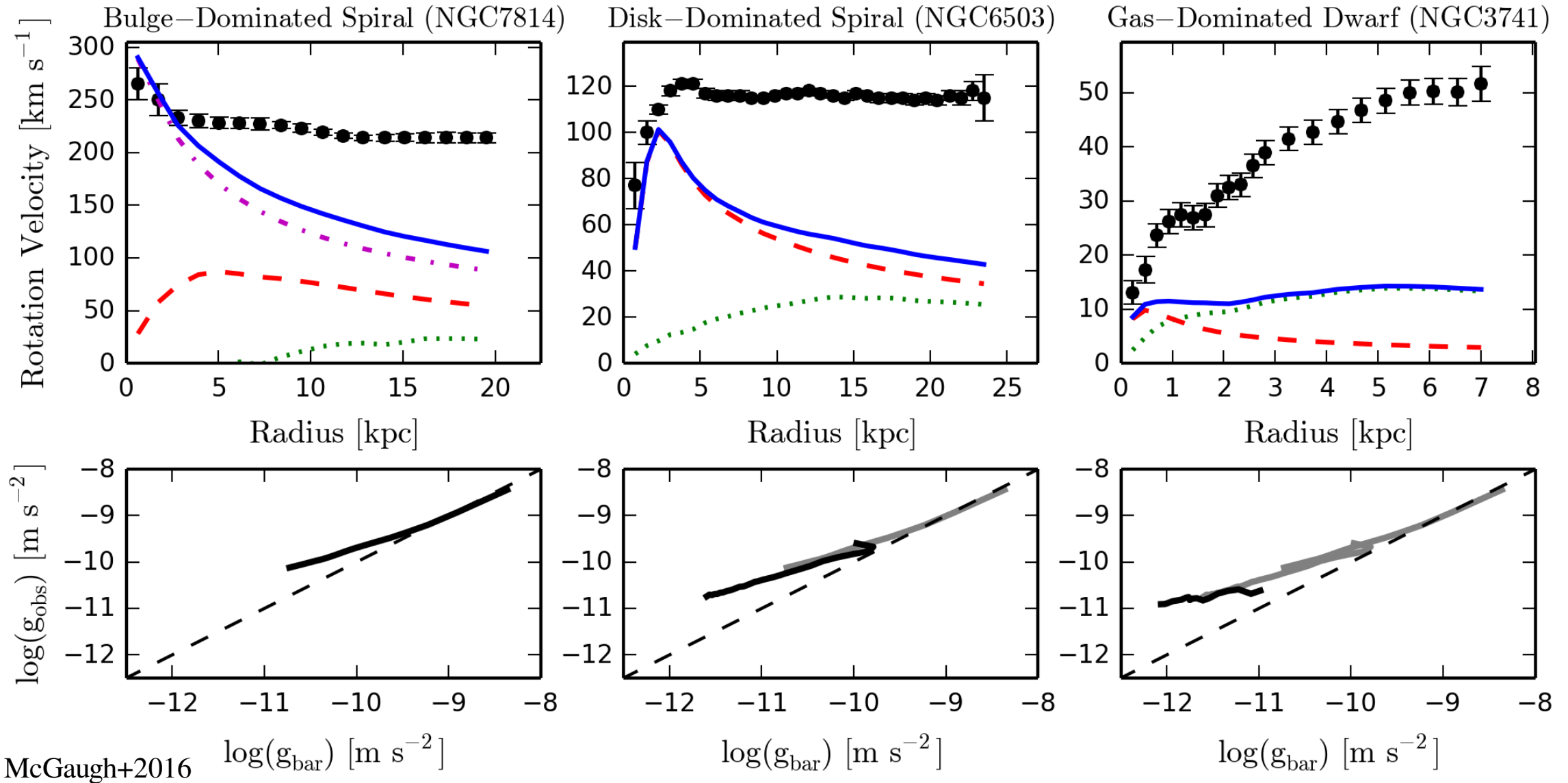
Key Properties:

- Acceleration scale $a_{\text{RAR}} \sim 10^{-10} \text{ m/s}^2$
- Small scatter (consistent with obs. errors)
- No residual dependencies (radius, etc.)
- Baryon distribution ↔ Rotation Curve

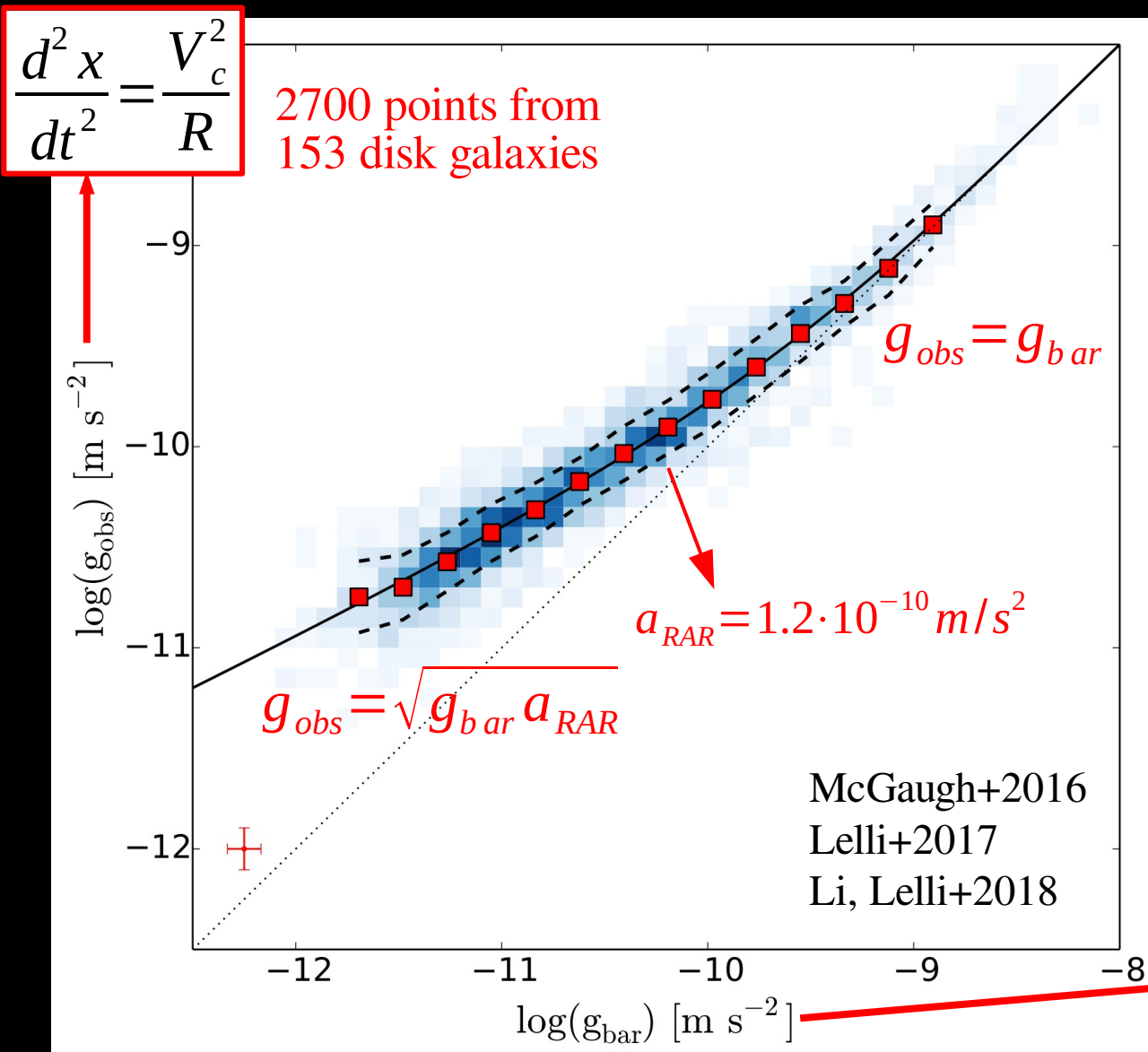
$$g_b = -\nabla \Phi_b$$

$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

Very different galaxies on the same RAR



RAR sets the DM halo profiles



$$g_{DM} = g_{obs} - g_{bar} = F(g_{bar})$$

$$M_{DM}(R) = \frac{R^2}{G} F(g_{bar})$$

No freedom to fit arbitrary DM halos!

“Cusp vs Core” is a symptom of a more serious general illness:

Baryon-DM coupling at any radii!

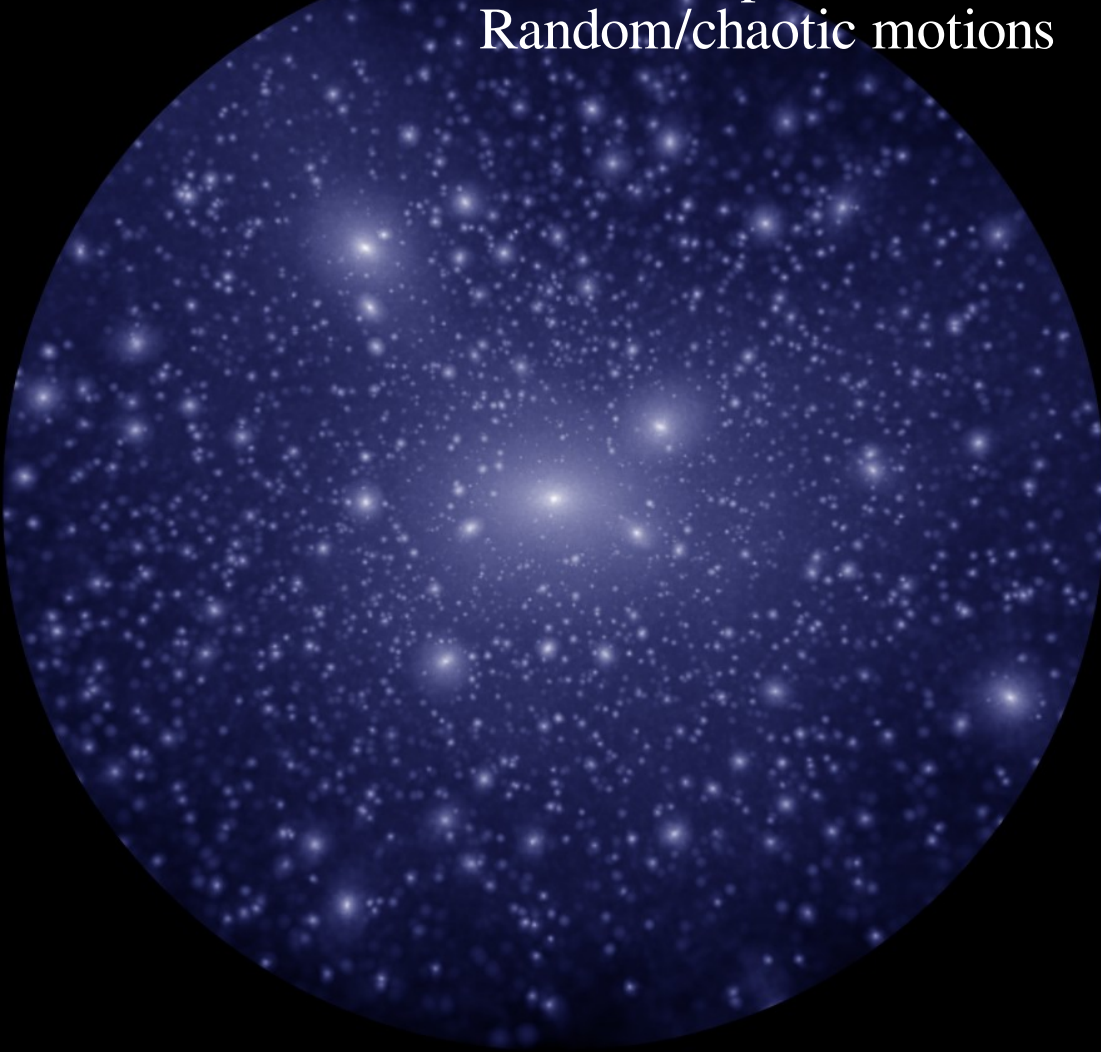
$$g_b = -\nabla \Phi_b$$

$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

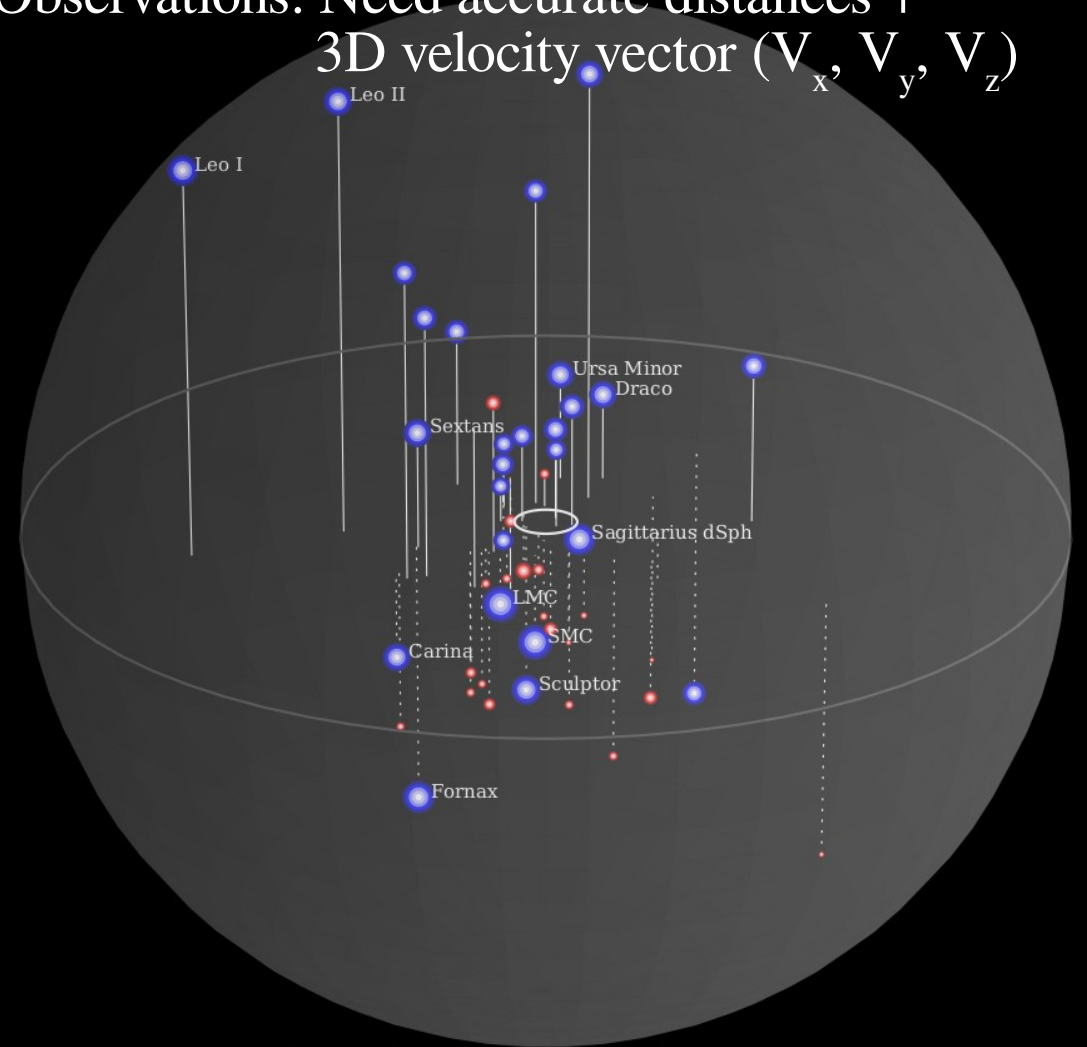
5. Planes of Satellites Problem (Satellites Phase-Space)

Distribution & Kinematics of Satellites

Λ CDM simulations: Near-isotropic distribution
Random/chaotic motions

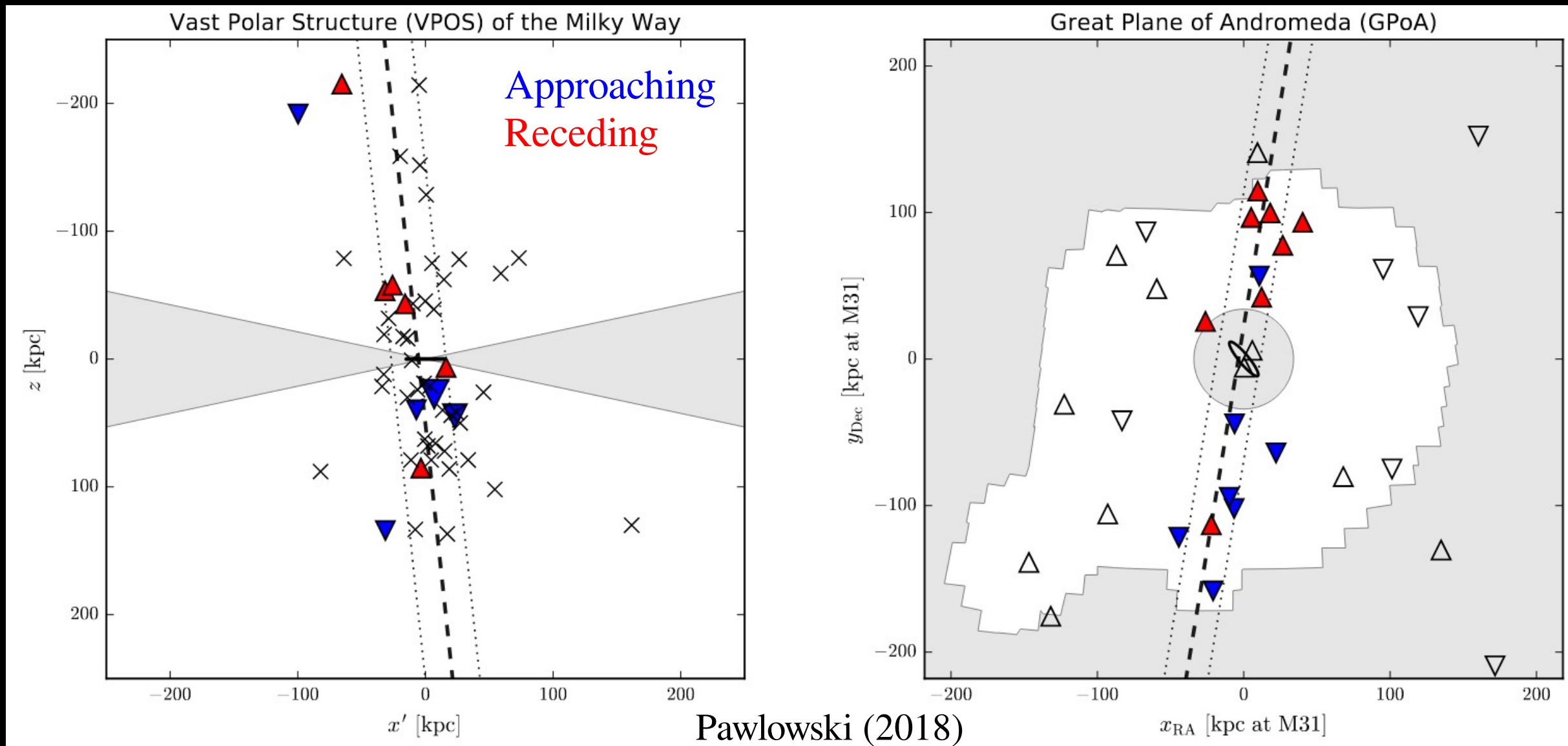


Observations: Need accurate distances +
3D velocity vector (V_x, V_y, V_z)

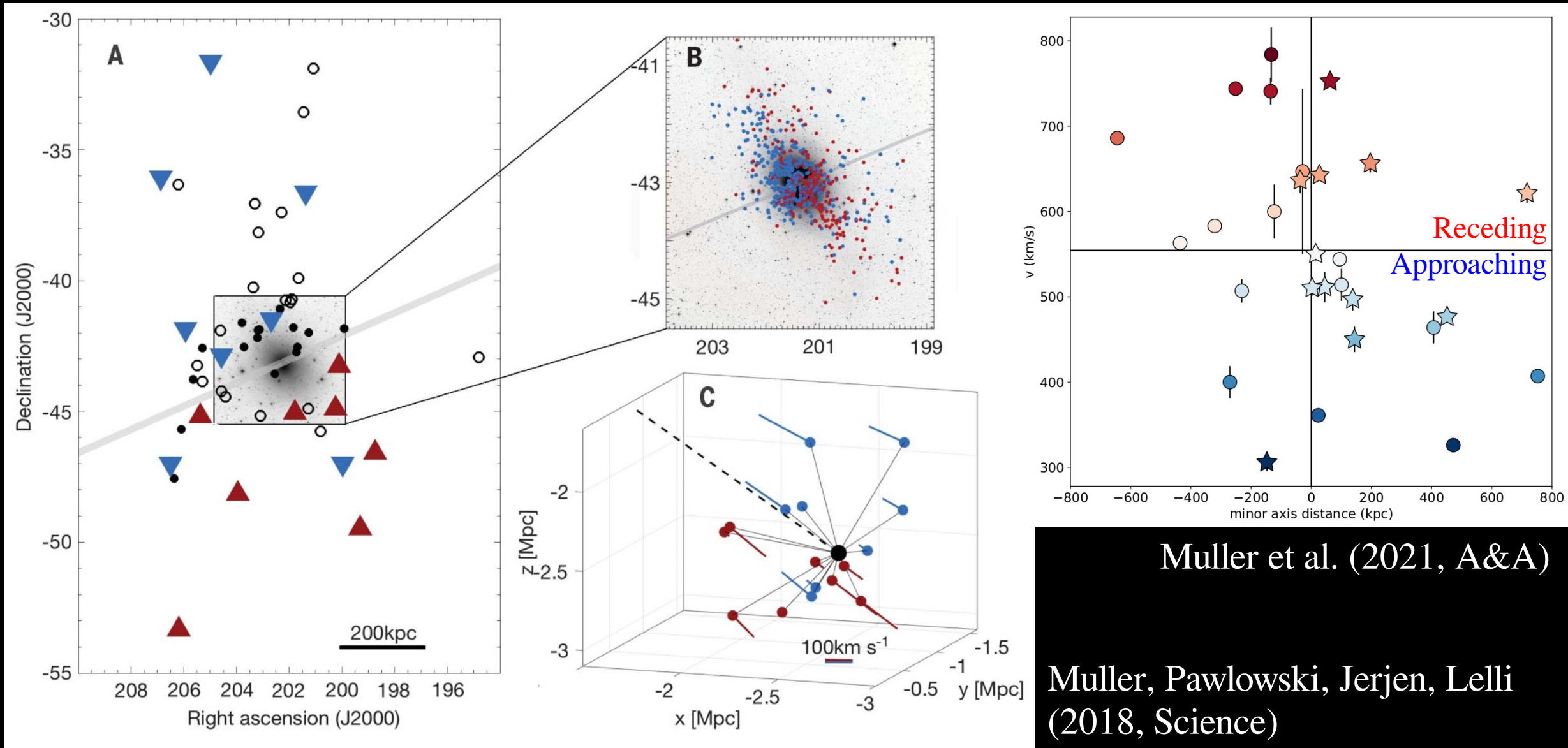


Pawlowski/Bullock/Boylan-Kolchin

Planes of Satellites in the Local Group



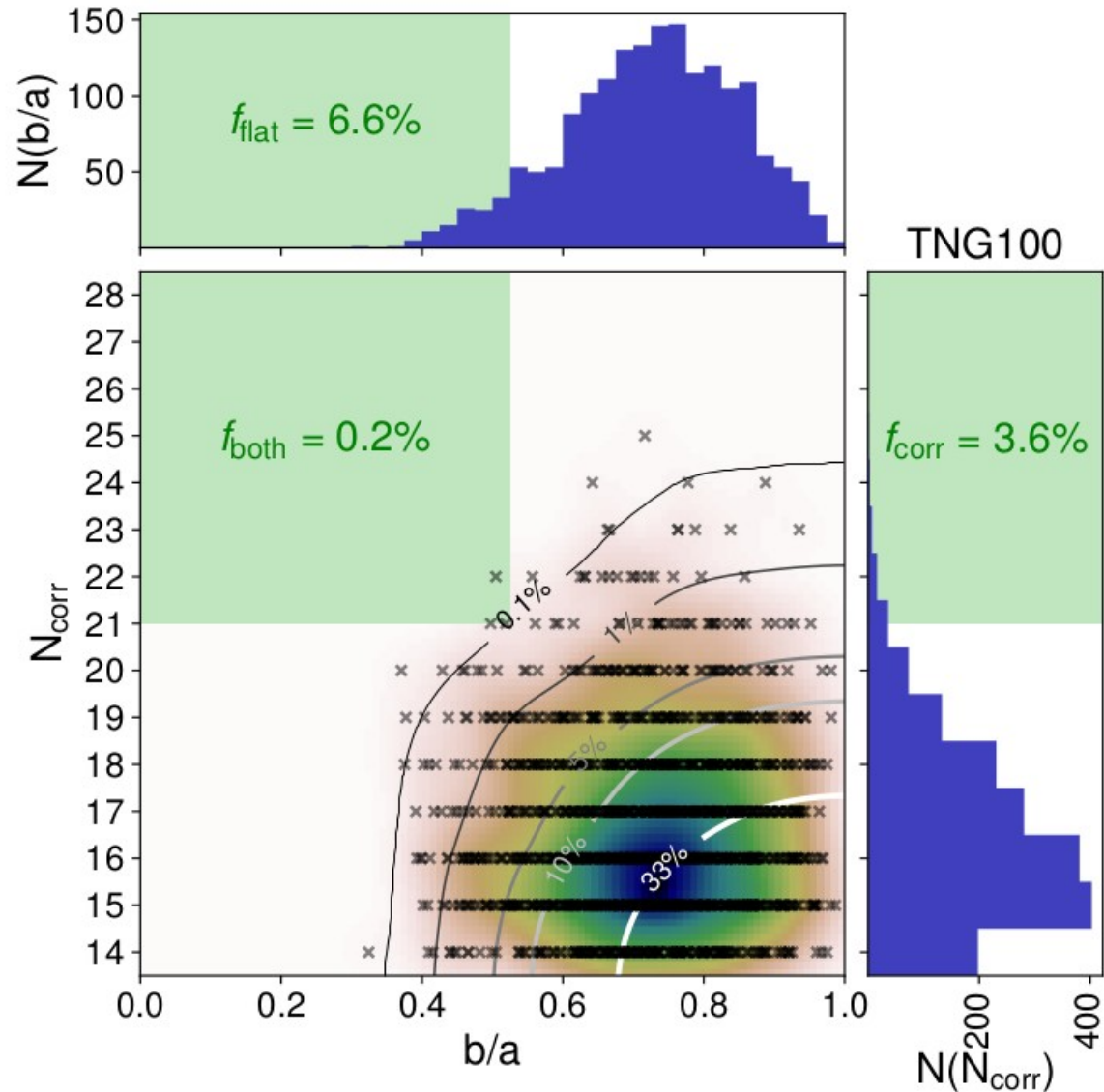
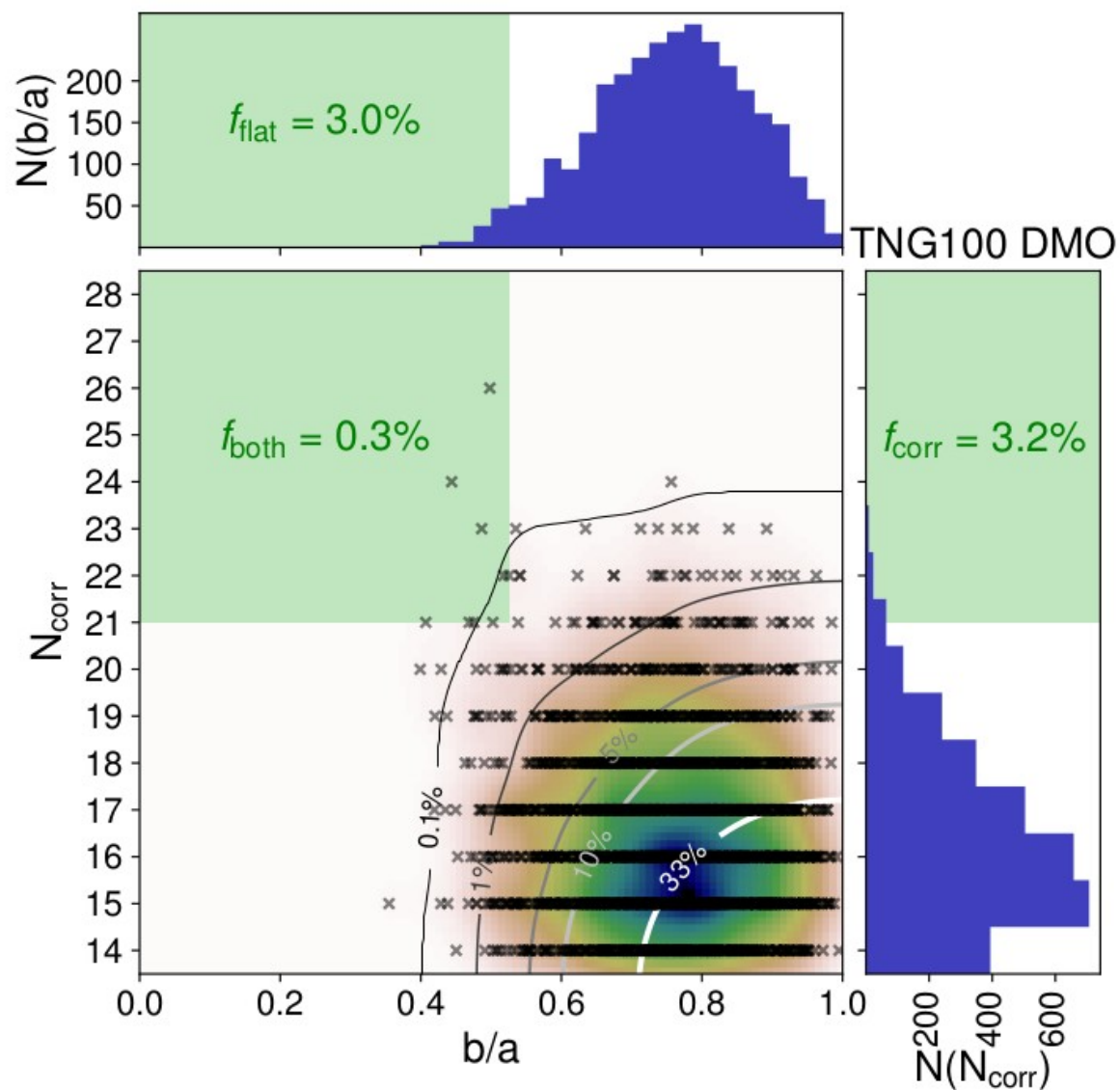
Planes of Satellites in Centaurus A



Muller et al. (2021, A&A)

Muller, Pawlowski, Jerjen, Lelli
(2018, Science)

Baryon Physics play NO role here



Small Scale Λ CDM Problems:

1. Missing Satellites Problem

2. Cusp vs Core Problem

3. Too-Big-Too-Fail Problem

Baryon Physics (stellar feedback)?

4. Regularity vs Diversity Problem \rightarrow Baryon-DM interaction?

5. Planes of Satellites Problem \rightarrow ?

Per tesi in astrofisica:

<http://astro.fisica.unifi.it/tesi/>