

The Tully-Fisher Relation

Federico Lelli

KES Lecture



Outline

1. Brief Historical Introduction

The HI 21-cm line and the Tully-Fisher relation

2. Physics Behind the TF relation

General implications for dark matter in galaxies

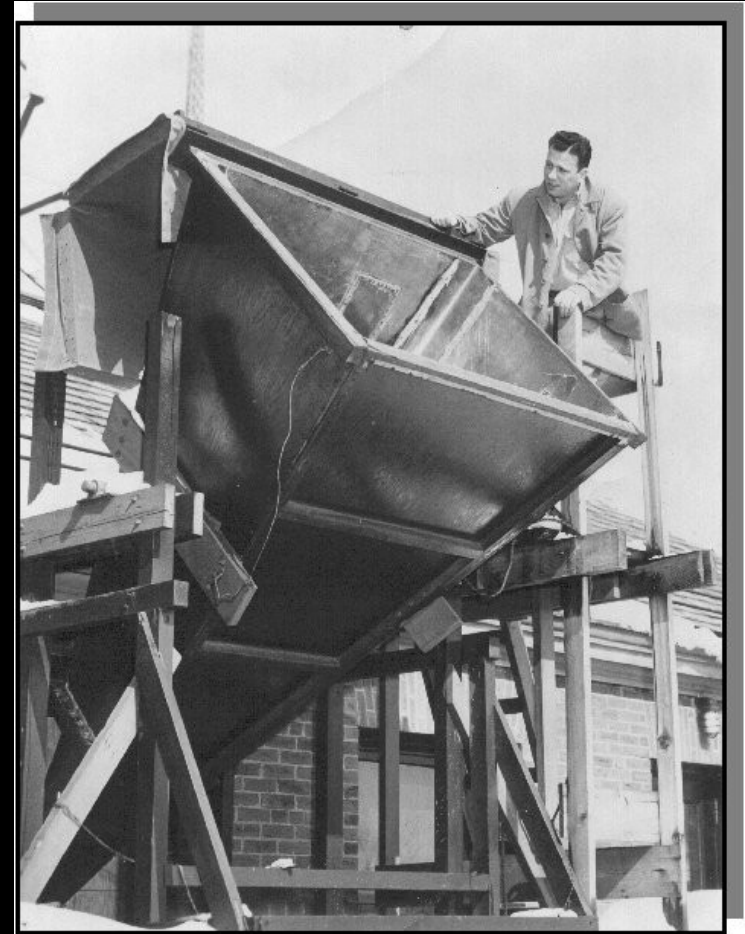
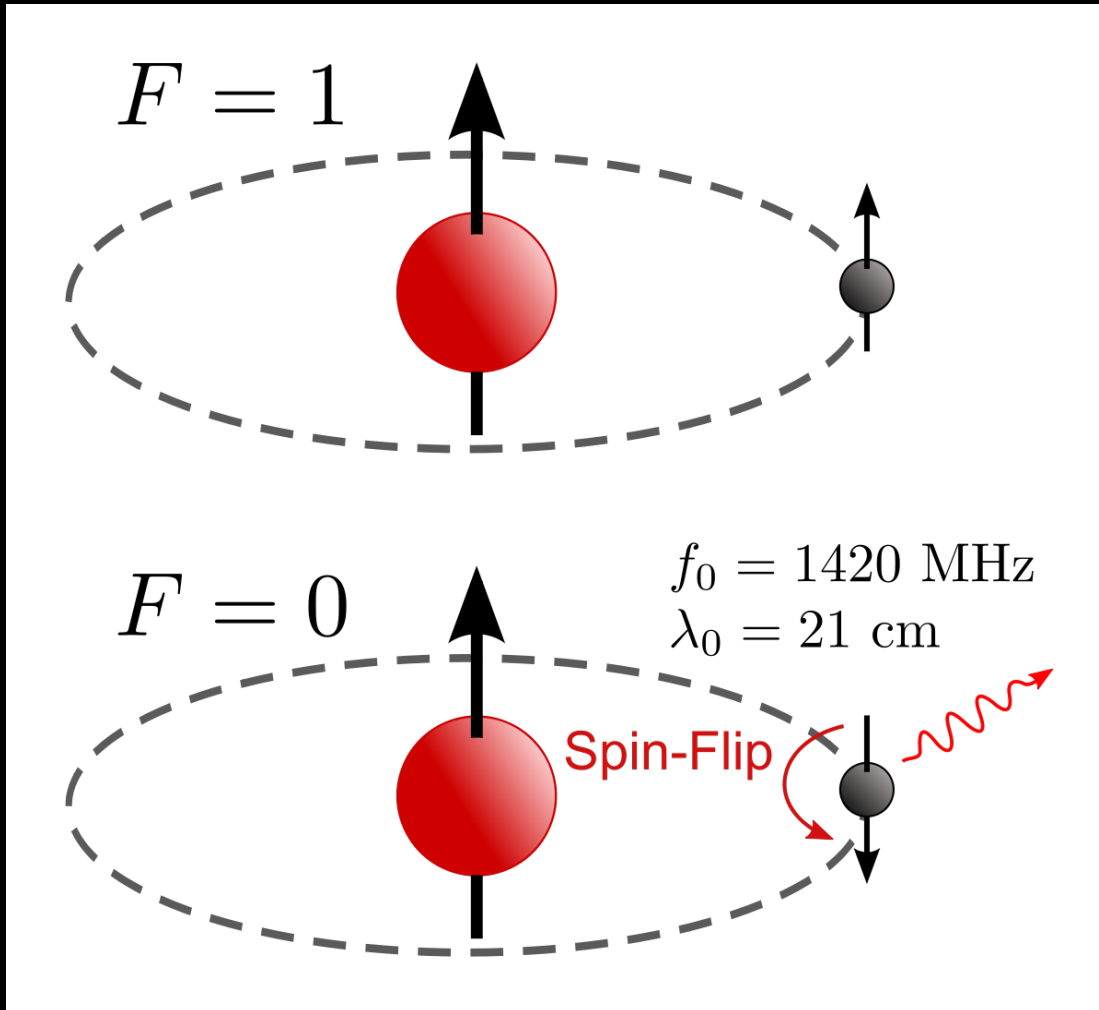
3. The TF relation in a LCDM context

General implications on missing baryons & more

1. Introduction

The 21-cm line of Atomic Hydrogen

- Hyperfine structure of Atomic Hydrogen (HI)
- Predicted to be observable by Van de Hulst (1944)
- First detected by Ewen & Percell (1951)



Ewen installing his antenna out of a window at Lyman Lab in Harvard

HI obs with single-dish radio telescopes

Resolution = λ/D if $\lambda=21\text{cm}$, we need a big D!

HI obs with single-dish radio telescopes

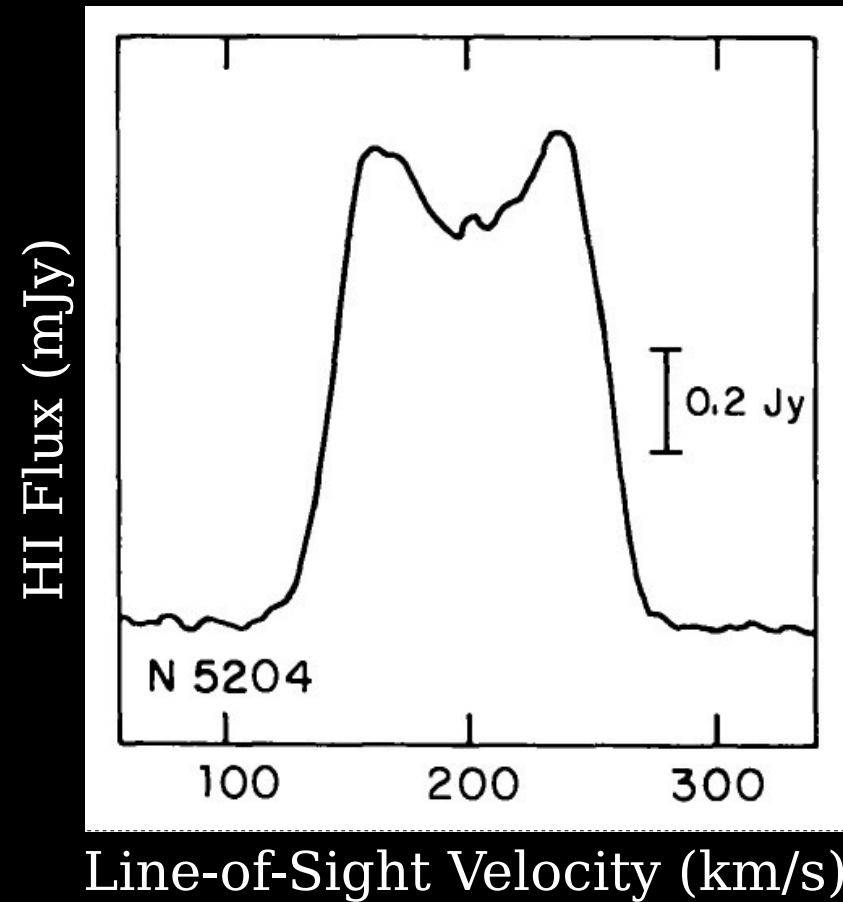
Resolution = λ/D if $\lambda=21\text{cm}$, we need a big D!

NRAO 91m and 43m telescopes, used by Fisher & Tully (1975)

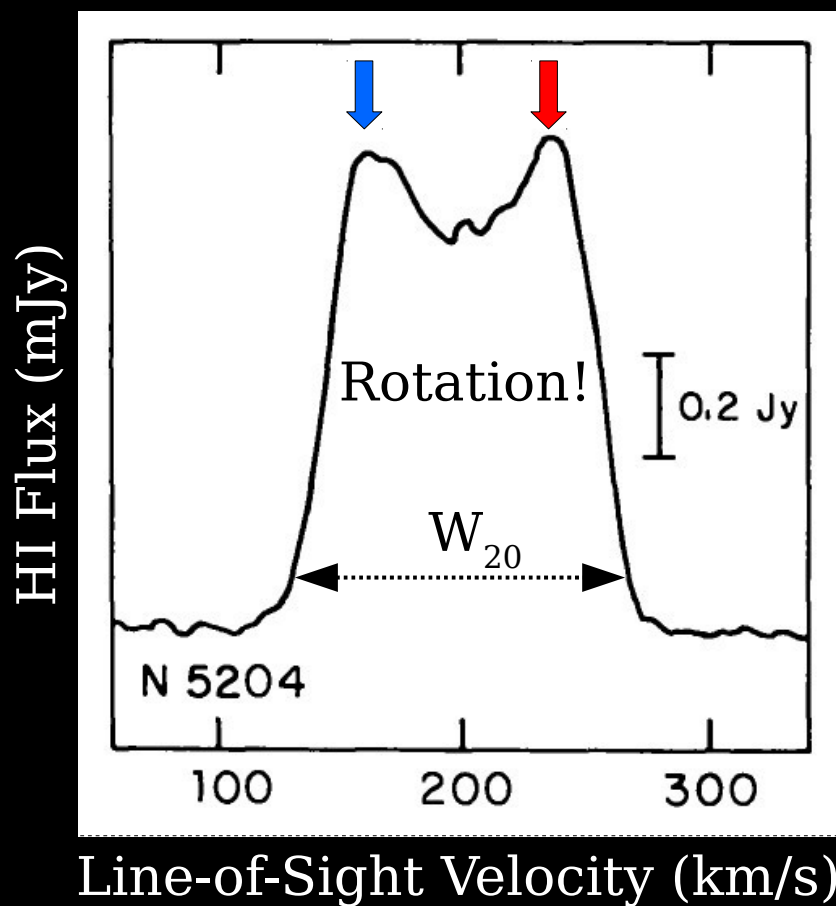
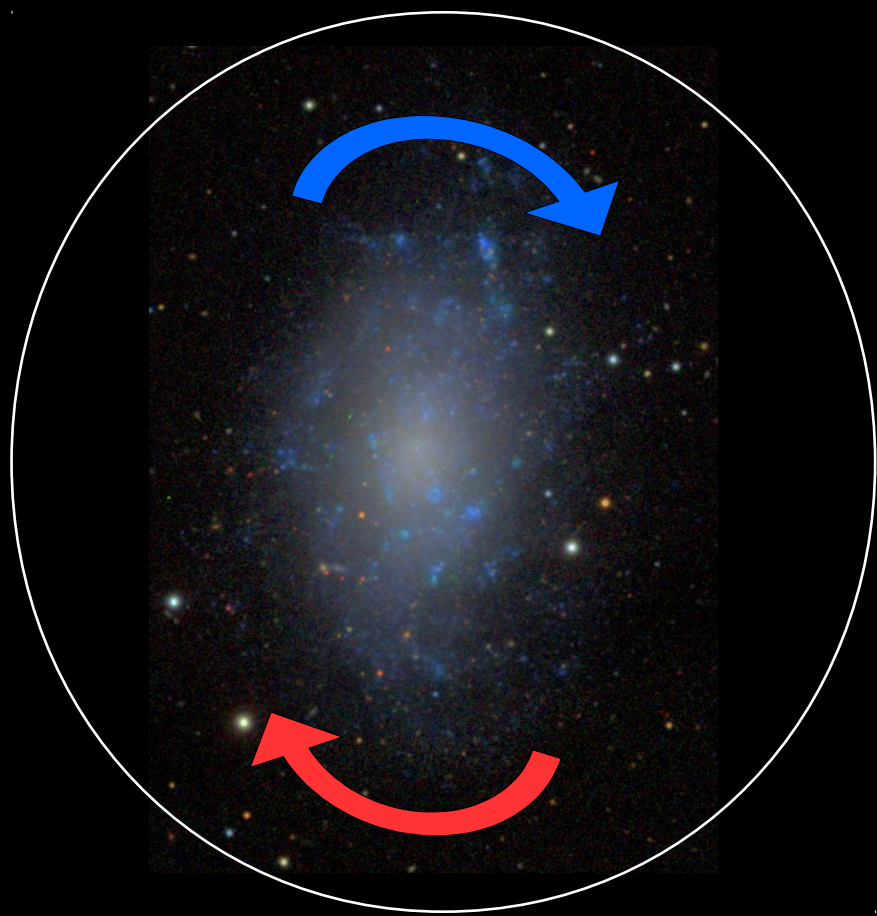


$D = 91 \text{ m} \rightarrow R \sim 8'$. Cannot resolve galaxies outside LG!
But the spectral resolution was good (down to $\sim 5 \text{ km/s}$)

HI integrated spectra for galaxies

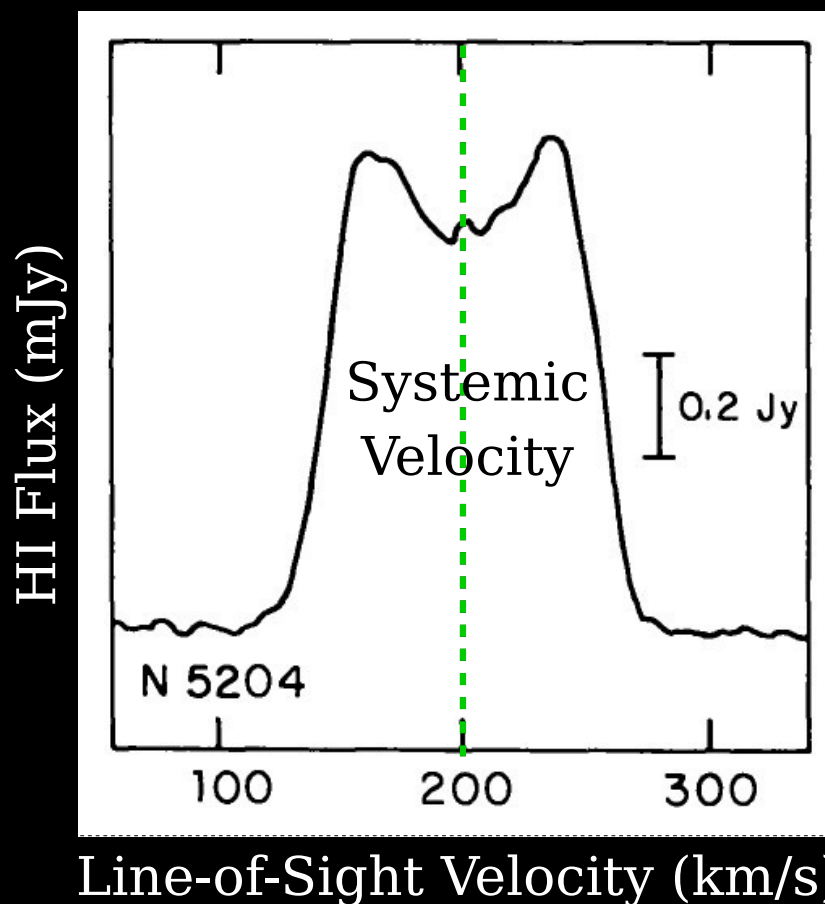


HI integrated spectra for galaxies



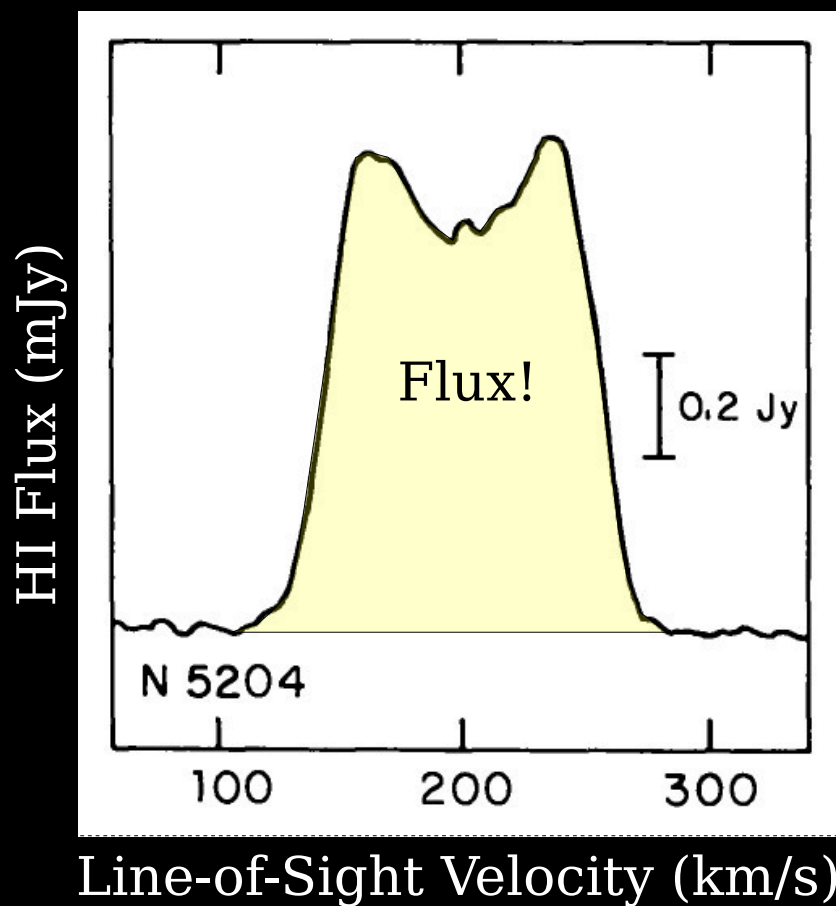
- HI Line-Width: W_{20} (20% of peak flux) ~ 2 rotation velocity

HI integrated spectra for galaxies



- HI Line-Width: W_{20} (20% of peak flux) ~ 2 rotation velocity
- Systemic Velocity / Redshift: $z \sim V_{\text{sys}} / c$ for low V_{sys}

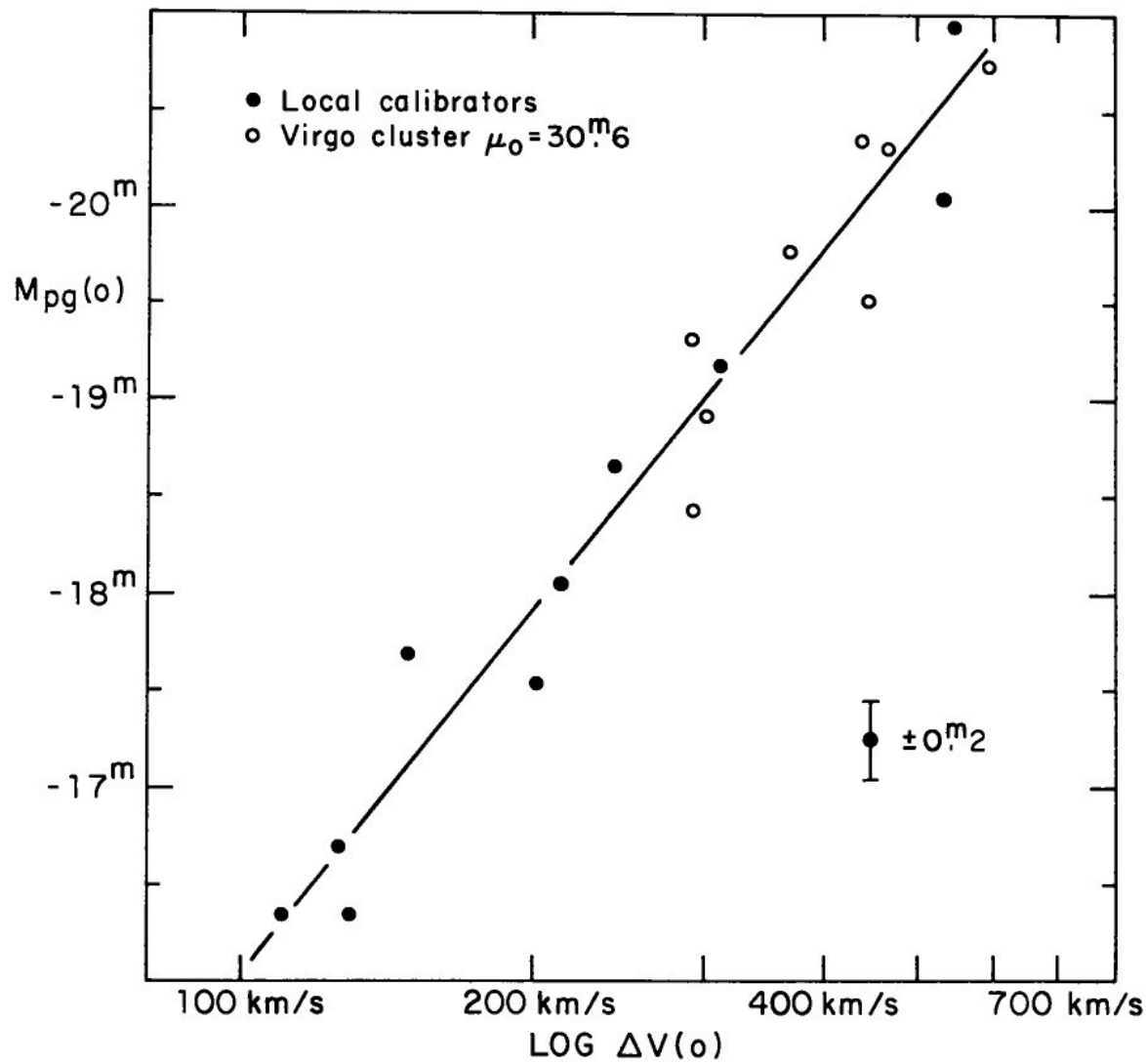
HI integrated spectra for galaxies



- HI Line-Width: W_{20} (20% of peak flux) ~ 2 rotation velocity
- Systemic Velocity / Redshift: $z \sim V_{\text{sys}} / c$ for low V_{sys}
- Total HI flux / HI mass: $M_{\text{HI}} = 236 D^2 [\text{Mpc}] S_{\text{HI}} [\text{mJy km/s}]$

The Original Tully-Fisher Relation (1977)

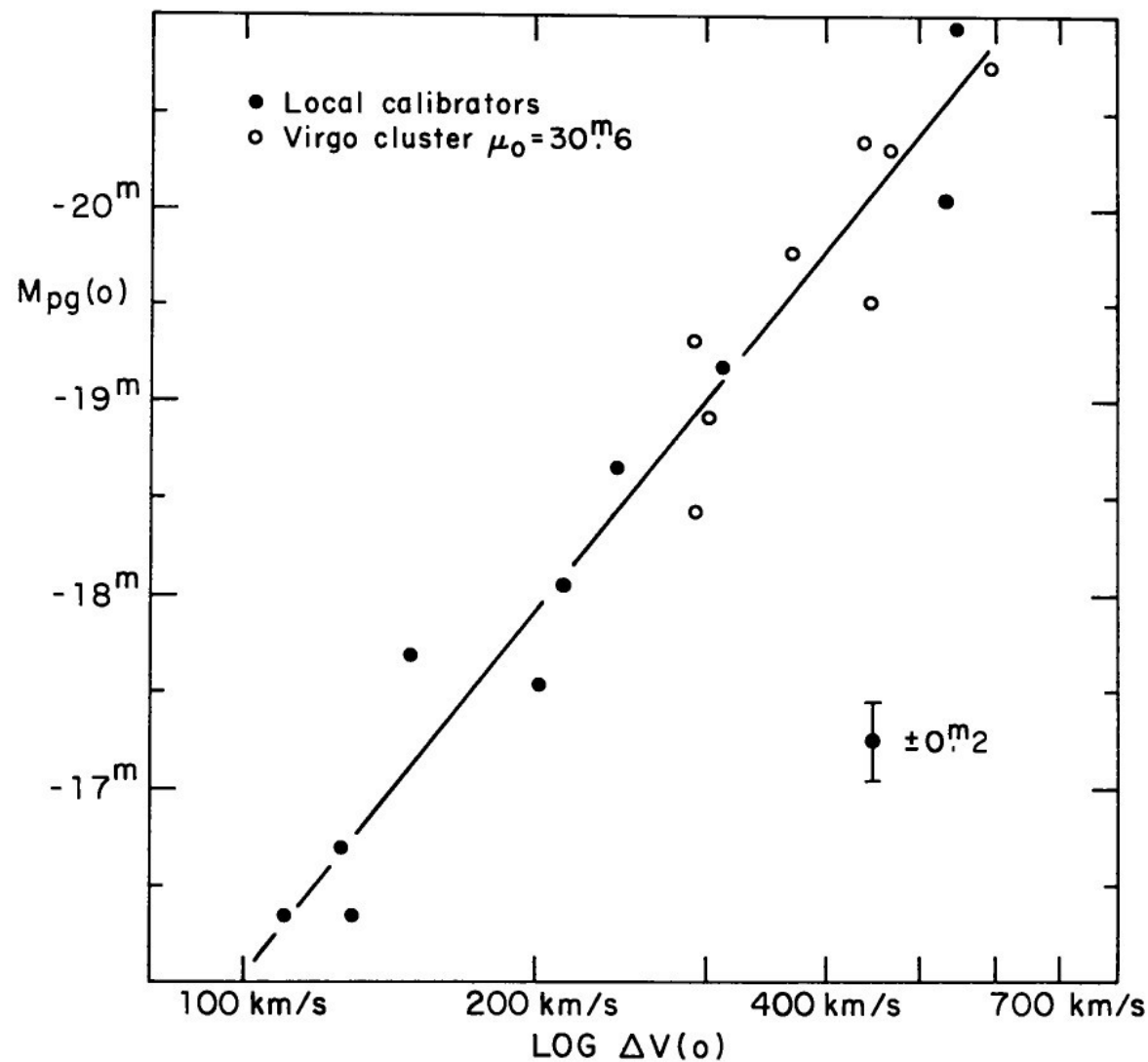
Absolute Magnitude (\propto Distance²)



HI Line-Width (Distance Independent)

The Original Tully-Fisher Relation (1977)

Absolute Magnitude (\propto Distance²)



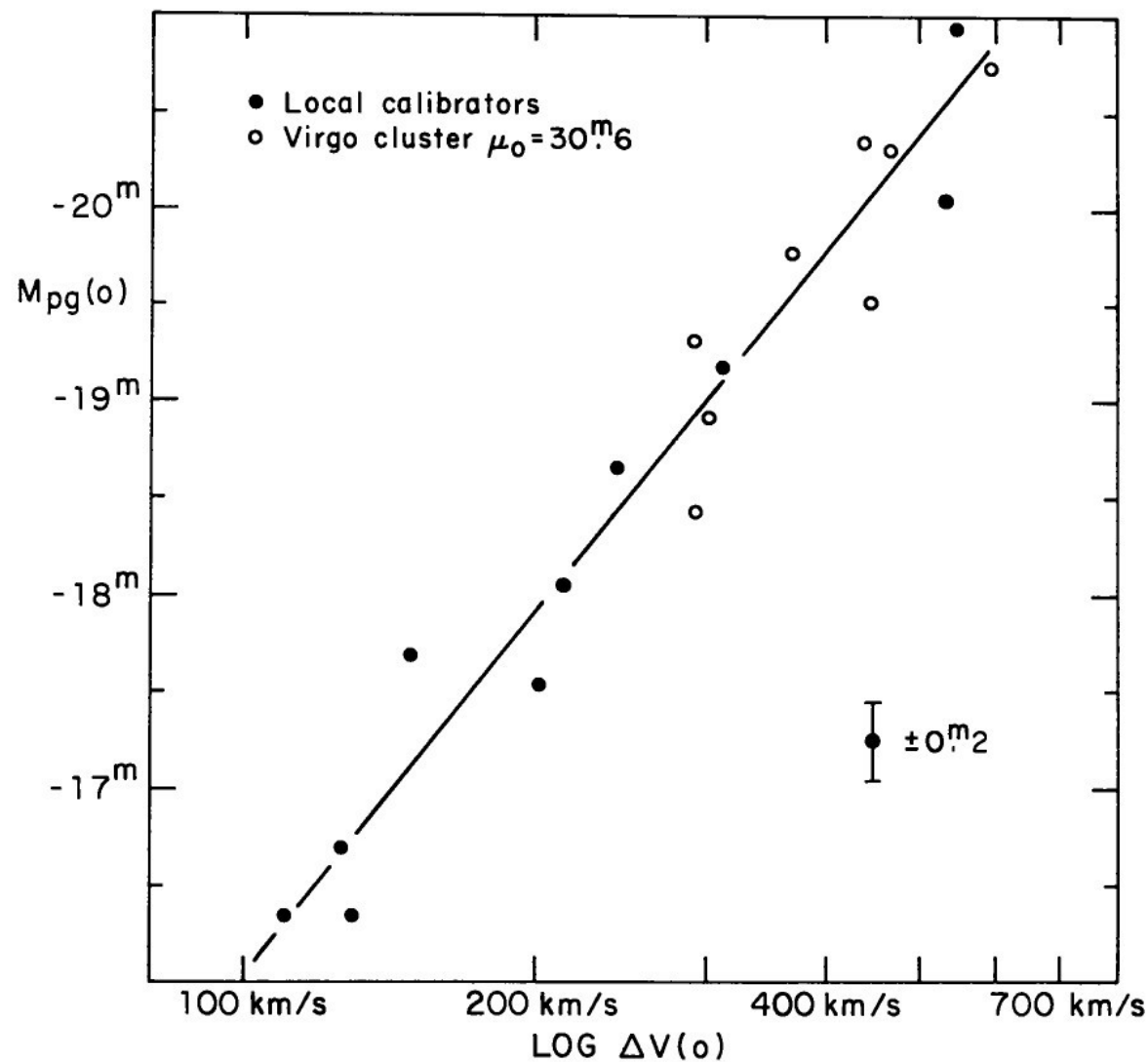
HI Line-Width (Distance Independent)

STEP 1:

Calibrate TF relation using galaxies with known distance (from Cepheids, TRGB, etc.)

The Original Tully-Fisher Relation (1977)

Absolute Magnitude (\propto Distance²)



HI Line-Width (Distance Independent)

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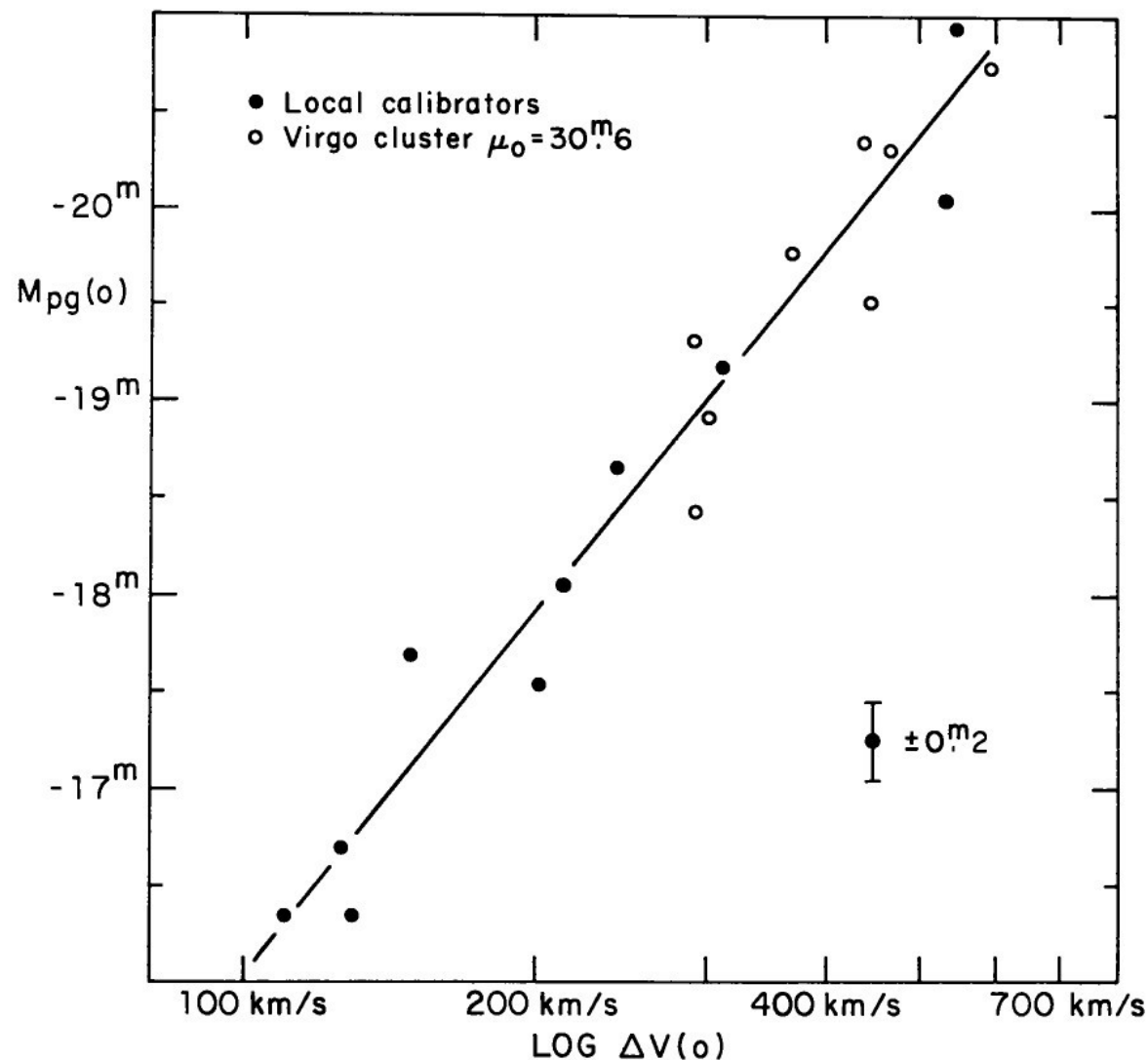
Calibrate TF relation using galaxies with known distance (from Cepheids, TRGB, etc.)

STEP 2:

Measure HI line-width (radio) & apparent mag (optical/IR) from large surveys

The Original Tully-Fisher Relation (1977)

Absolute Magnitude (\propto Distance²)



HI Line-Width (Distance Independent)

STEP 1:

Calibrate TF relation using galaxies with known distance (from Cepheids, TRGB, etc.)

STEP 2:

Measure HI line-width (radio) & apparent mag (optical/IR) from large surveys

STEP 3:

Infer distances (< 300 Mpc) for large galaxy samples (~ 18000 objs in Tully+2016)

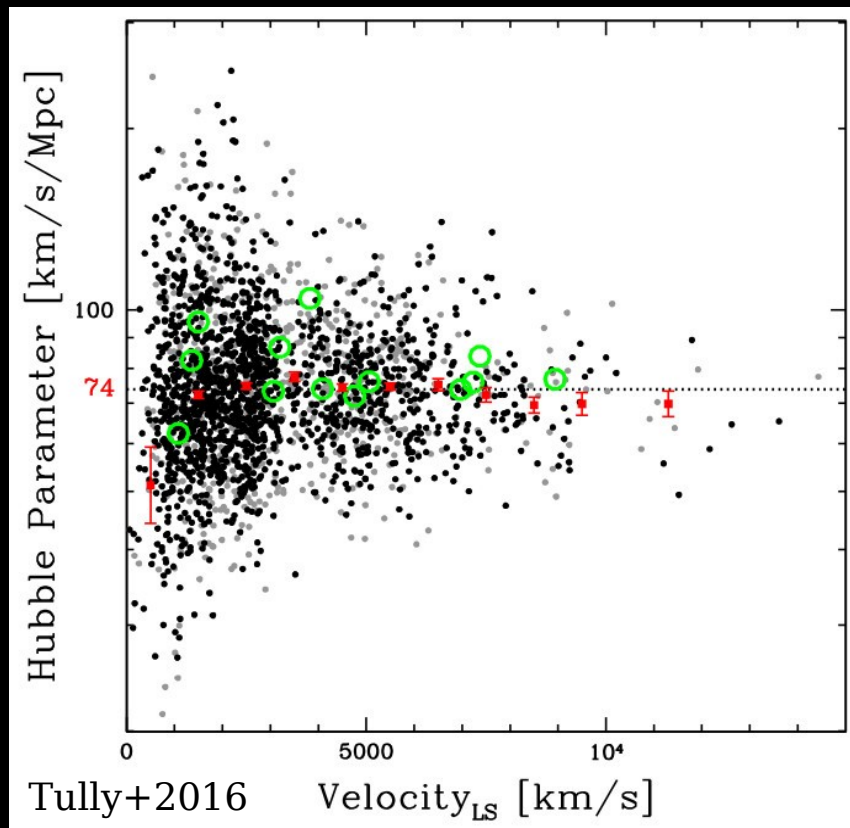
Classic Applications of the TF relation

1-Measure Hubble constant

$$V_{\text{sys}} \sim H_0 D + V_{\text{pec}} \text{ at low } z$$

$H_0 = 80 \text{ km/s/Mpc}$ (Tully & Fisher 1977)

$H_0 = 75 \pm 2 \text{ km/s/Mpc}$ (Tully+2016)



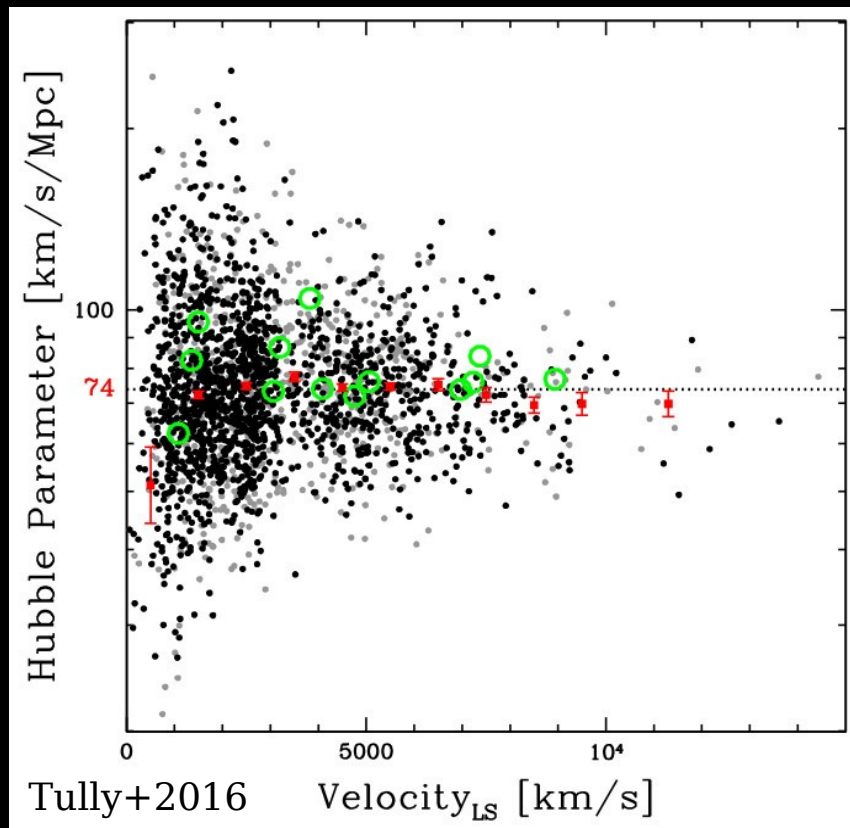
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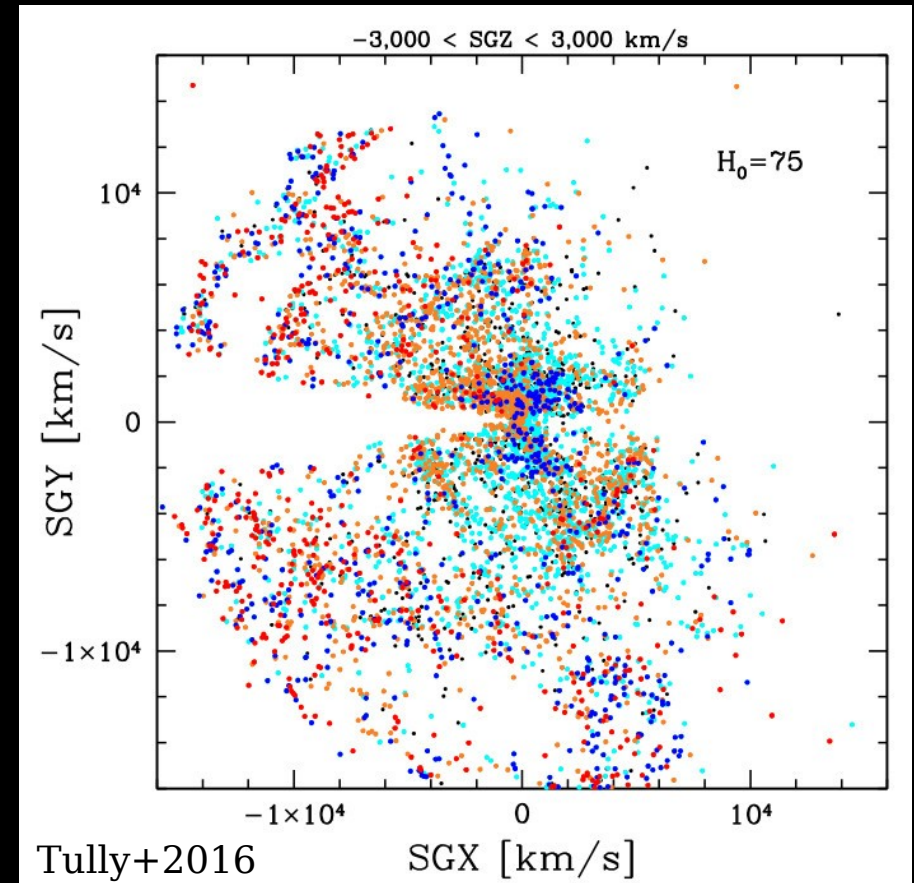
$H_0 = 75 \pm 2 \text{ km/s/Mpc}$ (Tully+2016)



2-Study Galaxy Flows

$$V_{\text{pec}} = (V_{\text{mod}} - H_0 D) / (1 + H_0 D/c)$$

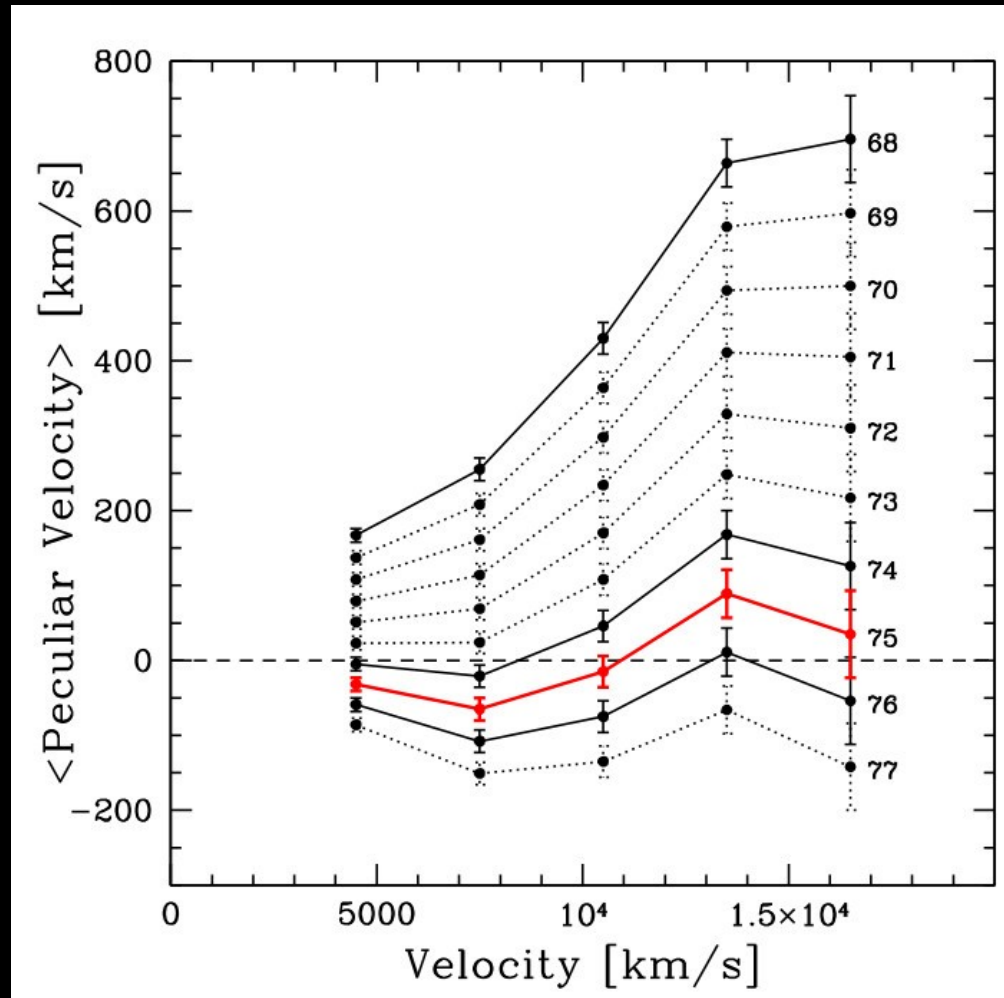
$$V_{\text{mod}} = f(z, D, \Omega_m, \Omega_\Lambda)$$



Peculiar Velocities & The Hubble Constant

$$V_{\text{pec}} = (V_{\text{mod}} - H_0 D) / (1 + H_0 D/c) \quad V_{\text{mod}} = f(z, D, \Omega_m, \Omega_\Lambda)$$

Fix Ω_m and Ω_Λ (or equivalently q_0), vary H_0 and get different V_{pec}



Tully+2016
 $H_0 = 75 \pm 2$

2. Physics Behind the Tully-Fisher relation

L_λ and W_{HI} are proxies for more
fundamental quantities!

Goal: find the quantities that
give the tighter relation

Luminosity \sim Stellar Mass

The TF relation is tighter in the NIR than in the optical
(e.g. Aaronson+1979, Verheijen 2001, Ponomareva+2017)

Luminosity \sim Stellar Mass

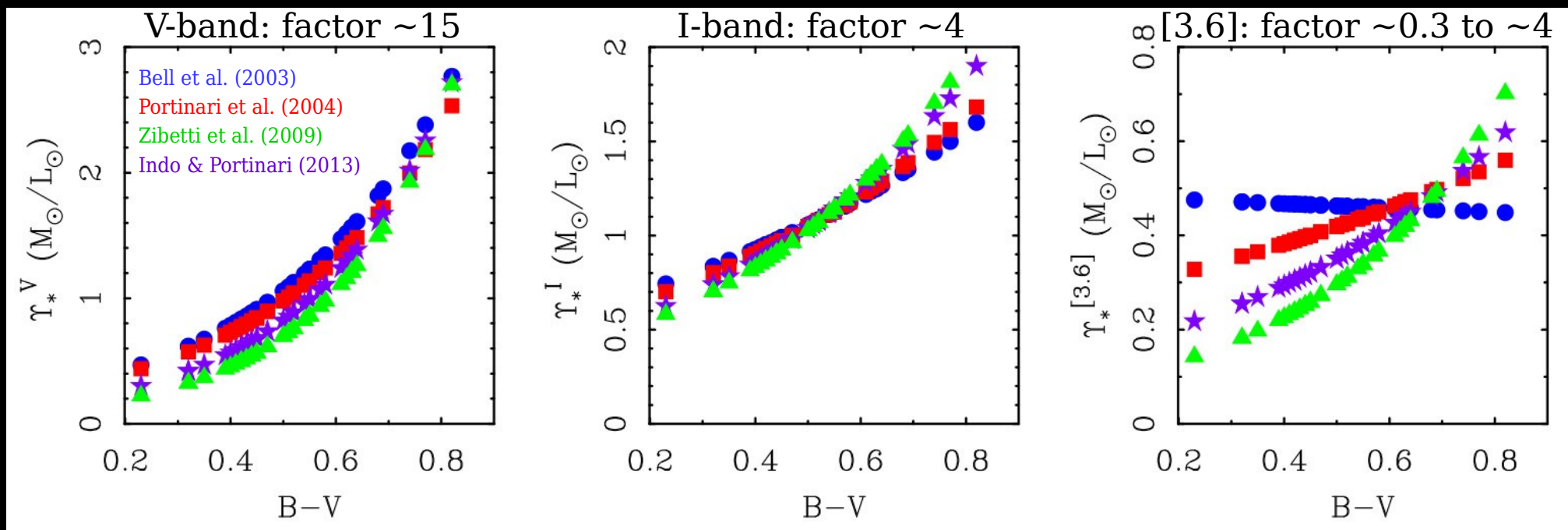
The TF relation is tighter in the NIR than in the optical (e.g. Aaronson+1979, Verheijen 2001, Ponomareva+2017)

$\Upsilon_* = M_*/L$ shows small galaxy-to-galaxy variations in the NIR (less sensitive to star-formation history, dust extinction, etc.)

Luminosity \sim Stellar Mass

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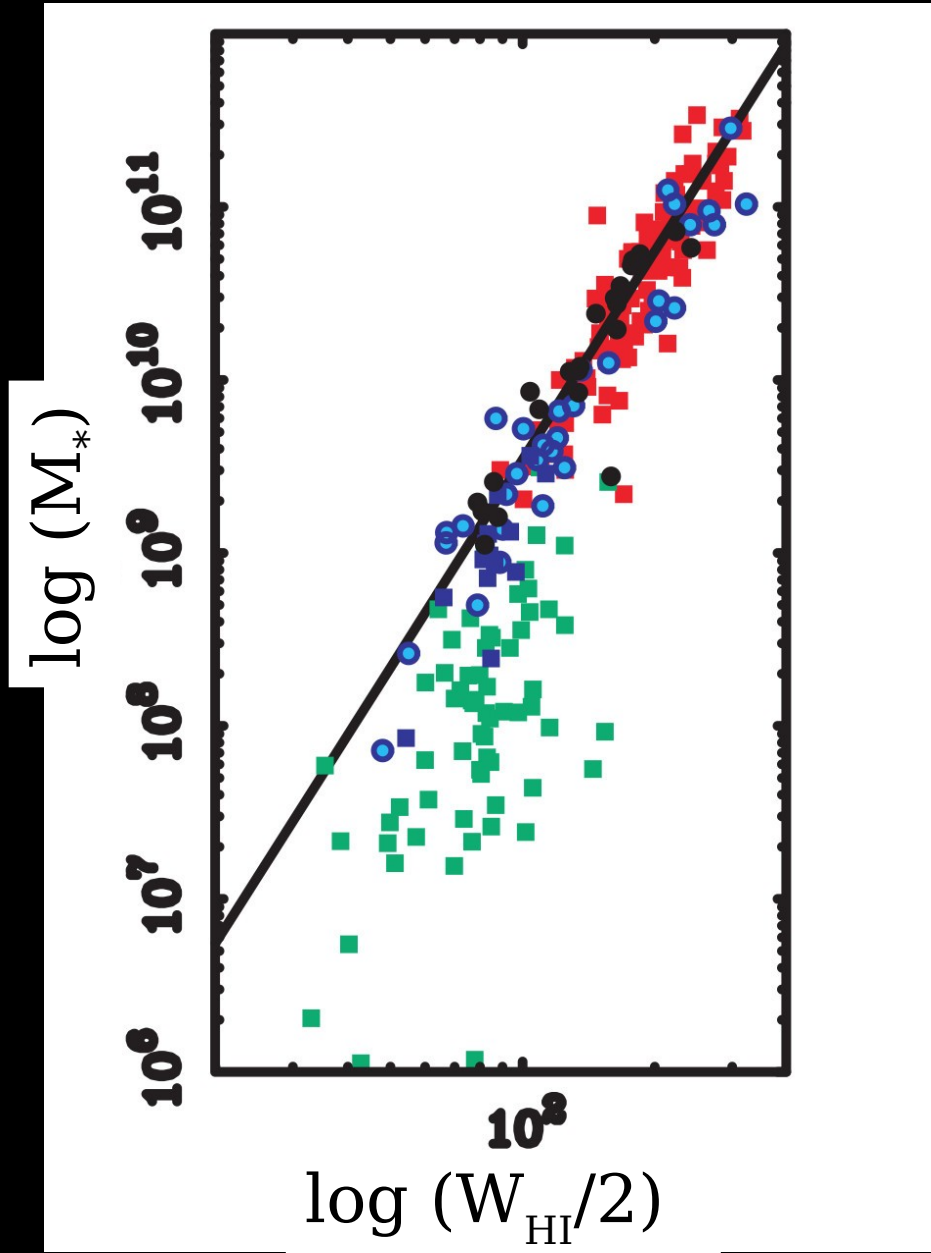


Predicted Υ_* -Color Relations from stellar population synthesis models

$\Upsilon_*^{[3.6]} \sim 0.5 M_\odot/L_\odot$ with $\sim 30\%$ scatter (e.g., Meidt+2014; Norris+2016; Schombert+2019)

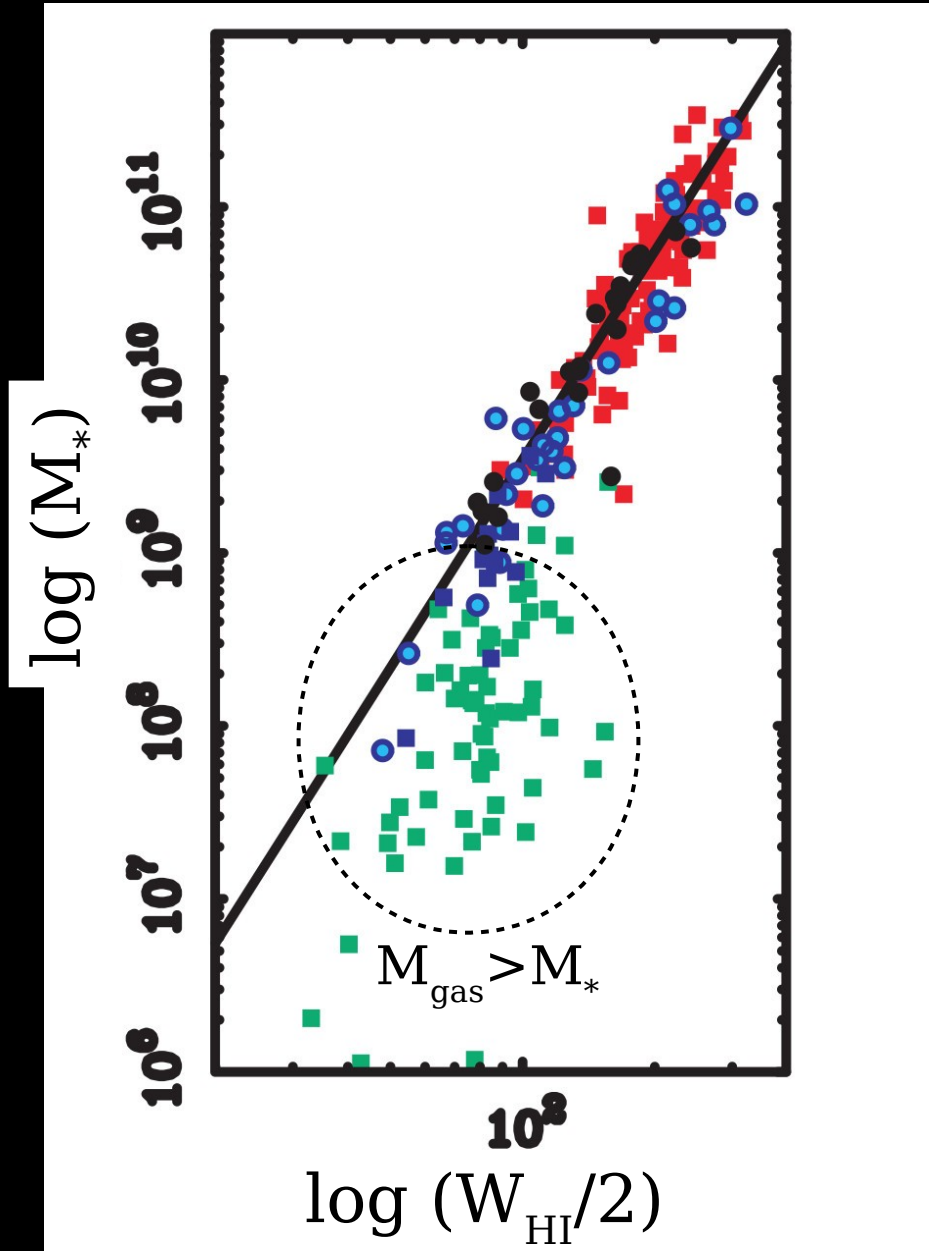
Stellar Mass is not enough!

Stellar-Mass TF Relation



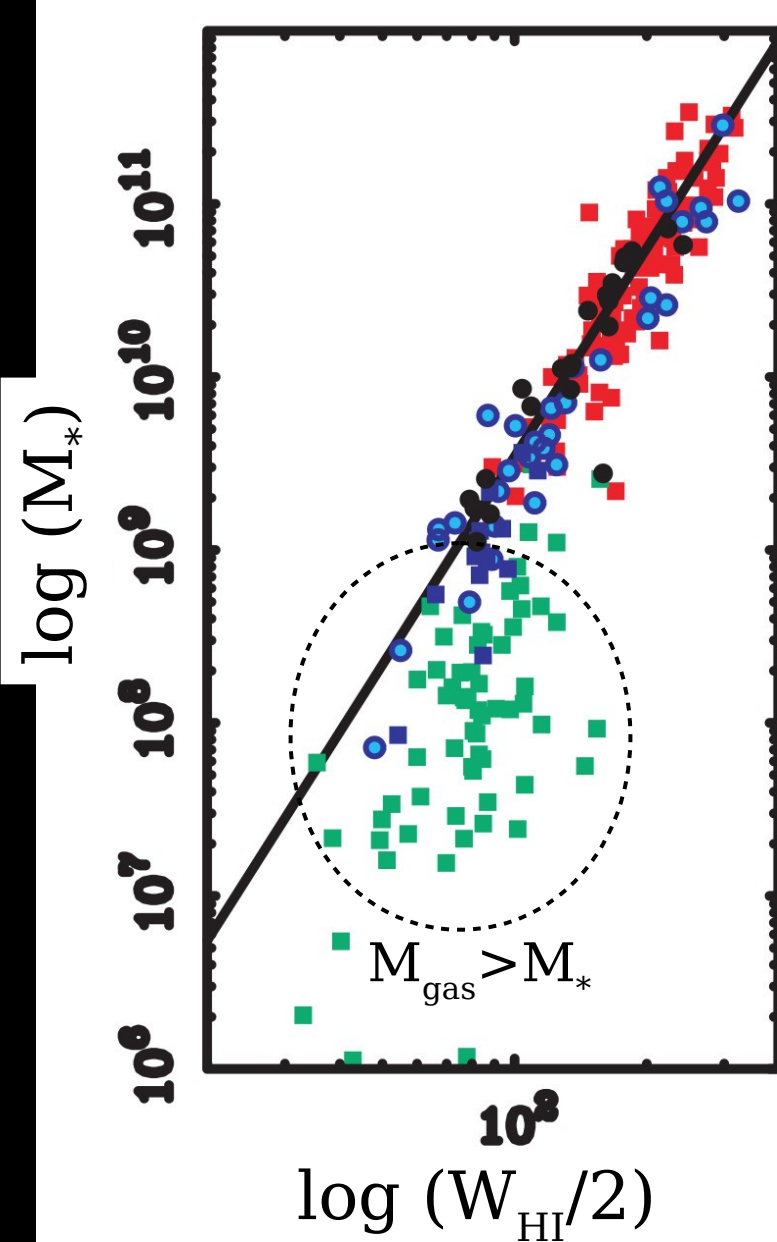
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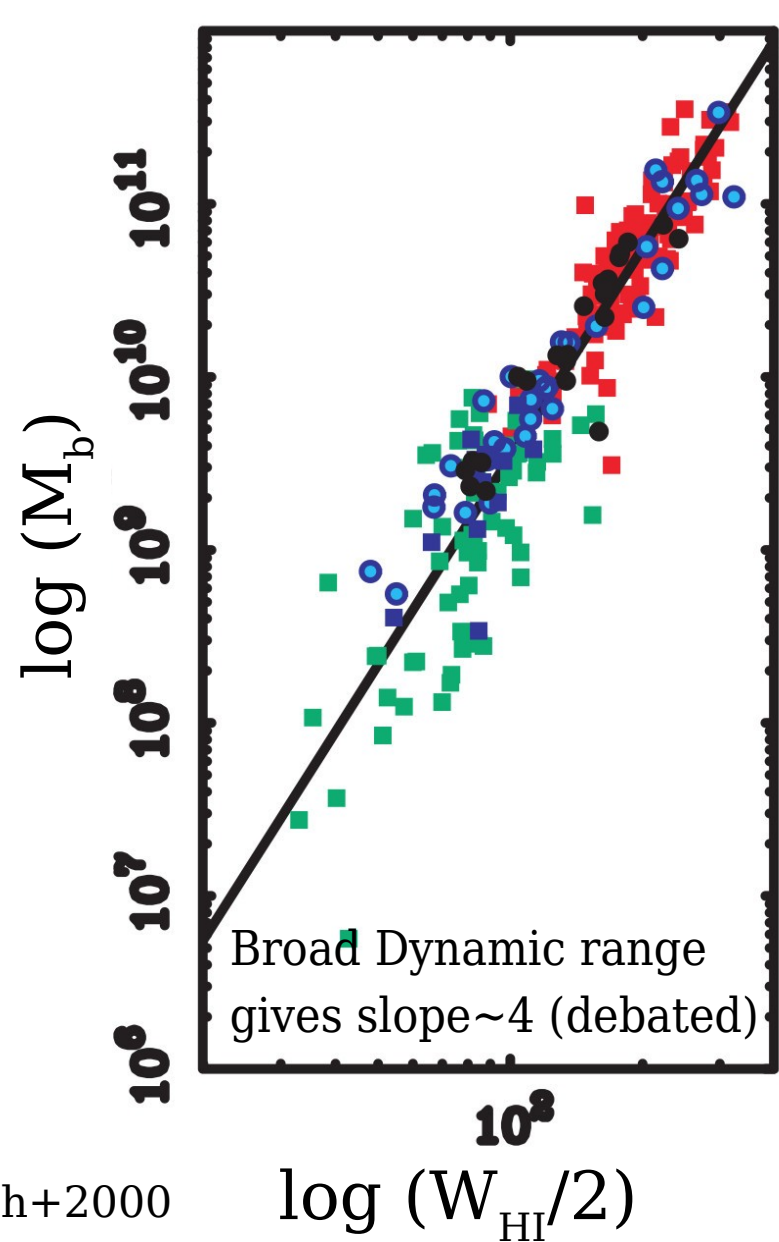
Baryonic Mass (stars+gas) is the key!

Stellar-Mass TF Relation

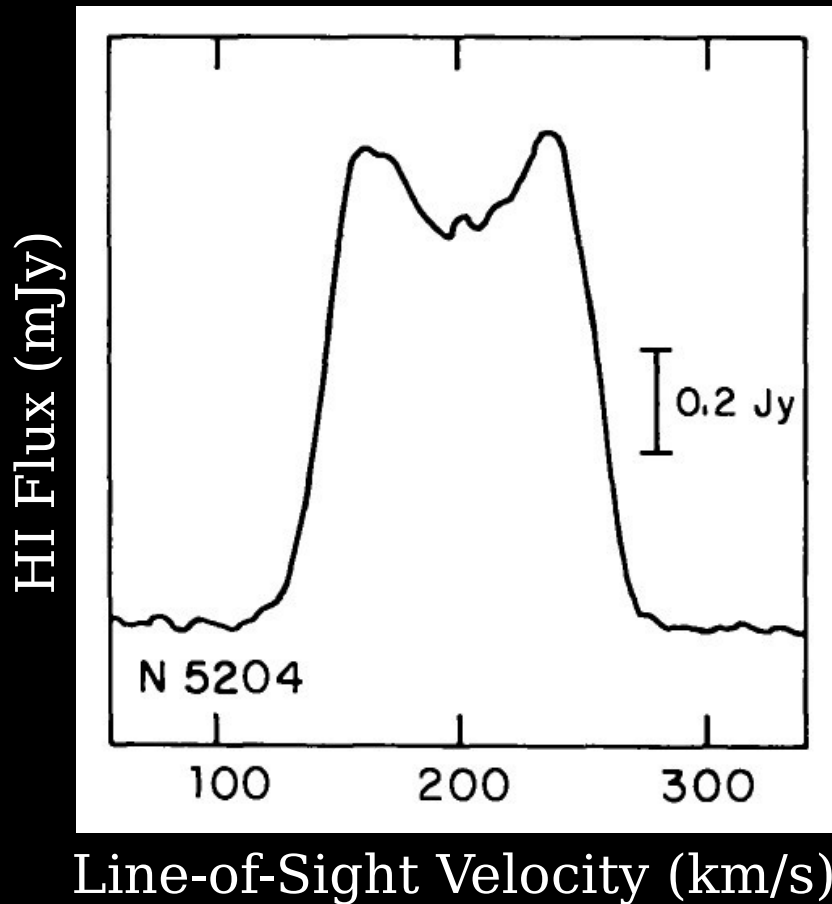


McGaugh+2000

Baryonic TF Relation



What's the HI line-width really measuring?



The HI line profile depends on $\Sigma_{\text{HI}}(R)$, $V_{\text{rot}}(R)$, inclination!

Need to spatially resolve HI distribution and kinematics!

HI obs with radio interferometers

$R \sim \lambda/B$ with $B = \text{max distance between two antennas}$

HI obs with radio interferometers

$R \sim \lambda/B$ with $B = \text{max distance between two antennas}$



WSRT (Netherlands)

HI resolution up to $\sim 15''$

Typical surveys done at $\sim 30''$



VLA (New Mexico)

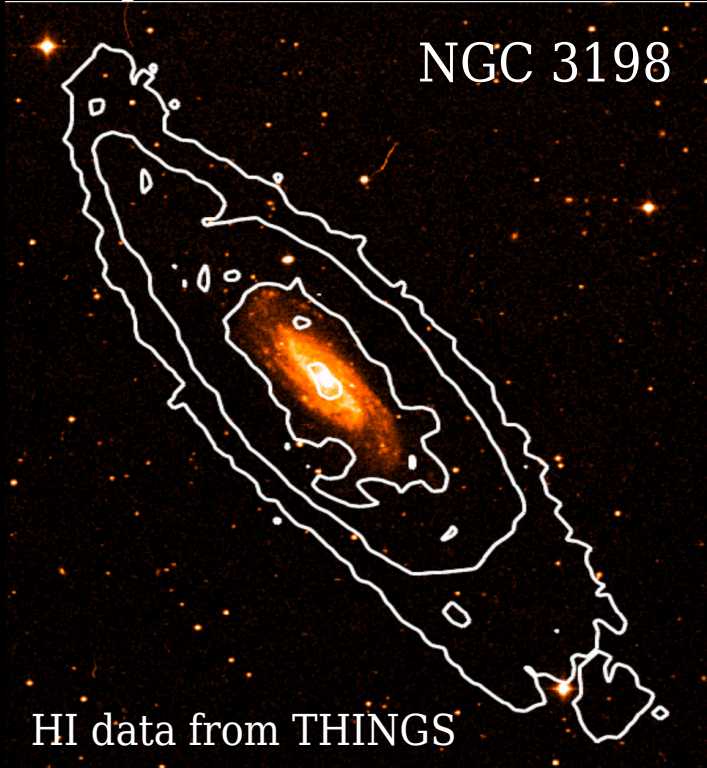
HI resolution up to $\sim 2''$

Typical surveys done at $5''-10''$

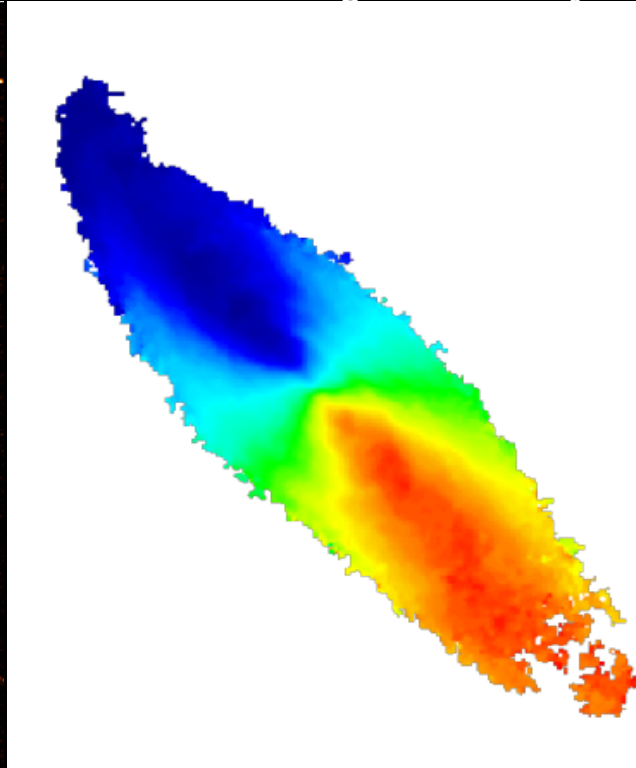
But HI interferometry is **time costly**! HI samples drop from ~ 18000 objects with single-dish observations (Tully+2016) to ~ 200 with interferometry (Lelli+2016).

HI distribution and kinematics

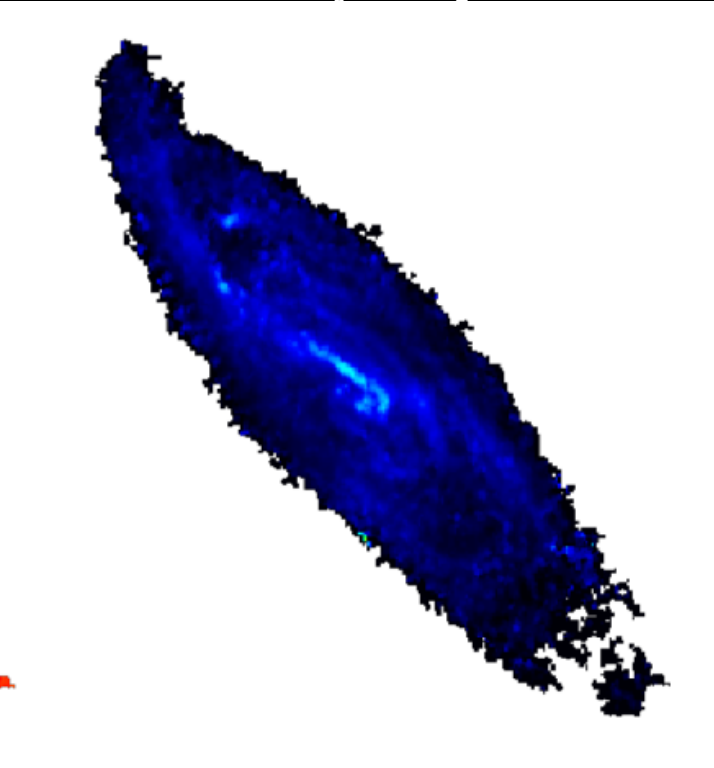
Optical + HI Distribution



HI line-of-sight velocity

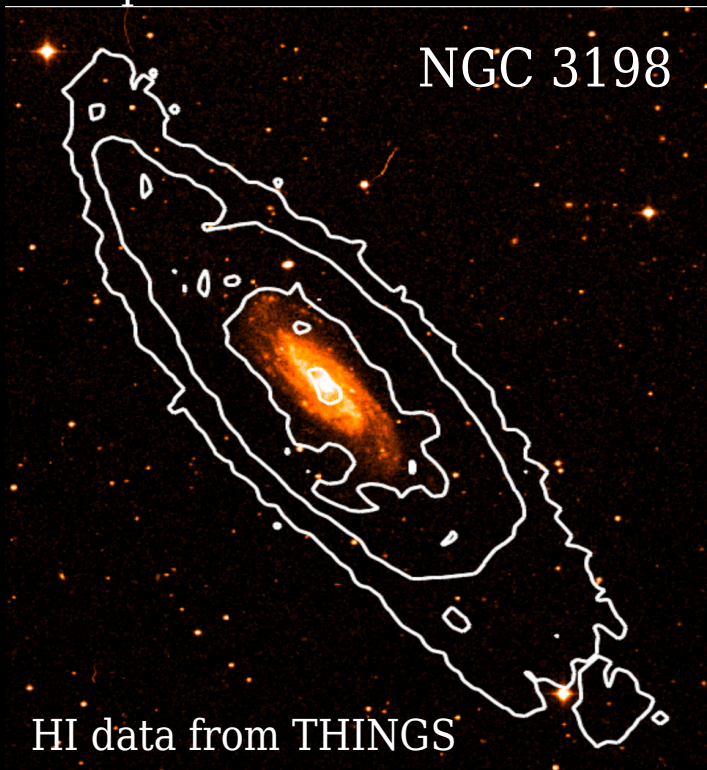


HI velocity "dispersion"

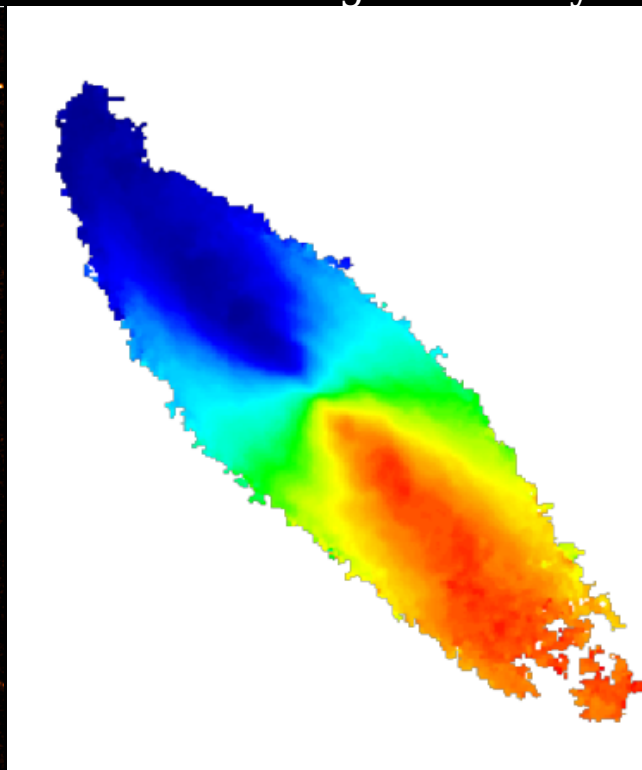


HI distribution and kinematics

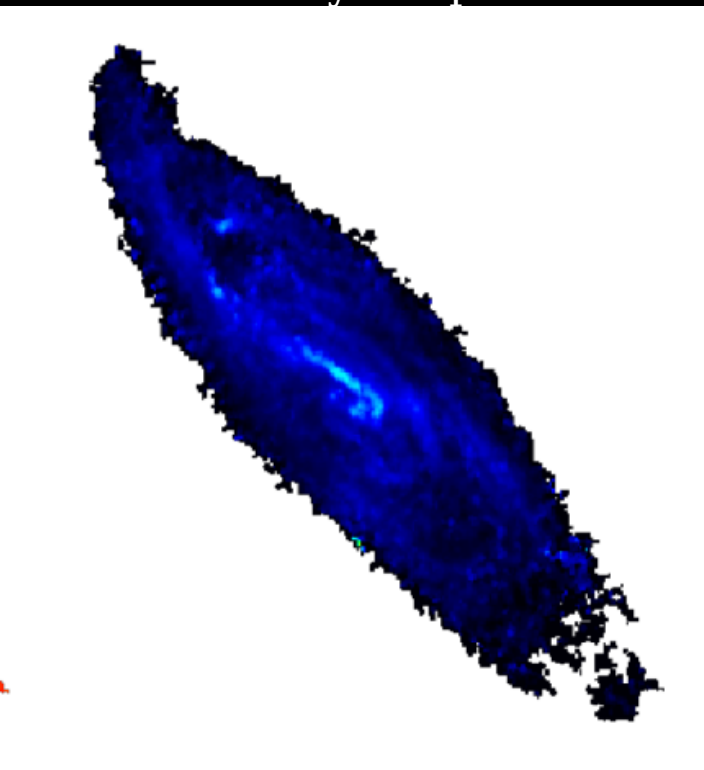
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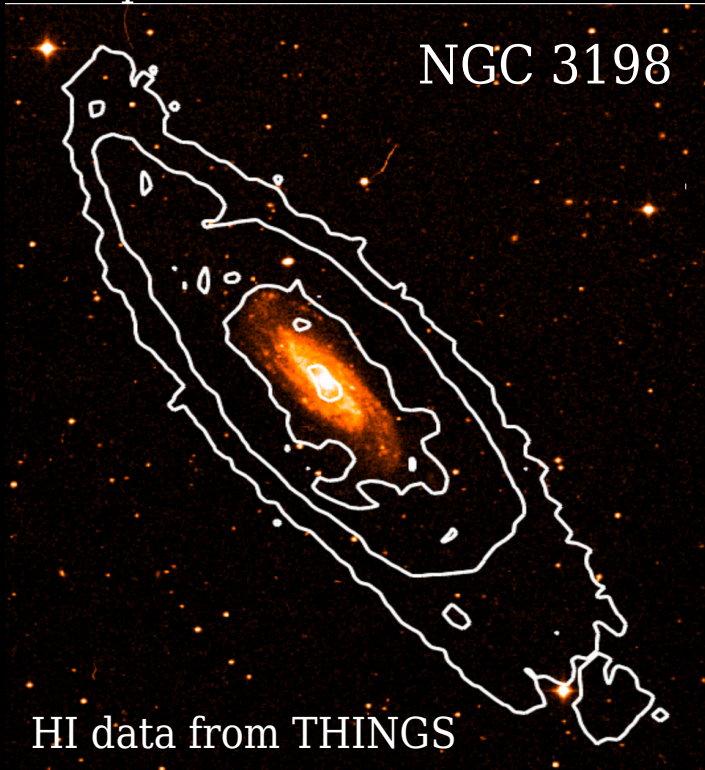
Key Points:

- HI distribution is more extended than stellar one (typically by a factor of 2)
- HI kinematics is generally consistent with rotation (non-circular motions small)
- HI velocity dispersion is $\sim 8-10$ km/s \rightarrow negligible pressure support (unlike stars)

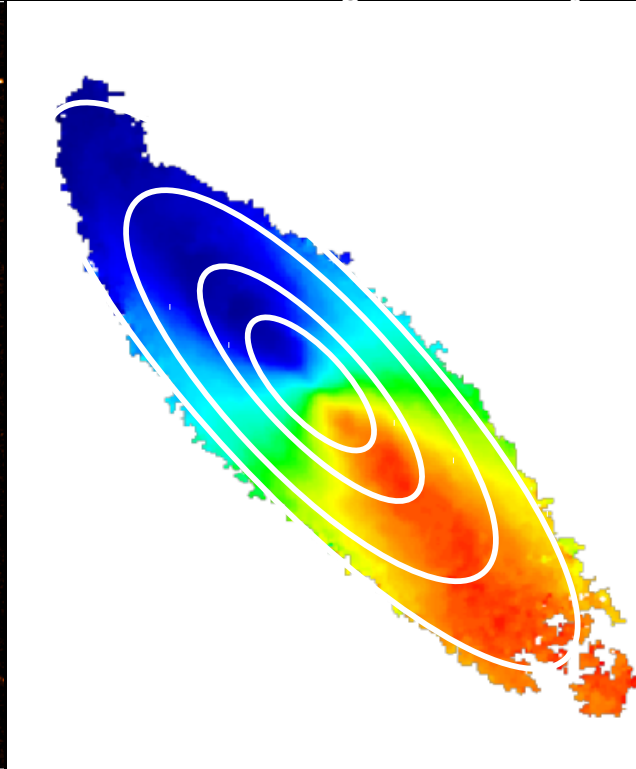
$$V_{\text{rot}} \sim V_{\text{circ}} = \text{sqrt}(R \, d\phi/dR)$$

HI distribution and kinematics

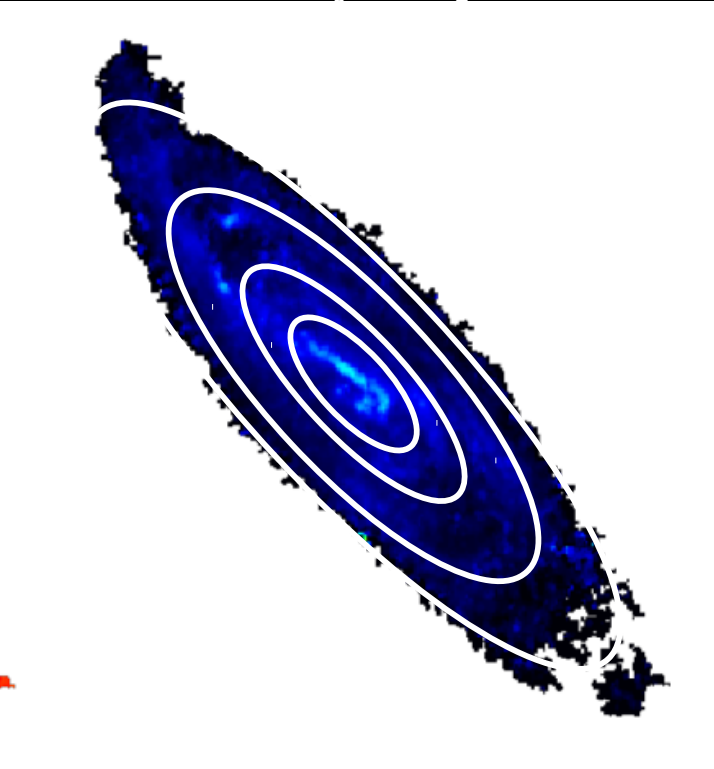
Optical + HI Distribution



HI line-of-sight velocity



HI velocity "dispersion"



How to derive a rotation curve:

- Divide galaxy into a set of concentric rings
- Deprojection from **sky plane** to **galaxy plane**

$$V_{\text{l.o.s.}} = V_{\text{sys}} + V_{\text{rot}} \sin(i) \cos(\theta)$$

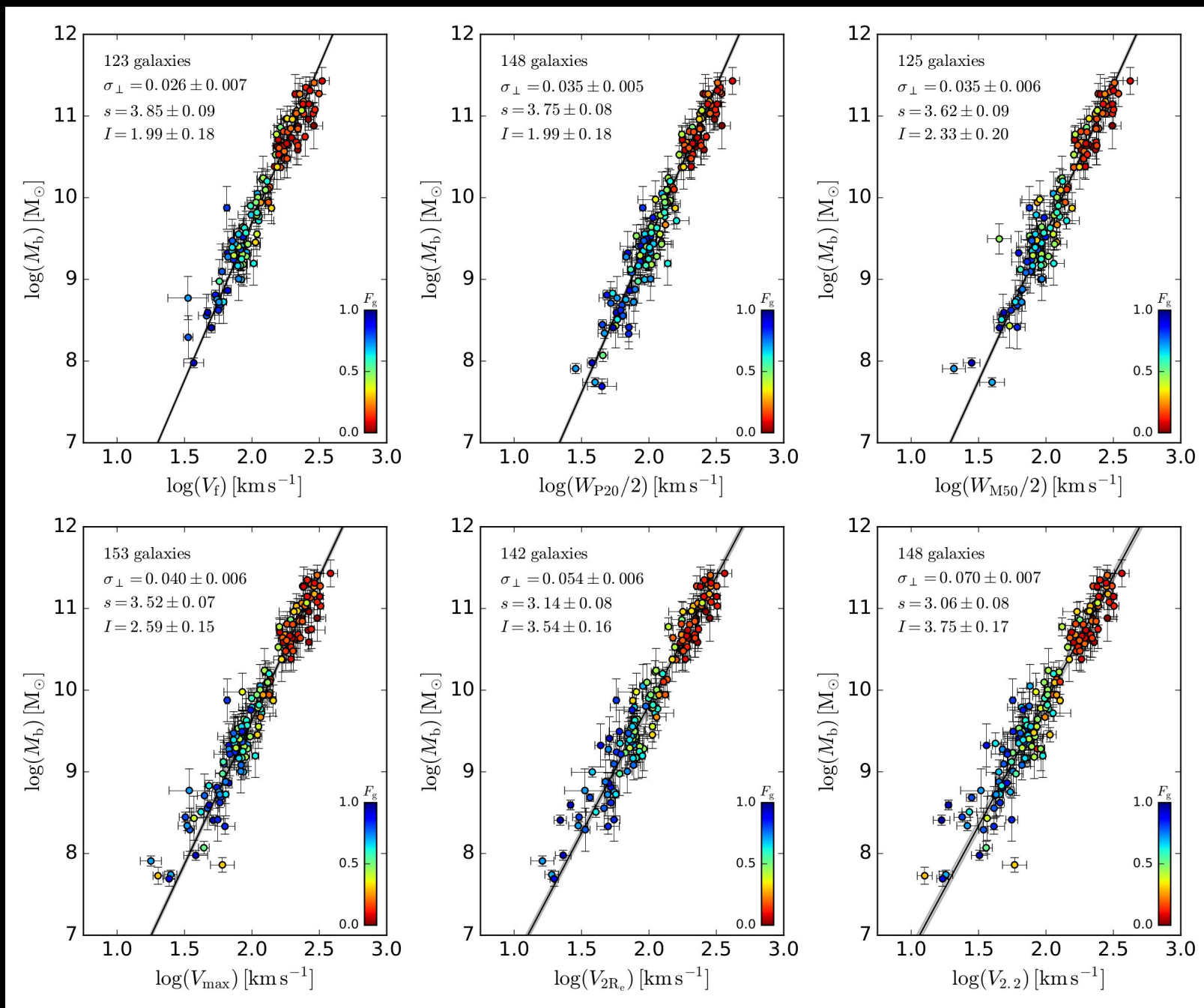
$$\cos(\theta) = \text{fnc}(\text{center}, \text{position angle})$$

i = disk inclination angle

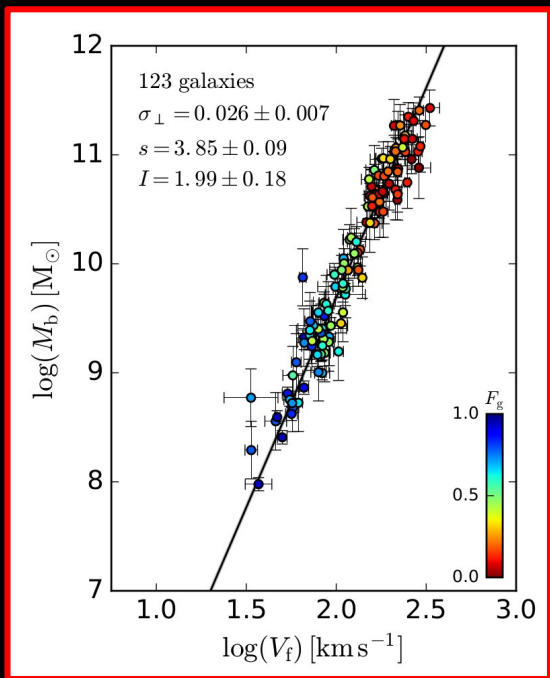
θ = azimuthal angle

V_{sys} = systemic velocity

BTFR for different velocity definitions

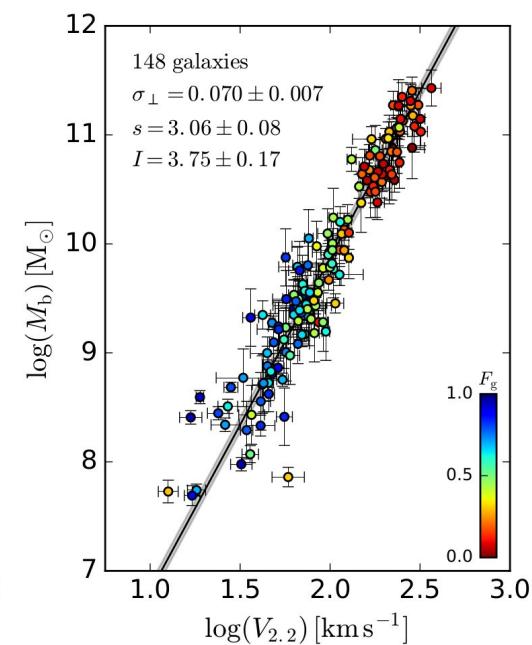
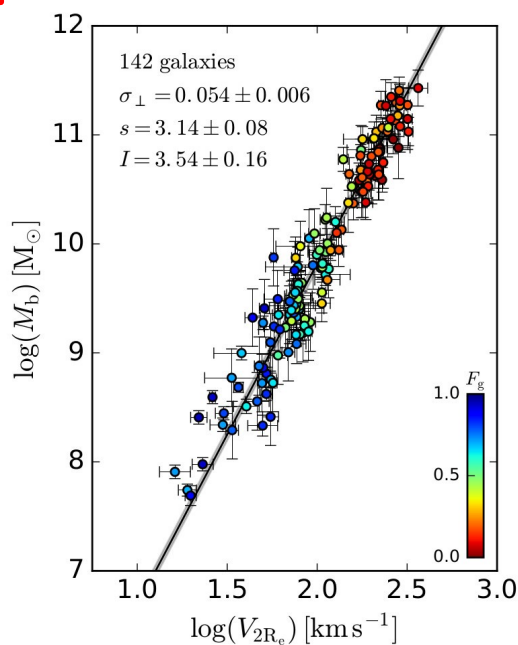
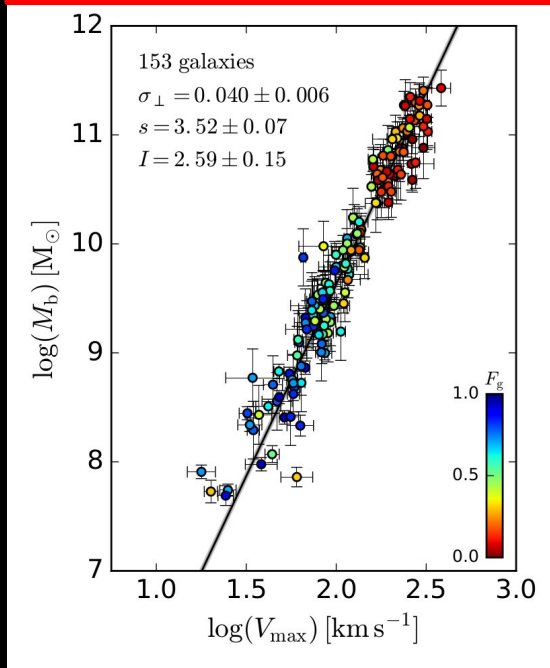


BTFR for different velocity definitions

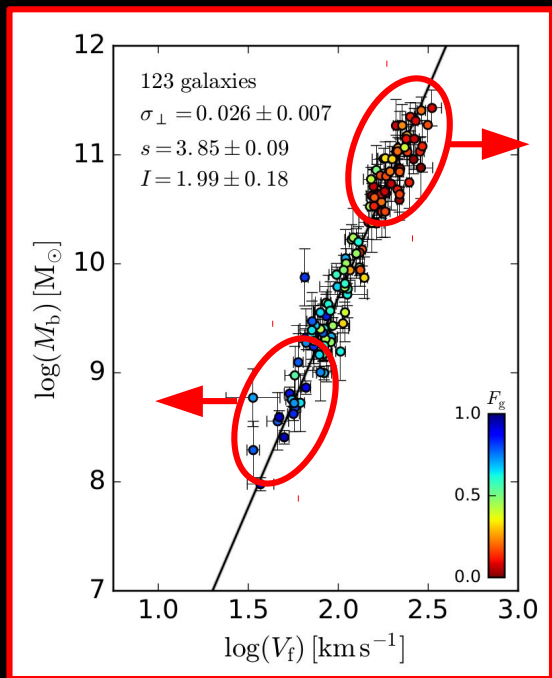


The flat rotation velocity (V_{flat}) gives the tightest and steepest BTFR!

(Verheijen 2001; Noordermeer & Verheijen 2007; McGaugh 2005; Ponomareva+2017; Lelli+2019)



BTFR for different velocity definitions



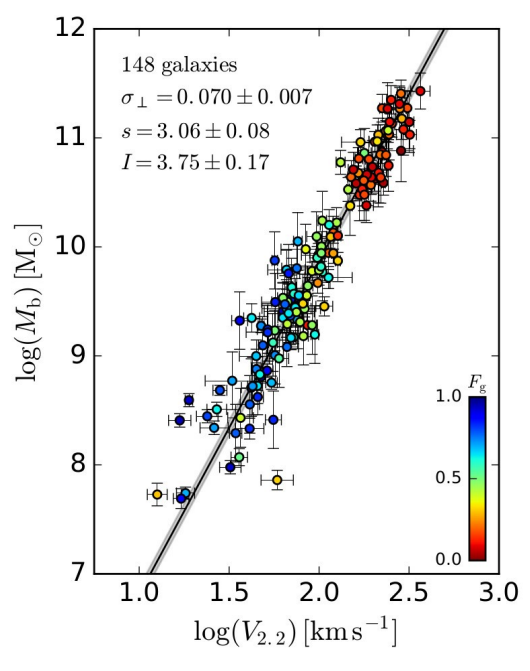
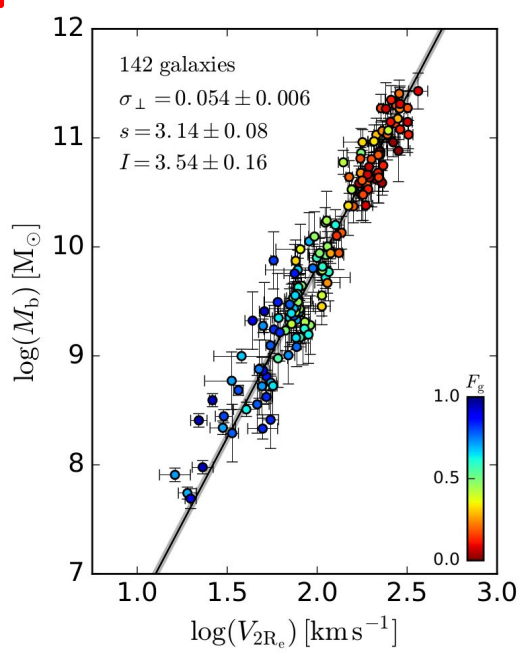
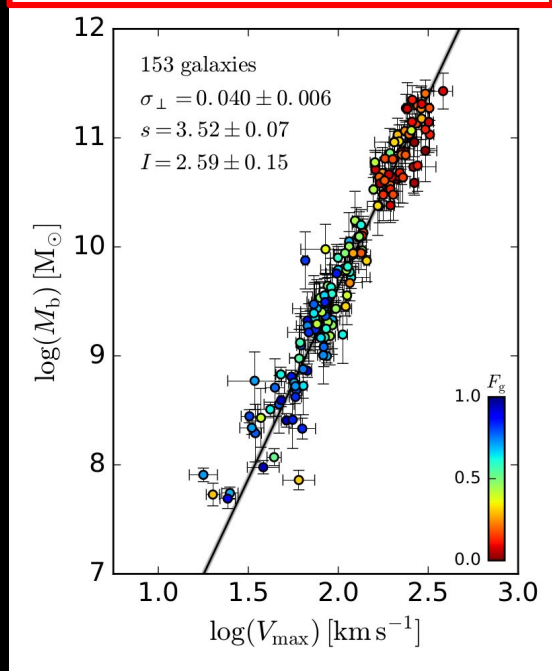
Why M_b - V_{flat} relation is steeper?

Rotation curve shapes!

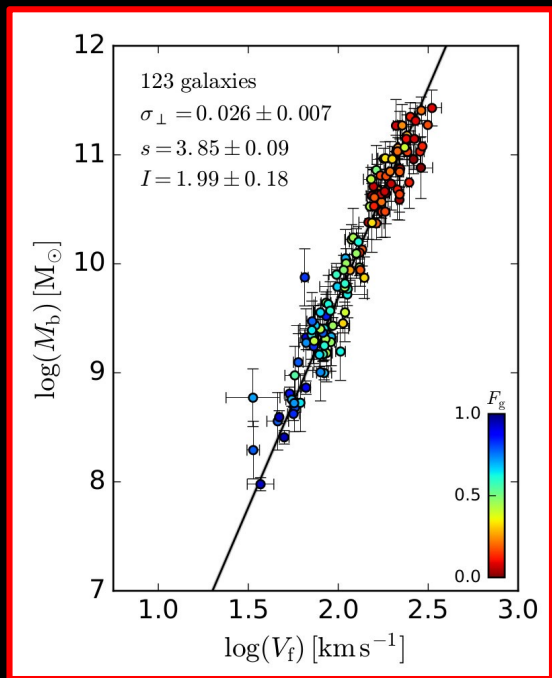
At high M_b : declining RCs $\rightarrow V_{\text{in}} > V_{\text{flat}}$

At low M_b : rising RCs $\rightarrow V_{\text{in}} < V_{\text{flat}}$

Inner velocities give shallower BTFR



BTFR for different velocity definitions



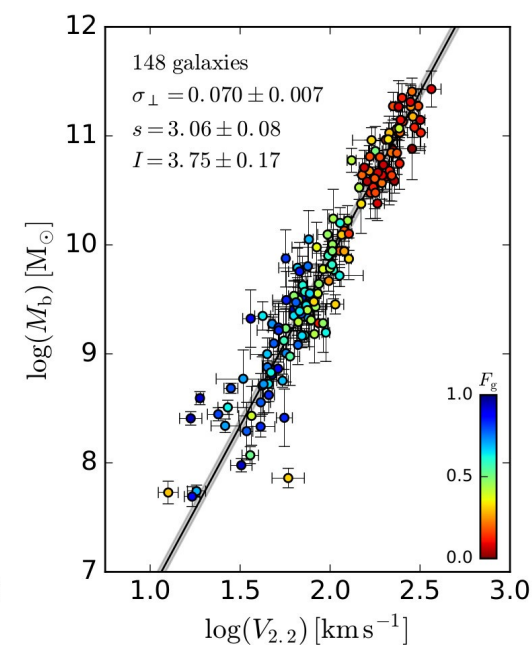
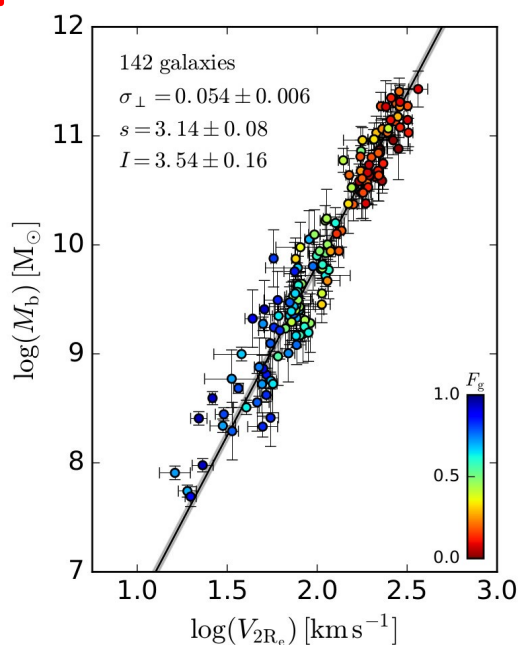
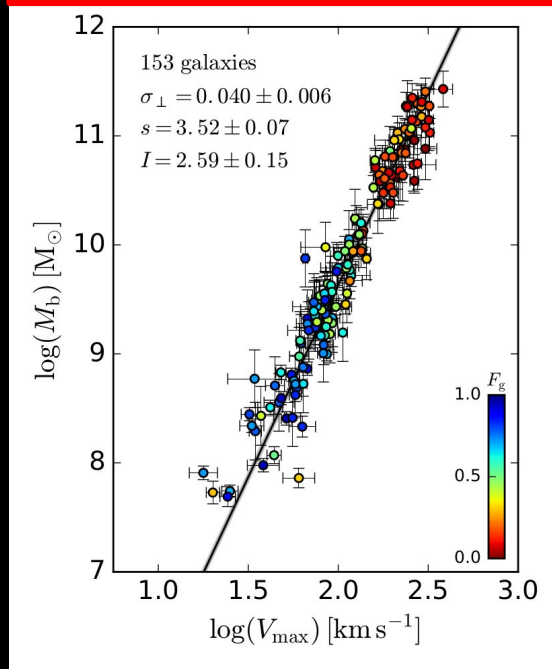
Why M_b - V_{flat} relation is tighter?

Counter-intuitive result!

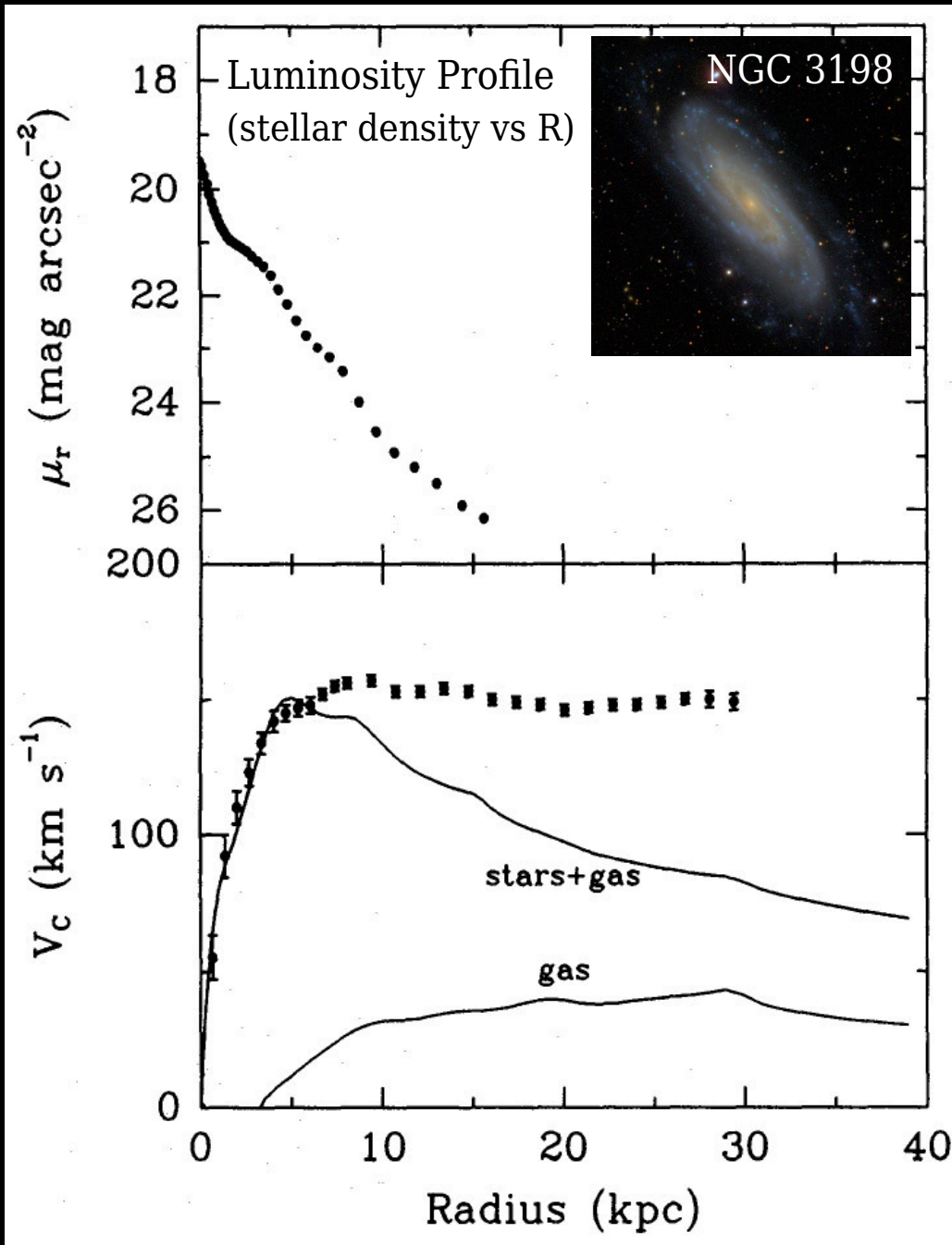
Baryons important near the center...

but M_b best correlate with V_{flat}

(set by the dark matter halo)!



Mass Models for Disk Galaxies



Solve Poisson's Equation for each baryonic component ($i = \text{stars, gas}$)

$$\nabla^2 \Phi_i(R, z) = 4\pi G \rho_i(R, z)$$

Assume nominal disk thickness

$$\rho_i(R, z) = \mu_i(R) \nu_i(z)$$

Find expected circular velocity

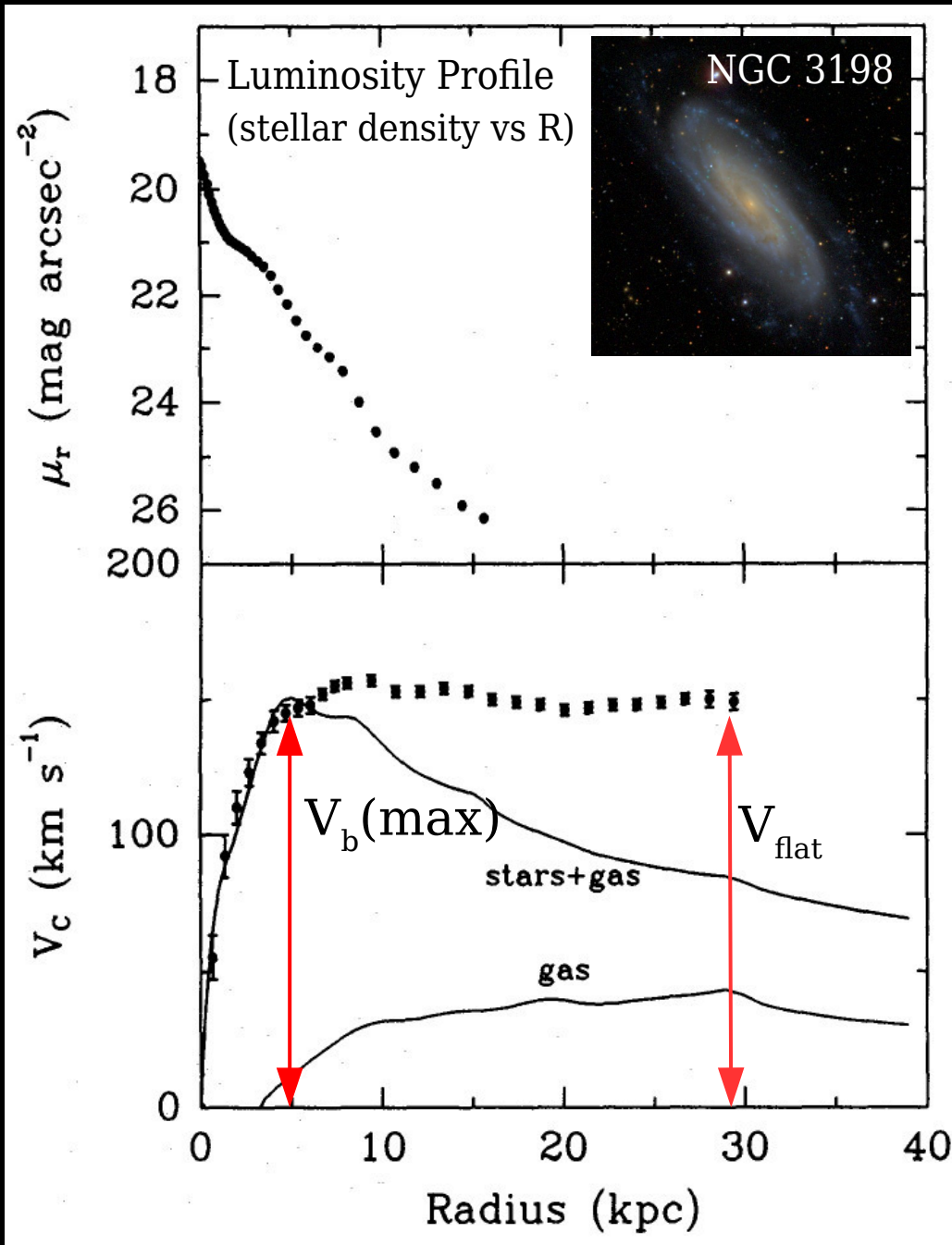
$$\frac{V_i^2(R, z=0)}{R} = - \frac{\partial \Phi_i(R, z=0)}{\partial R}$$

Sum over all baryonic contributions

$$V_b^2(R) = (M/L) V_{star}^2 + V_{gas}^2$$

van Albada et al. (1985); Begeman (1987)

Mass Models for Disk Galaxies

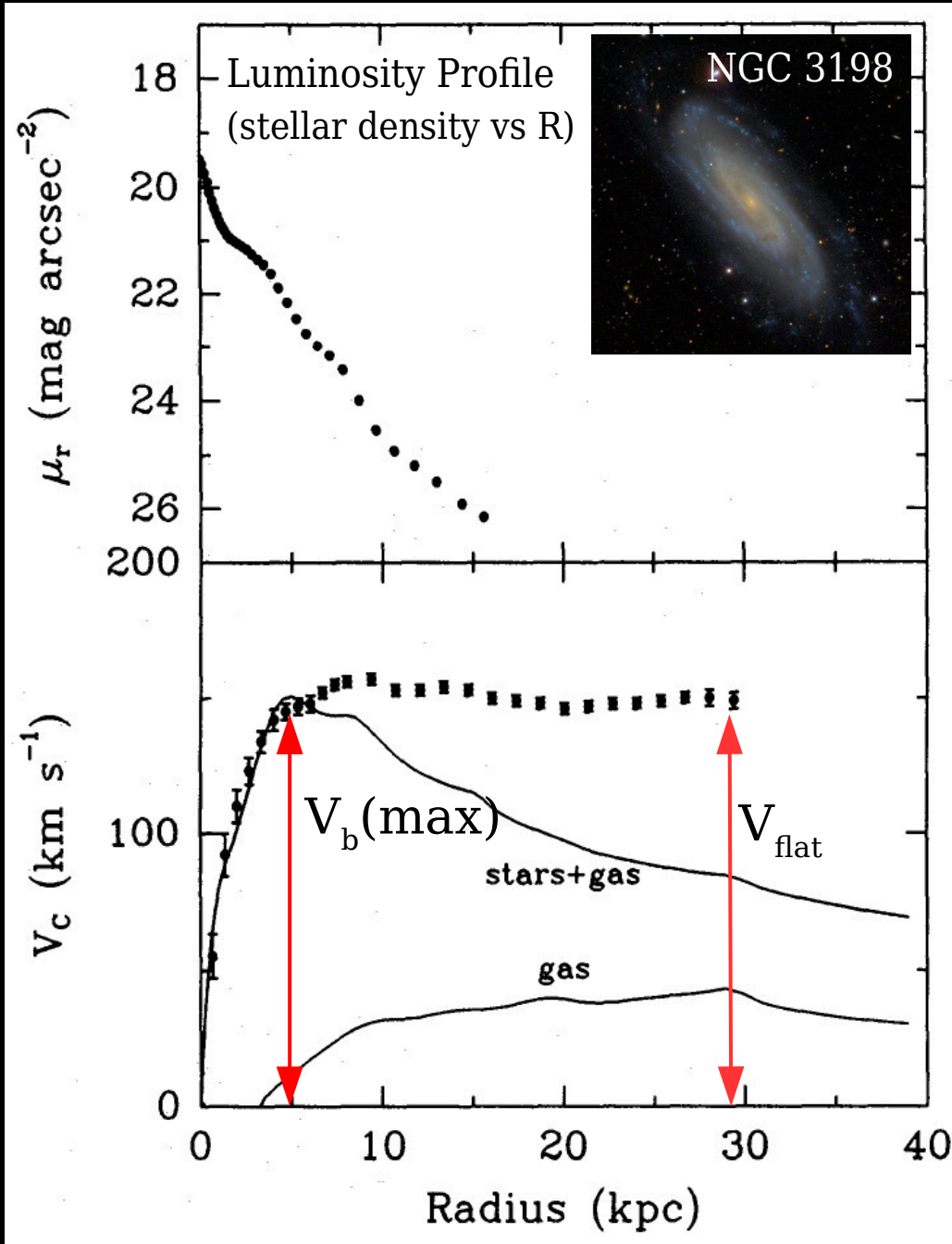


If we assume that all galaxy disks are maximal, then BTFR is trivial!

$$V_{\text{flat}} = V_b(\text{max}) = \sqrt{\alpha G M_b / R}$$

$\alpha = O(1)$ due to disk geometry

Mass Models for Disk Galaxies



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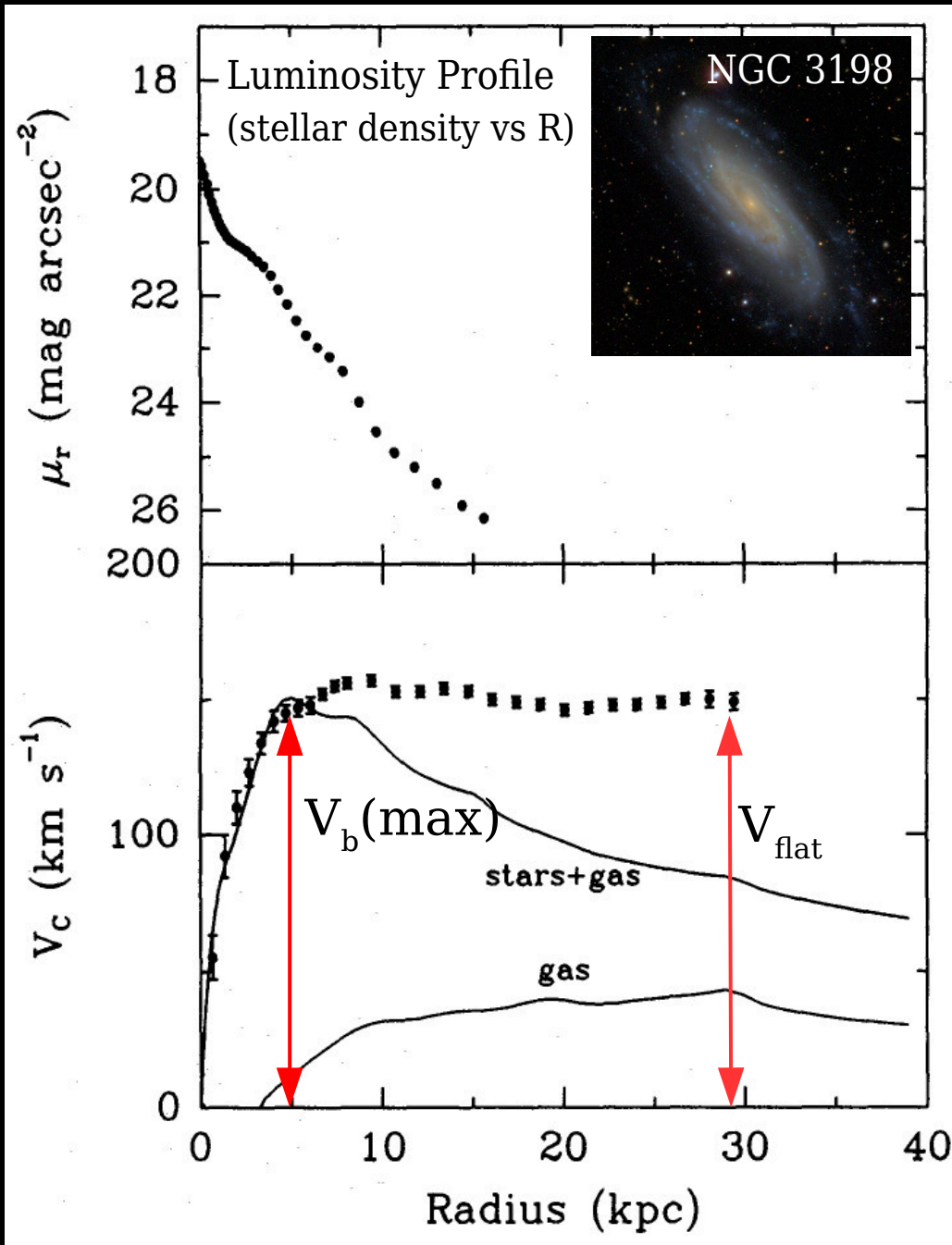
$\alpha = O(1)$ due to disk geometry

If you square this twice:

$$V_{\text{flat}}^4 = V_b(\text{max})^4 = (\alpha G)^2 \Sigma_b M_b$$

Normalization set by Σ !

Mass Models for Disk Galaxies



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Normalization set by Σ !

- **Pre-90s:** Σ thought to be constant for galaxy disks (Freeman's Law)
- **Post-90s:** LSB disks emerged (Schombert 1992; McGaugh 1994)
- **Prediction:** LSB galaxies should follow a different TF relation!

The Tully–Fisher relation for low surface brightness galaxies: implications for galaxy evolution

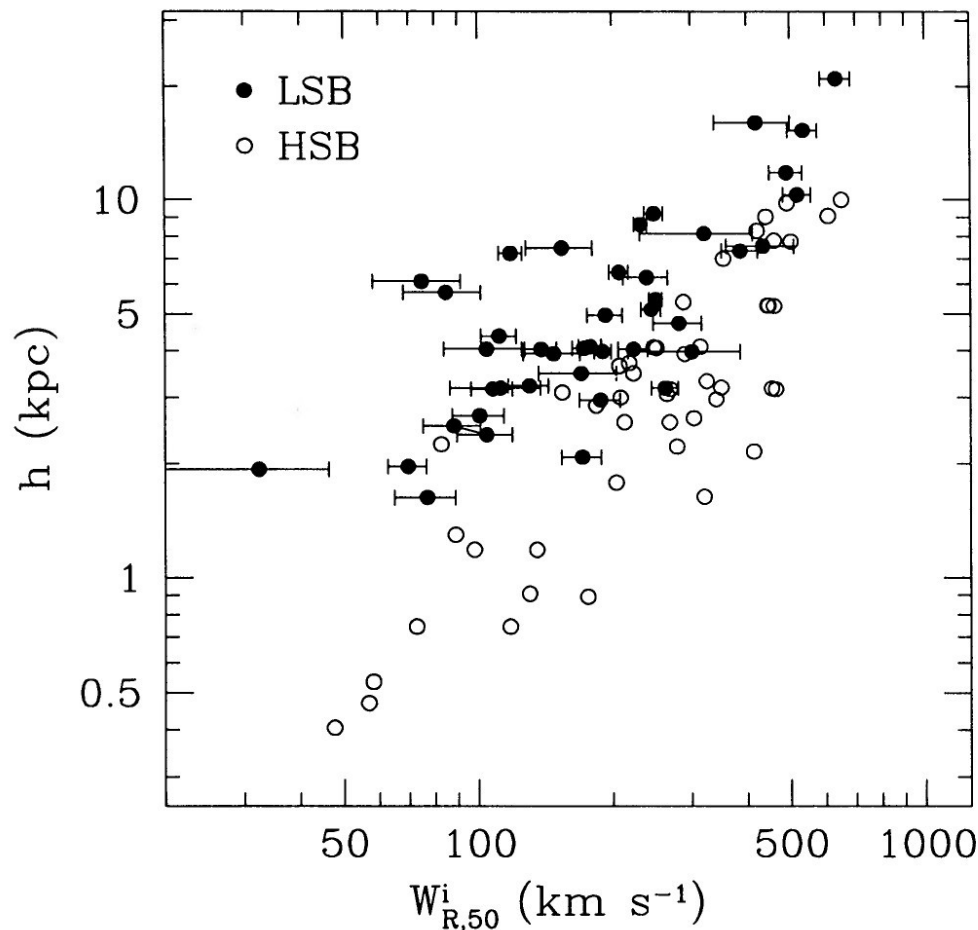
M. A. Zwaan,¹ J. M. van der Hulst,¹ W. J. G. de Blok¹ and S. S. McGaugh²

¹*Kapteyn Astronomical Institute, PO Box 800, 9700 AV Groningen, The Netherlands*

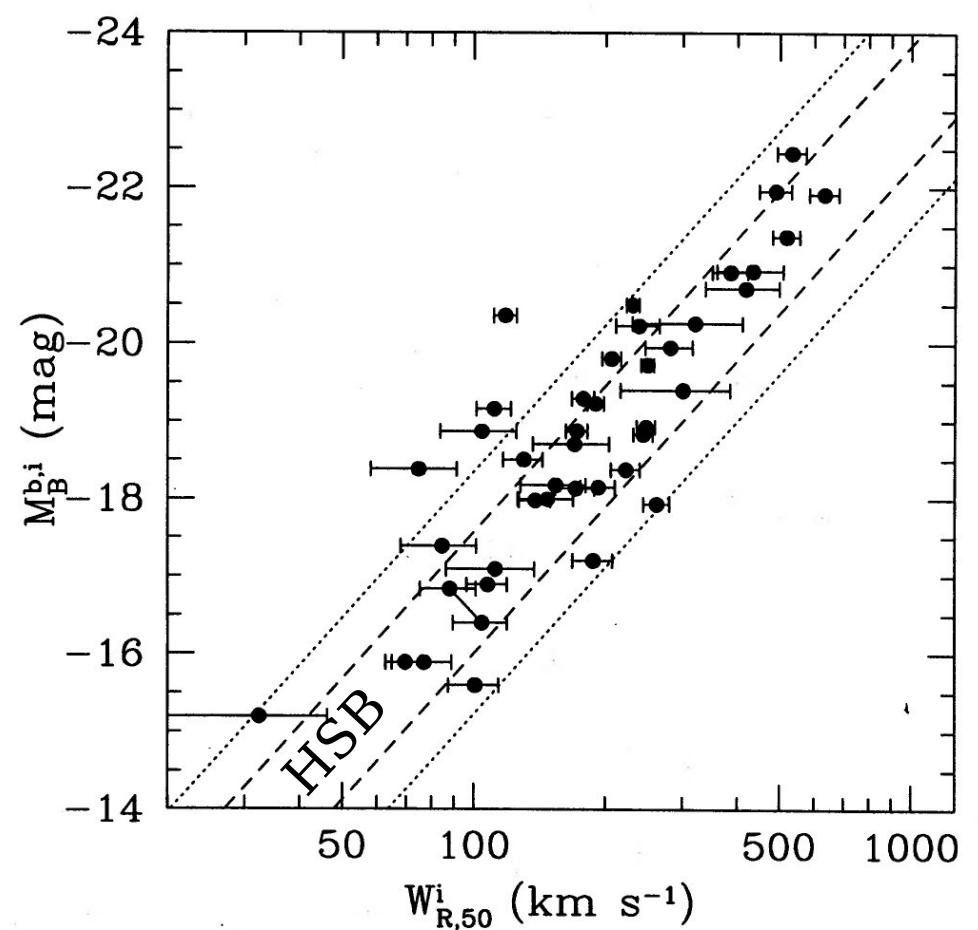
²*Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

1995

LSB vs HSB: different sizes!



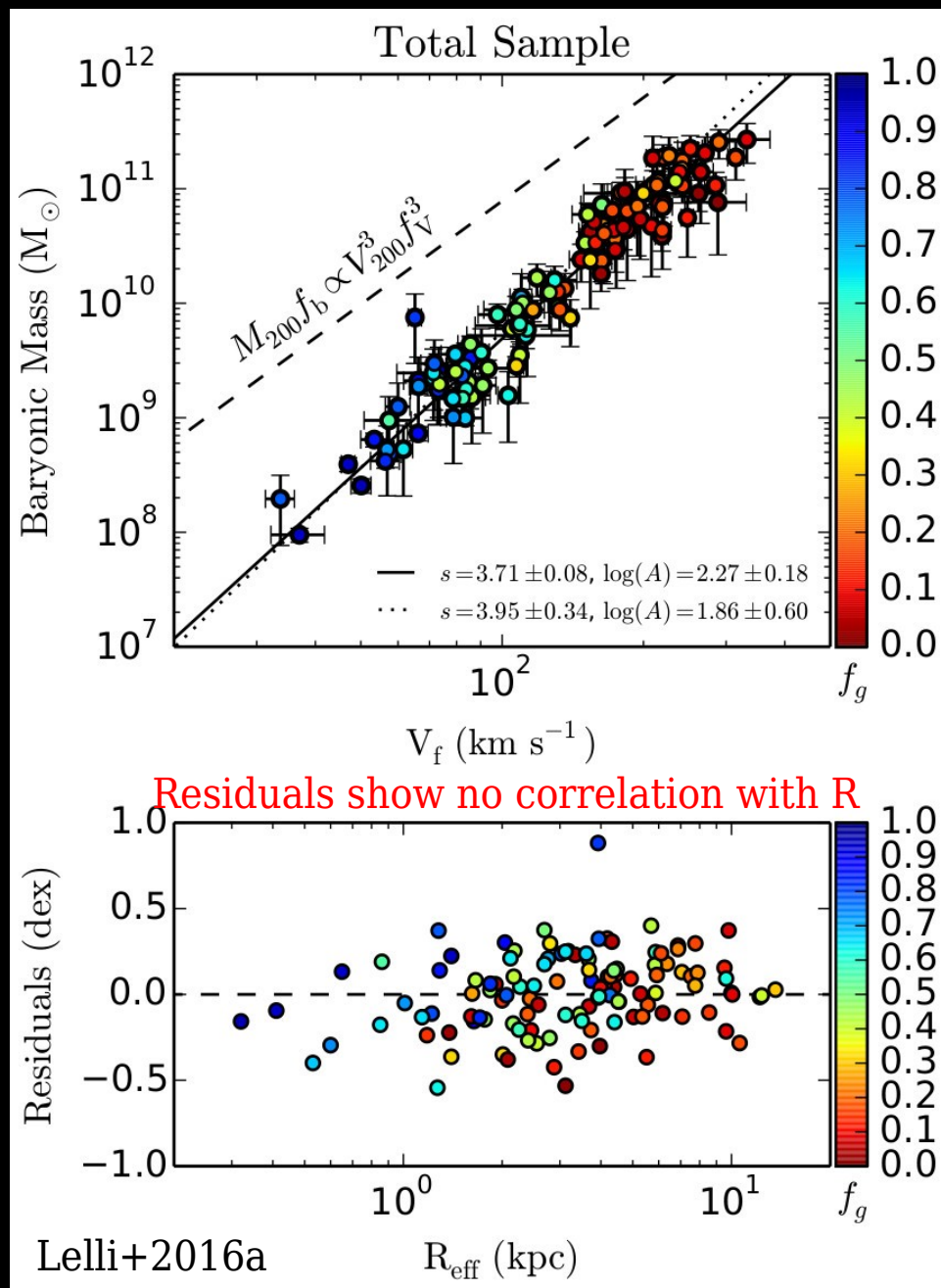
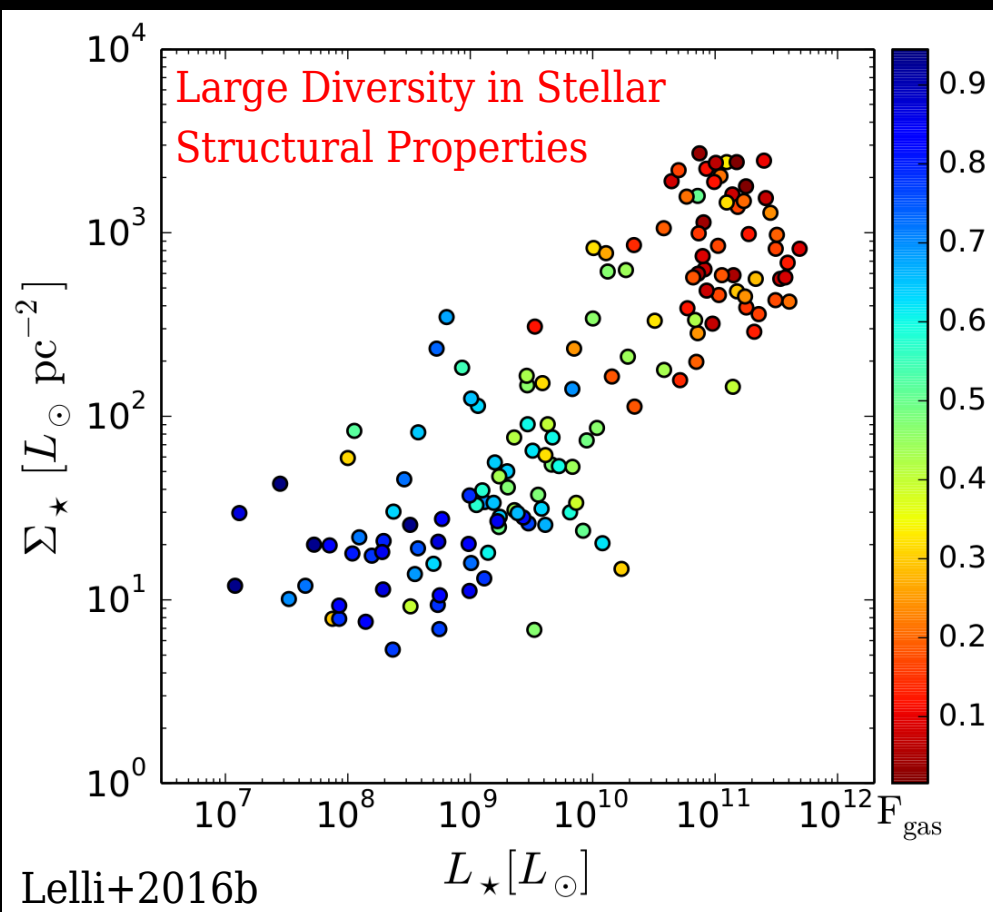
But LSB are on the same TFR as HSB!



HSBs and LSBs lie on the same BTFR



Database of 175 disk galaxies with HI interferometry and [3.6] photometry

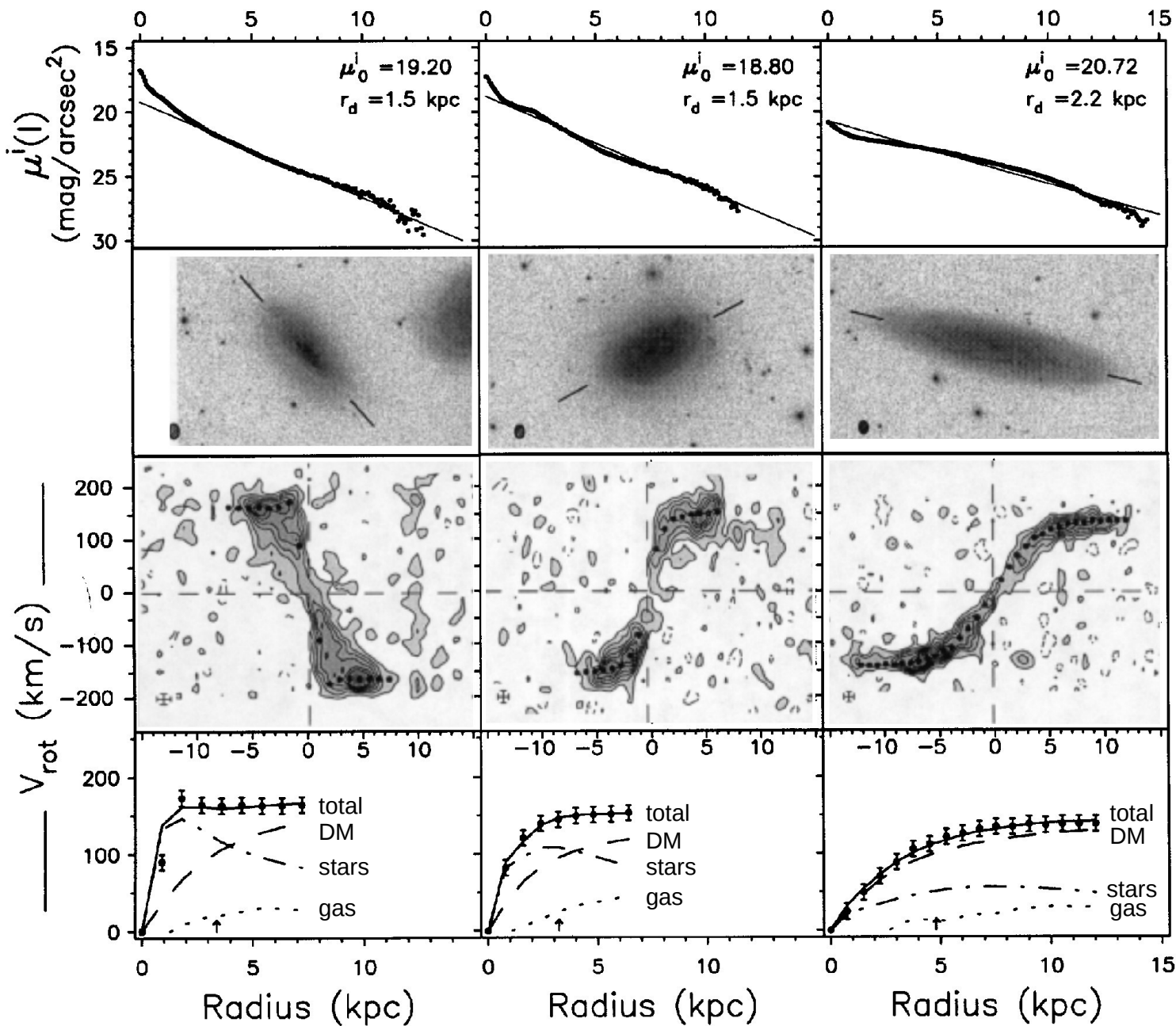


A galaxy triplet on the BTFR

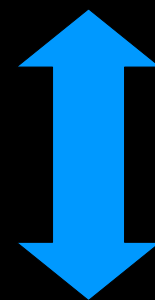
UGC 6973
HSB + Bulge

NGC 3949
HSB

NGC 3917
LSB



Same M_{bar} & V_{flat}
but different SB

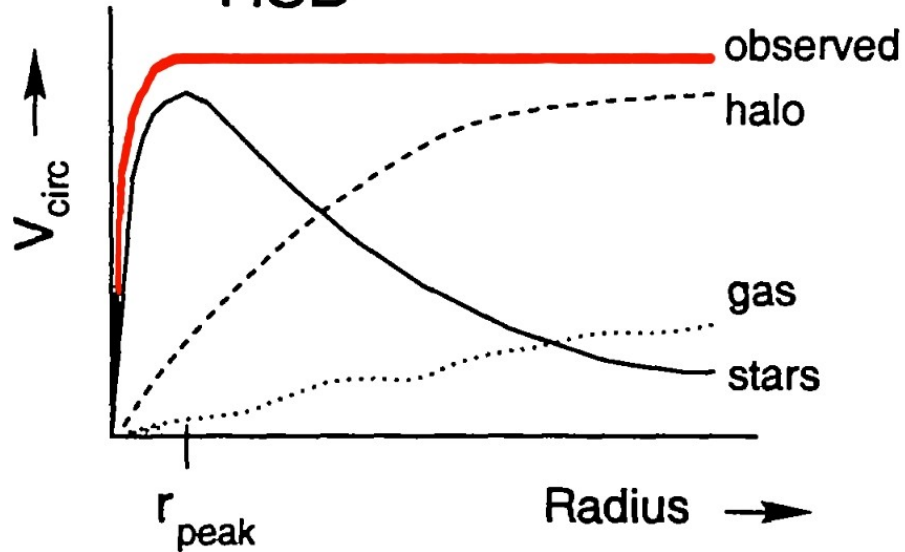


Different
Rotation Curves
& Mass Models

Tully & Verheijen (1997)

The HSB - LSB dichotomy

HSB



HSB galaxies:

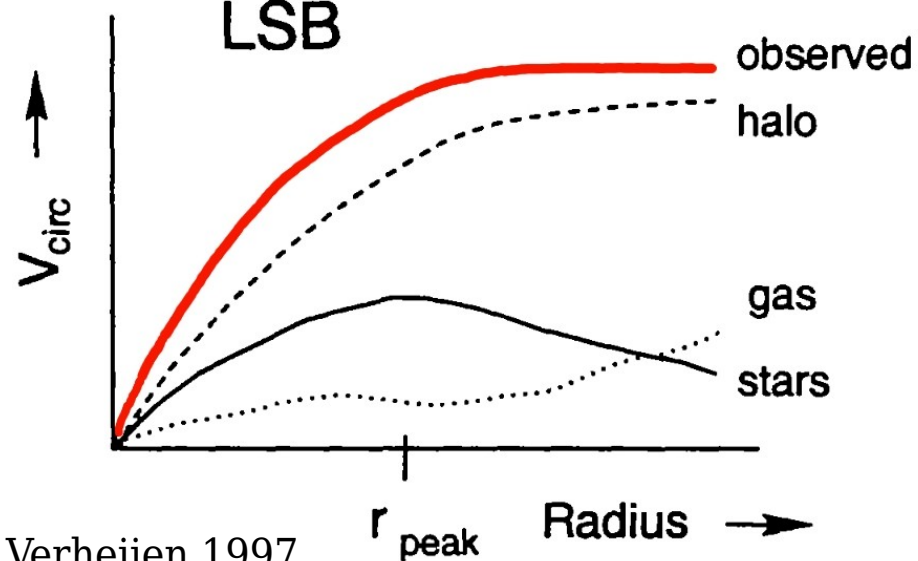
- Steeply rising rotation curves
- Maximum disk hypothesis

Realistic M_*/L .



Baryons dominate inner galaxy regions

LSB



LSB galaxies:

- Slowly rising rotation curves
- DM dominates at small R

Verheijen 1997

Deriving the Tully-Fisher relation 2.0:

$$\frac{V_{rot}^2}{R} = \frac{\alpha G M_{tot}}{R^2}$$

Deriving the Tully-Fisher relation 2.0:

$$\frac{V_{rot}^2}{R} = \frac{\alpha G M_{tot}}{R^2} \longrightarrow V_{rot}^4 = (\alpha G)^2 \frac{\sum_b M_b}{f_b^2} \quad f_b = \frac{M_b}{M_{tot}}$$

Deriving the Tully-Fisher relation 2.0:

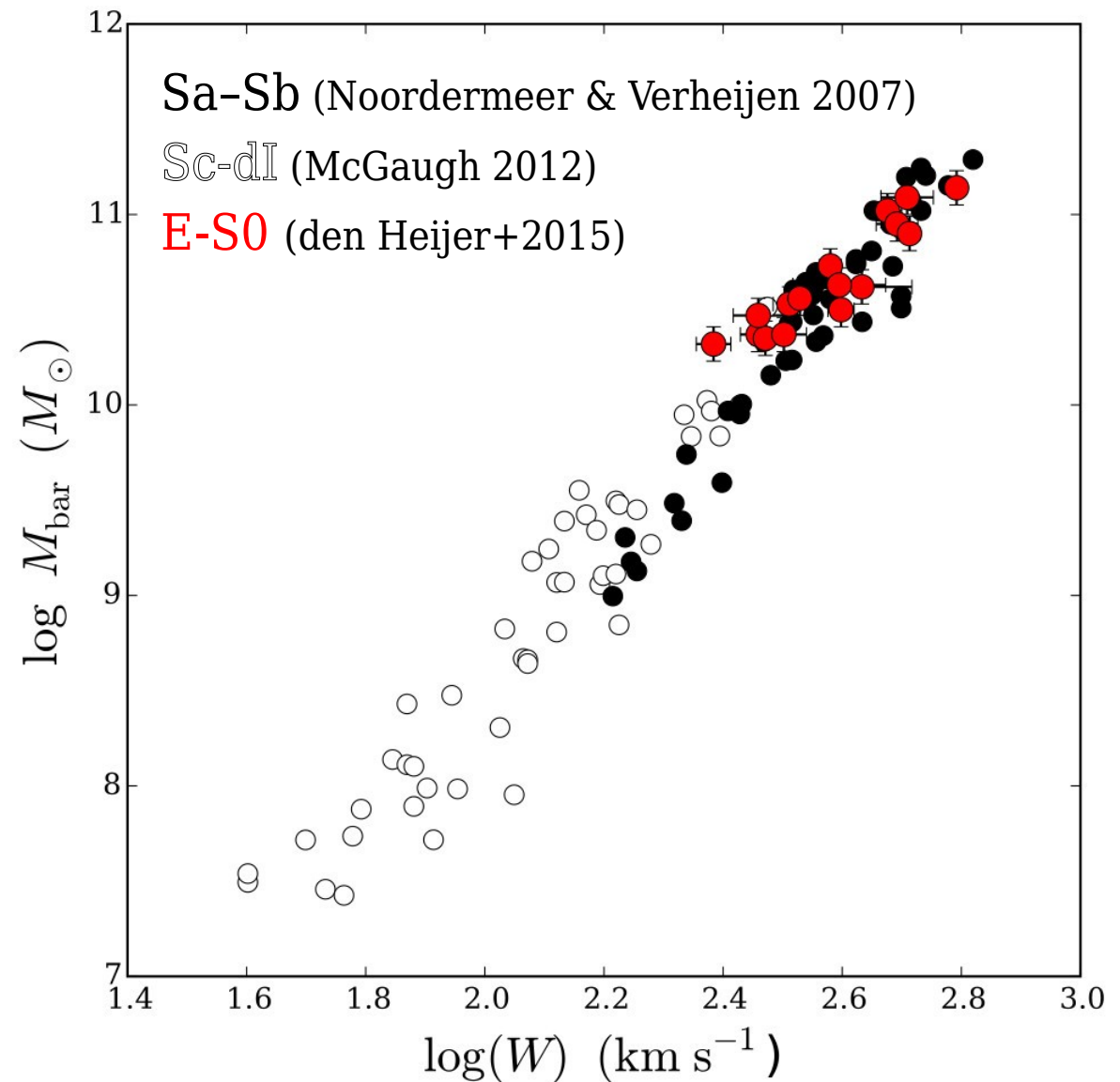
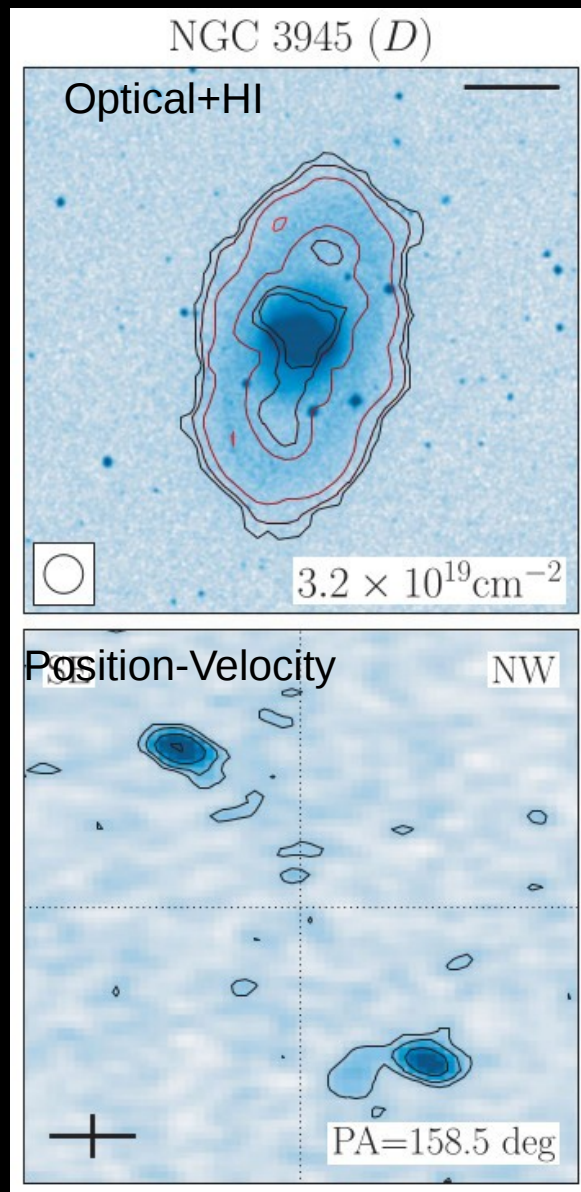
$$\frac{V_{rot}^2}{R} = \frac{\alpha G M_{tot}}{R^2} \longrightarrow V_{rot}^4 = (\alpha G)^2 \frac{\Sigma_b}{f_b^2} M_b \quad f_b = \frac{M_b}{M_{tot}}$$

The tightness of the BTFR implies that $\frac{\Sigma_b}{f_b^2} \simeq const$

Fine-tuning problem at fixed baryonic mass:

As the average baryonic surface density decreases, the DM content must increase by a precise amount.

Early-type galaxies (E and S0) follow BTFR!



ETGs with outer, extended HI discs (Serra+2012, den Heijer 2015)

3. The Tully-Fisher relation in a LCDM context

Deriving the Tully-Fisher relation 3.0:

$$(1) \quad M_{\Delta} = \frac{4\pi}{3} R_{\Delta}^3 \cdot \Delta \cdot \rho_{crit} \quad \rho_{crit} = \frac{3H_0^2}{8\pi G}$$

Cosmological definition
of dark matter halo mass
(typically $\Delta=200$)

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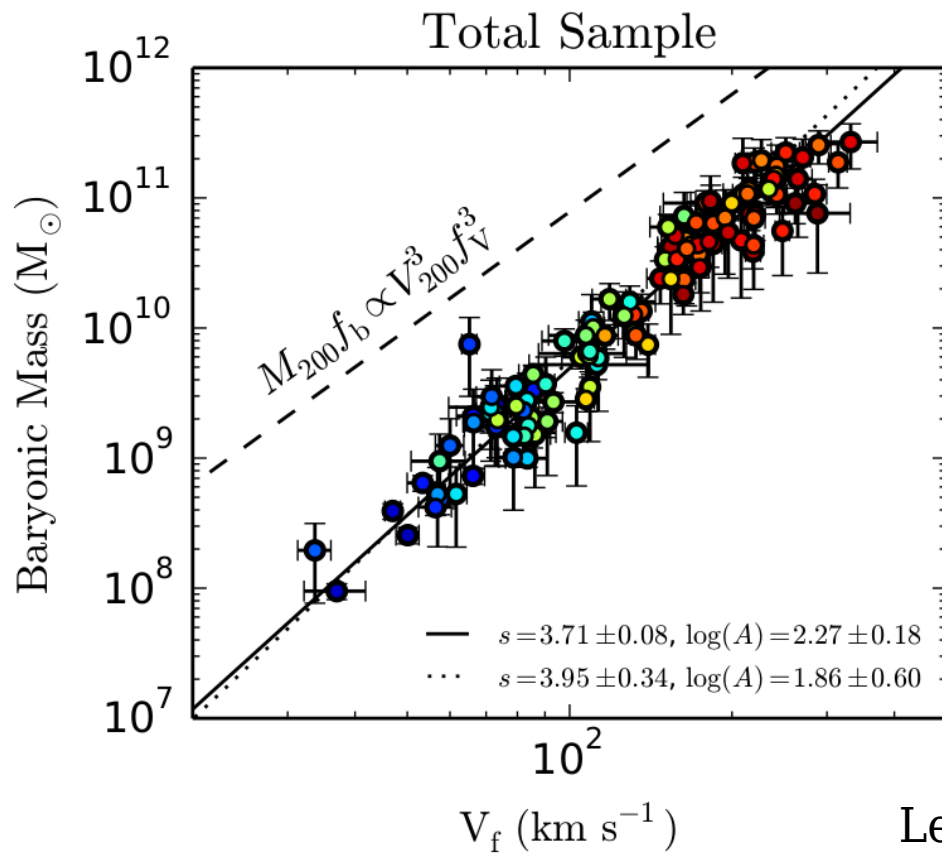
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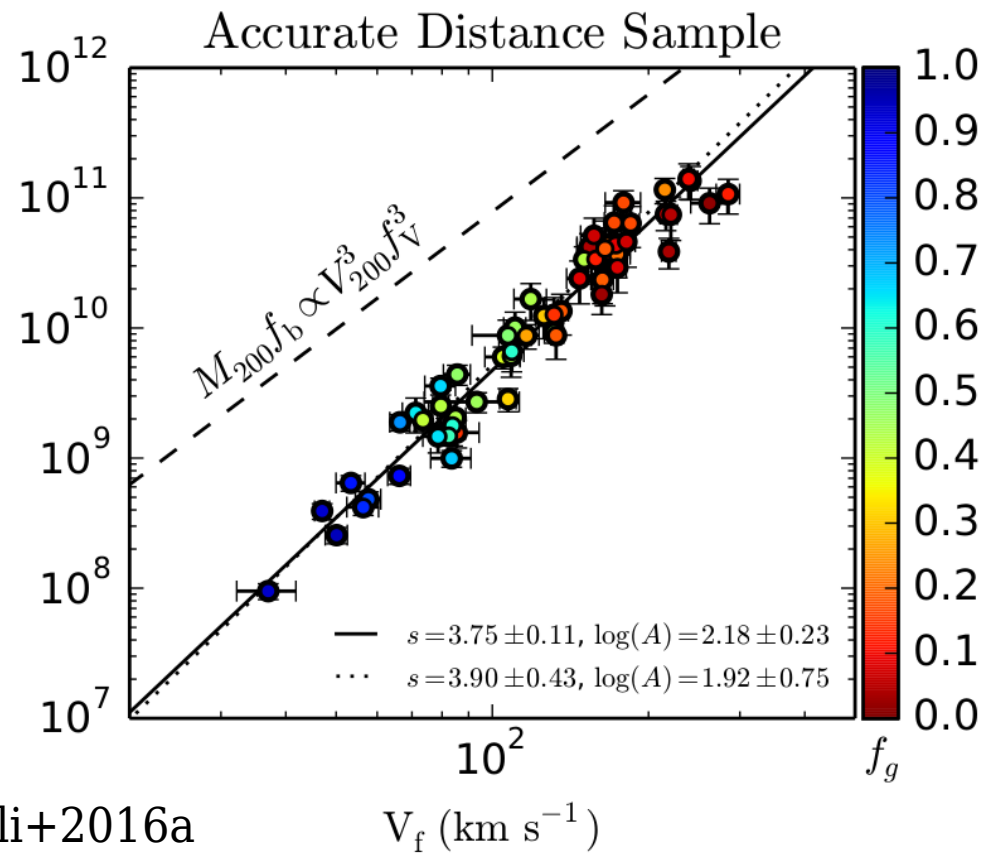
Working Hypothesis: $F_b = F_{cosmic}$ [CMB & galaxy clusters]

$F_V = 1$ [surely wrong but $O(1)$ is ok]

Baryonic Tully-Fisher Relation vs LCDM



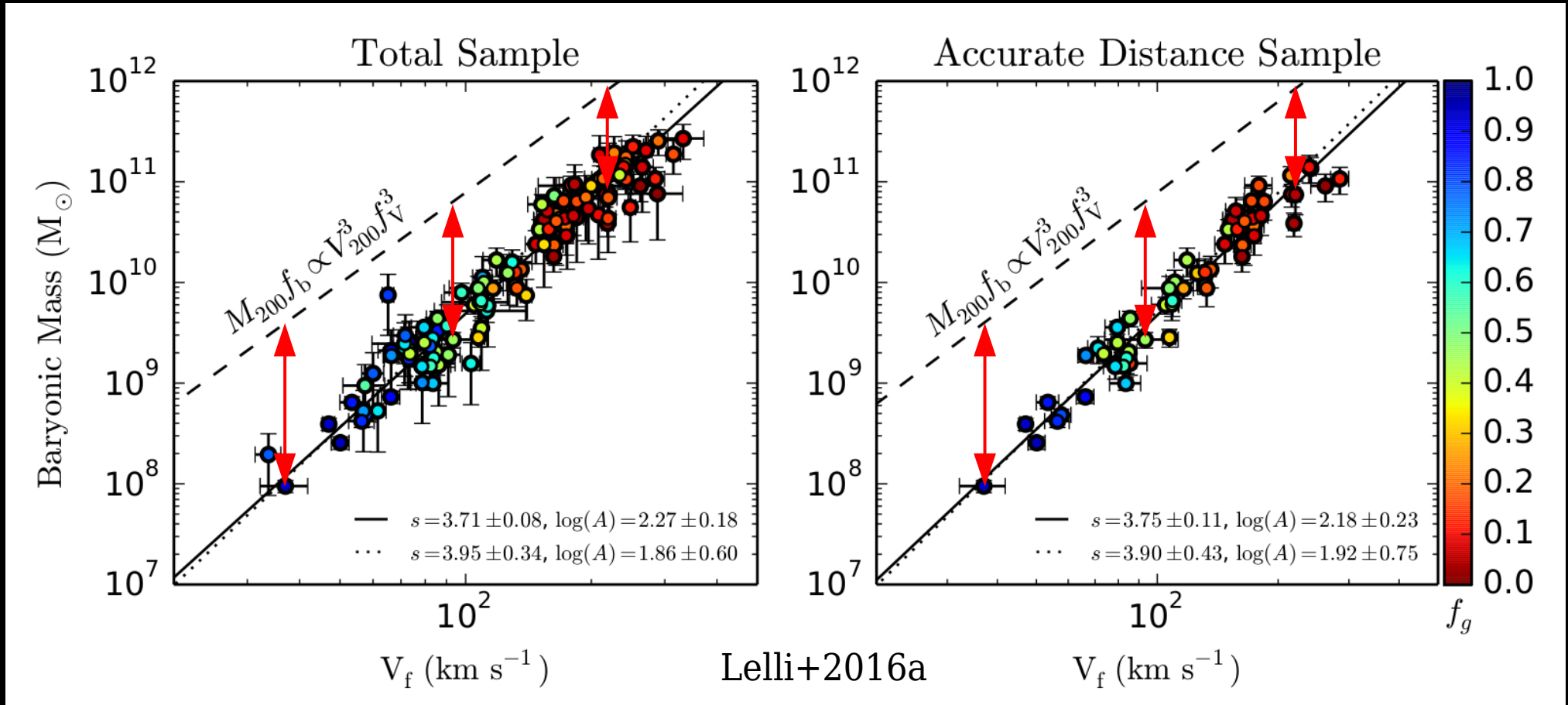
Lelli+2016a



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Baryonic Tully-Fisher Relation vs LCDM

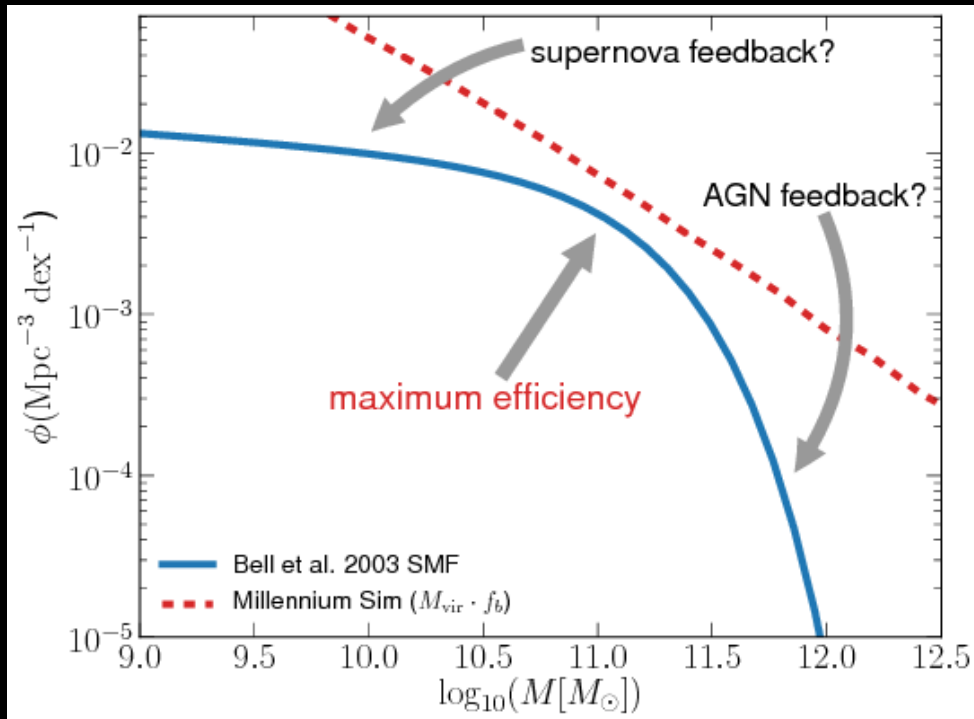


To fix normalization: $F_b < F_{\text{CMB}} \rightarrow$ missing baryons (hot gas?)

To fix slope: F_b must systematically vary with V_{flat} (or M_{200})

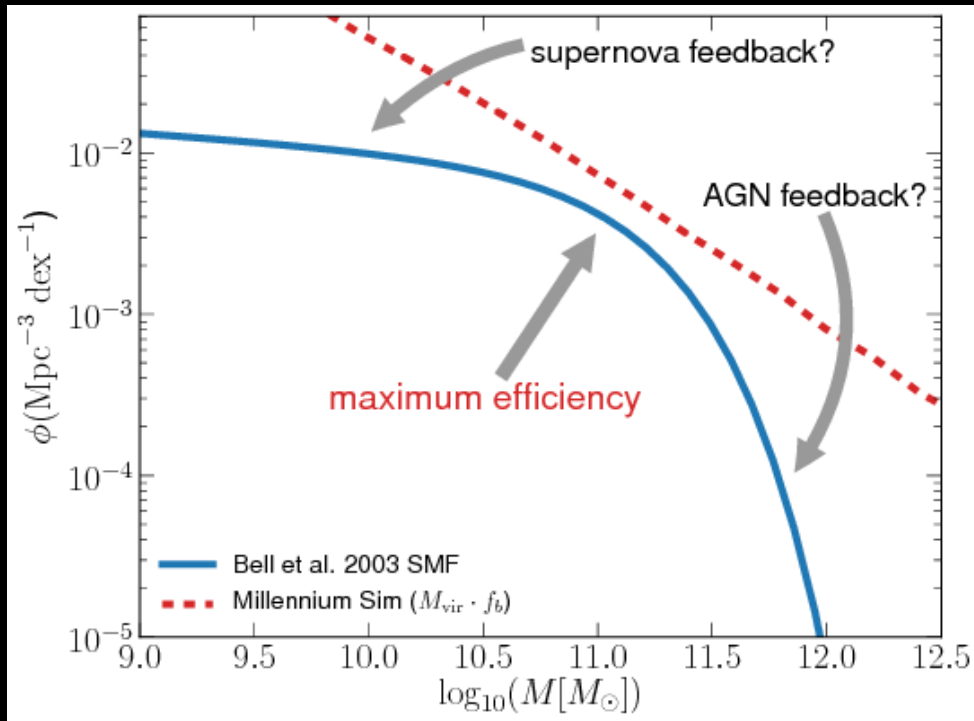
Small scatter (<25%): additional fine-tuning problem!

The Stellar Mass Function Problem



A constant M_*/M_h can't reproduce the observed stellar mass function!

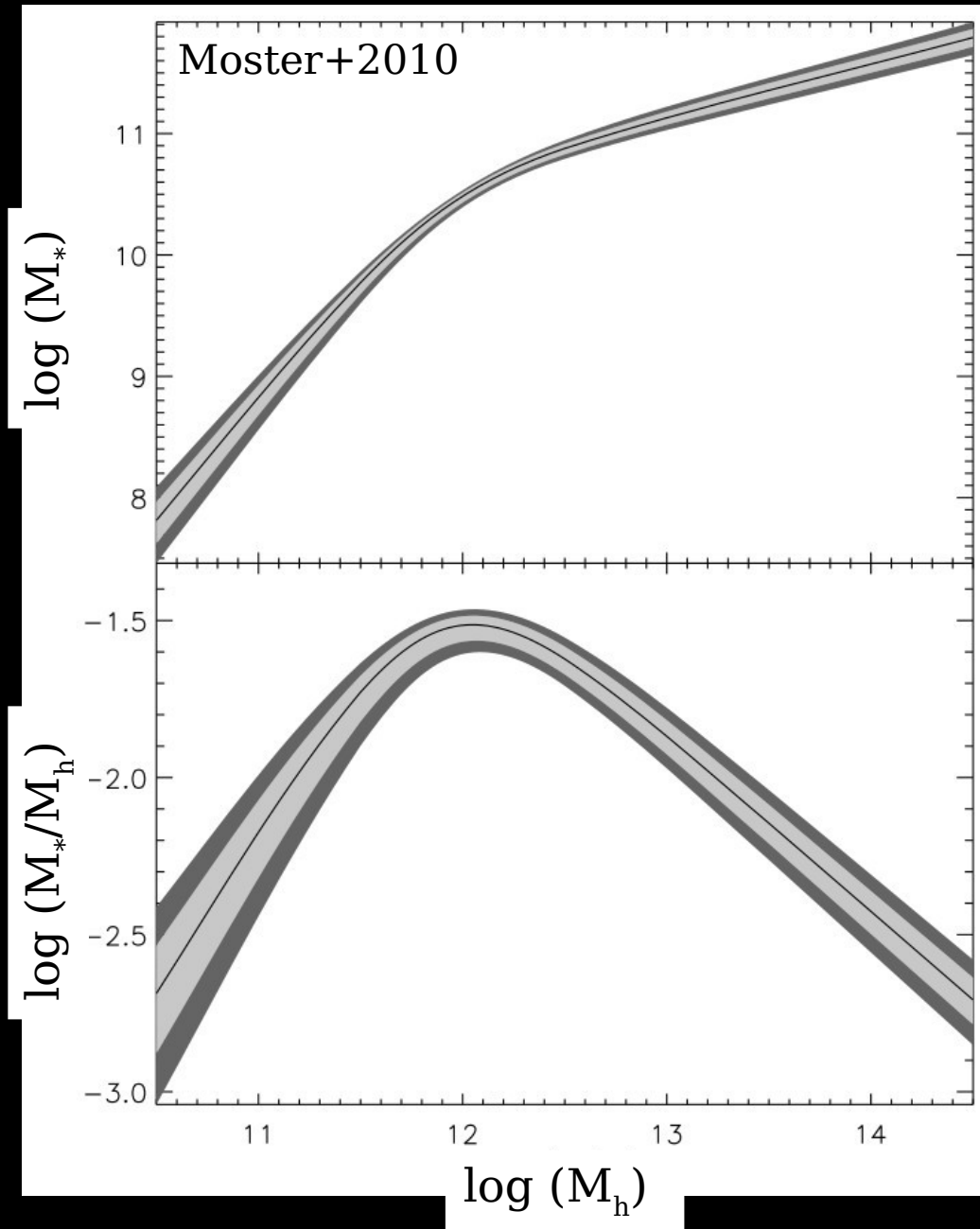
The Stellar Mass Function Problem



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Basics of Abundance Matching (AM):

- Order galaxies and halos by mass
- Assign the most massive galaxy to the most massive halo, and so on.
- Derive M_*-M_h relation



Interesting test for LCDM models:

(1) Assume M_* - M_h relation from Abundance Matching

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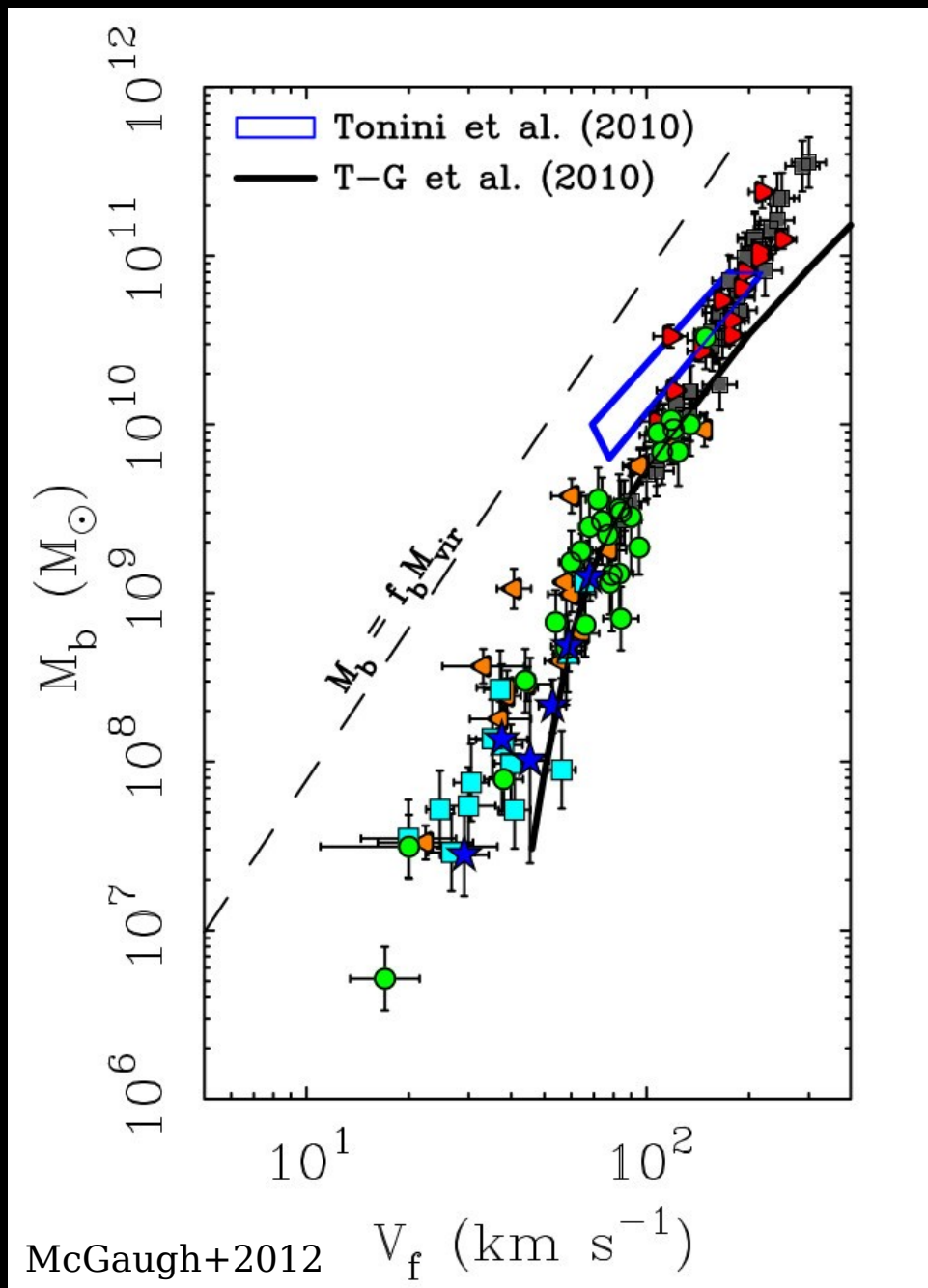
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- (2) Assume a DM halo profile (e.g., NFW)
- (3) Assume M_h - c relation of DM halos (from sims)
- (4) Model the baryonic distribution with some recipe (e.g., angular momentum partition) or even better take it directly from the data (e.g. Desmond+2018)!

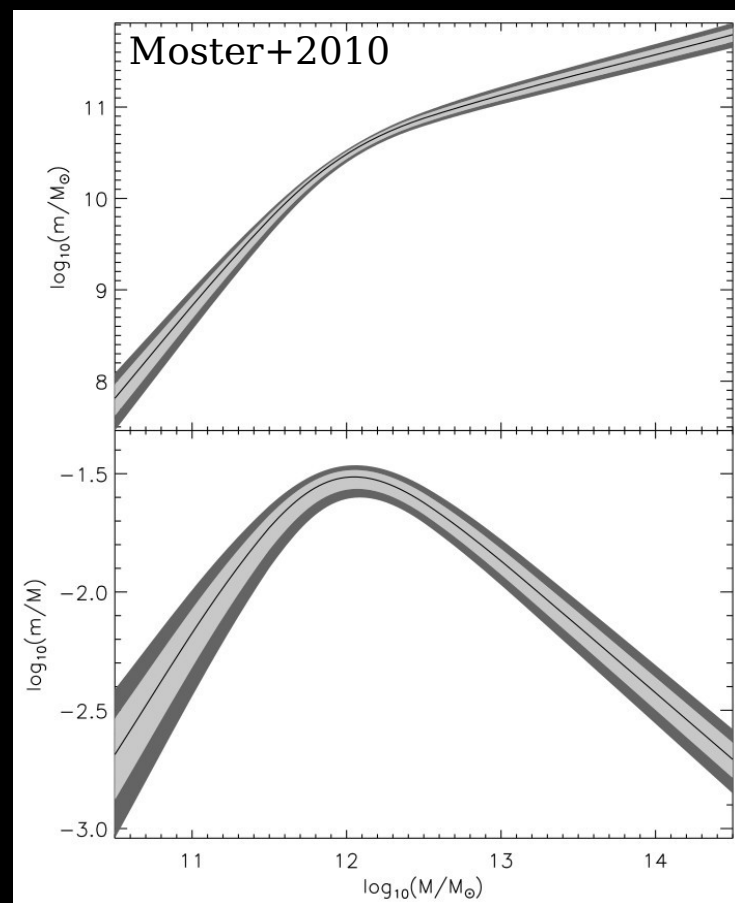
→ Calculate model rotation curves and BTFR!

Basic AM models versus Observations

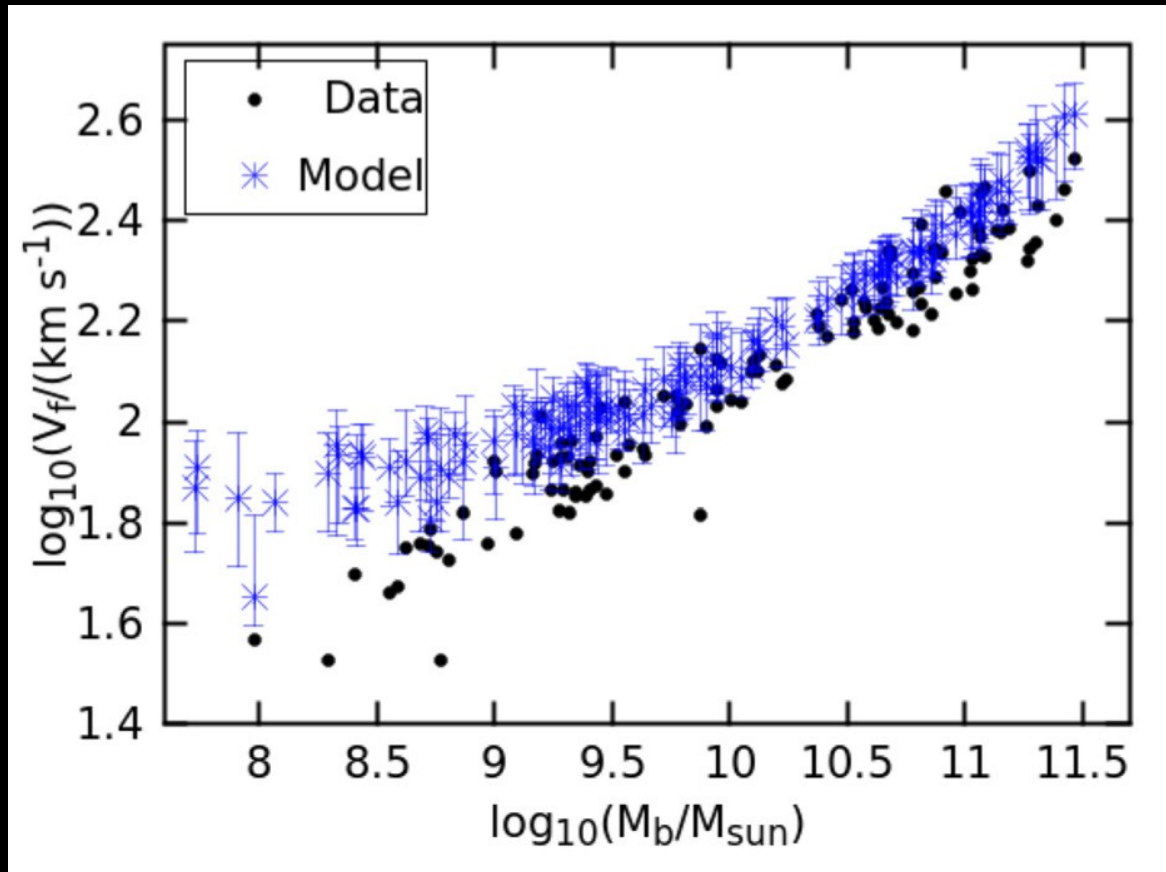


MIXED RESULTS:

- Normalization is OK: Good!
- Strong curvature: Bad!
- Unavoidable: M_* - M_h relation is **non-linear** in AM models!



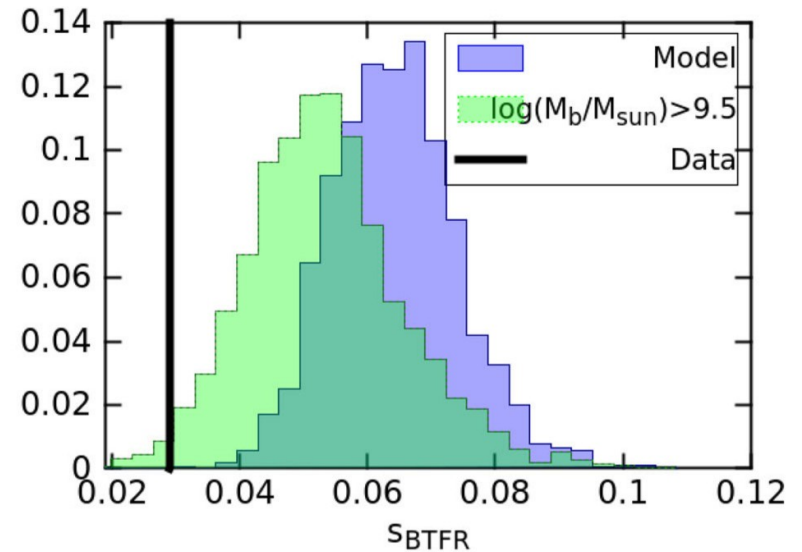
BTFR scatter is also a key test!



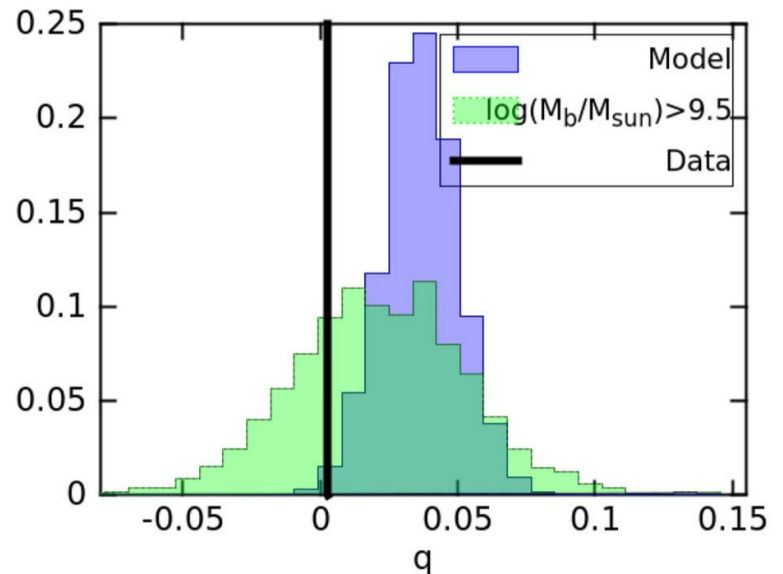
Desmond (2018):

- Abundance matching on SPARC galaxies
- Baryon distribution taken from obs.
- Differences must be due to DM halo!
- Repeat N-times to account for variance

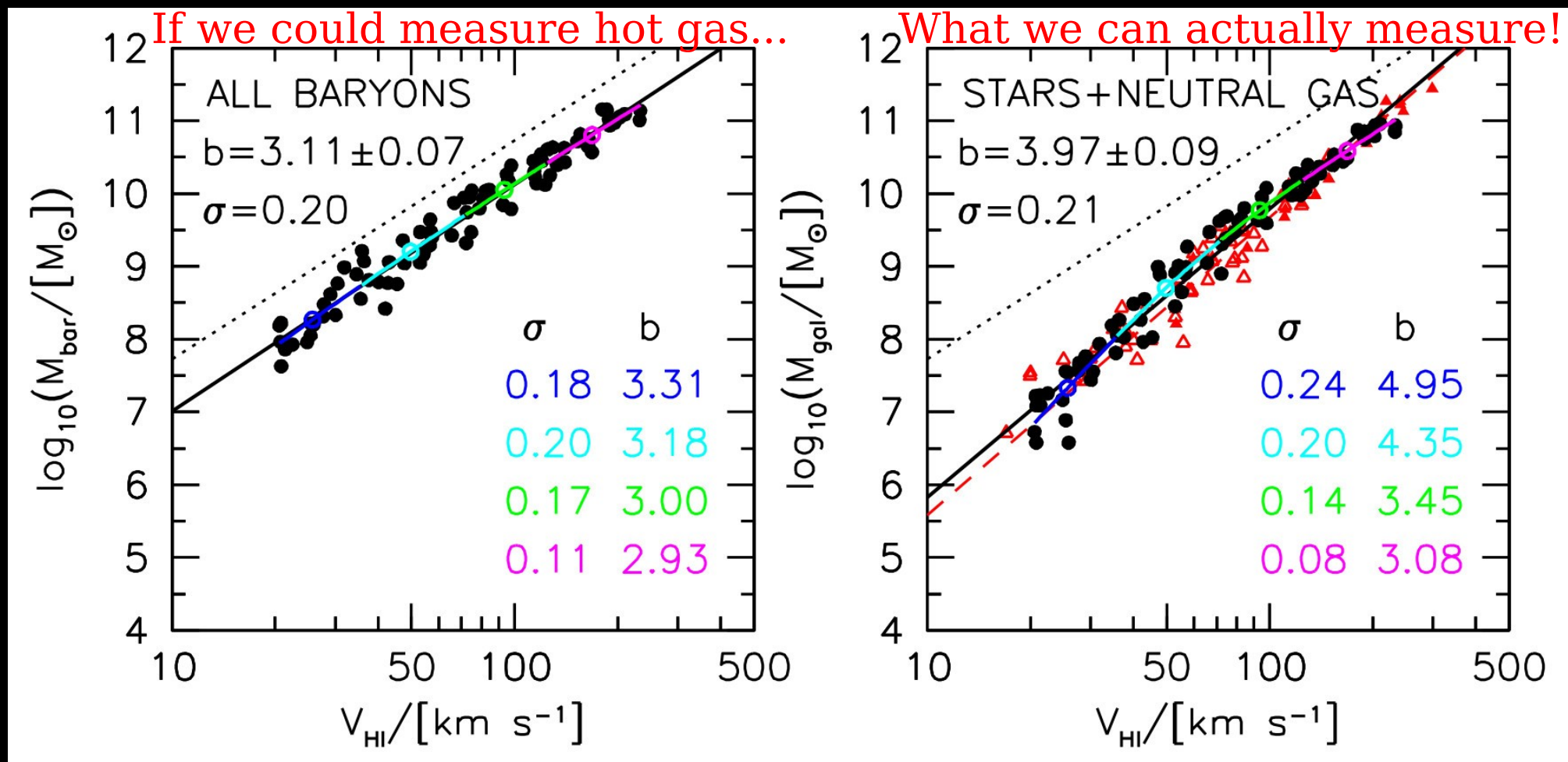
Scatter is 3.6σ too high!



Curvature is 3σ too strong!



BTFR from hydrodynamical simulations



NIHAO zoom-in cosmological simulations of galaxy formation (Dutton+2017)
BTFR curvature has almost disappeared and the scatter small.

This is remarkable... but how is this possible?

Where did the M_*/M_h scatter and the characteristic M_* go?

Messages to take home:

1. TF relation is not just a distance indicator!

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3. A blessing and a curse for LCDM models

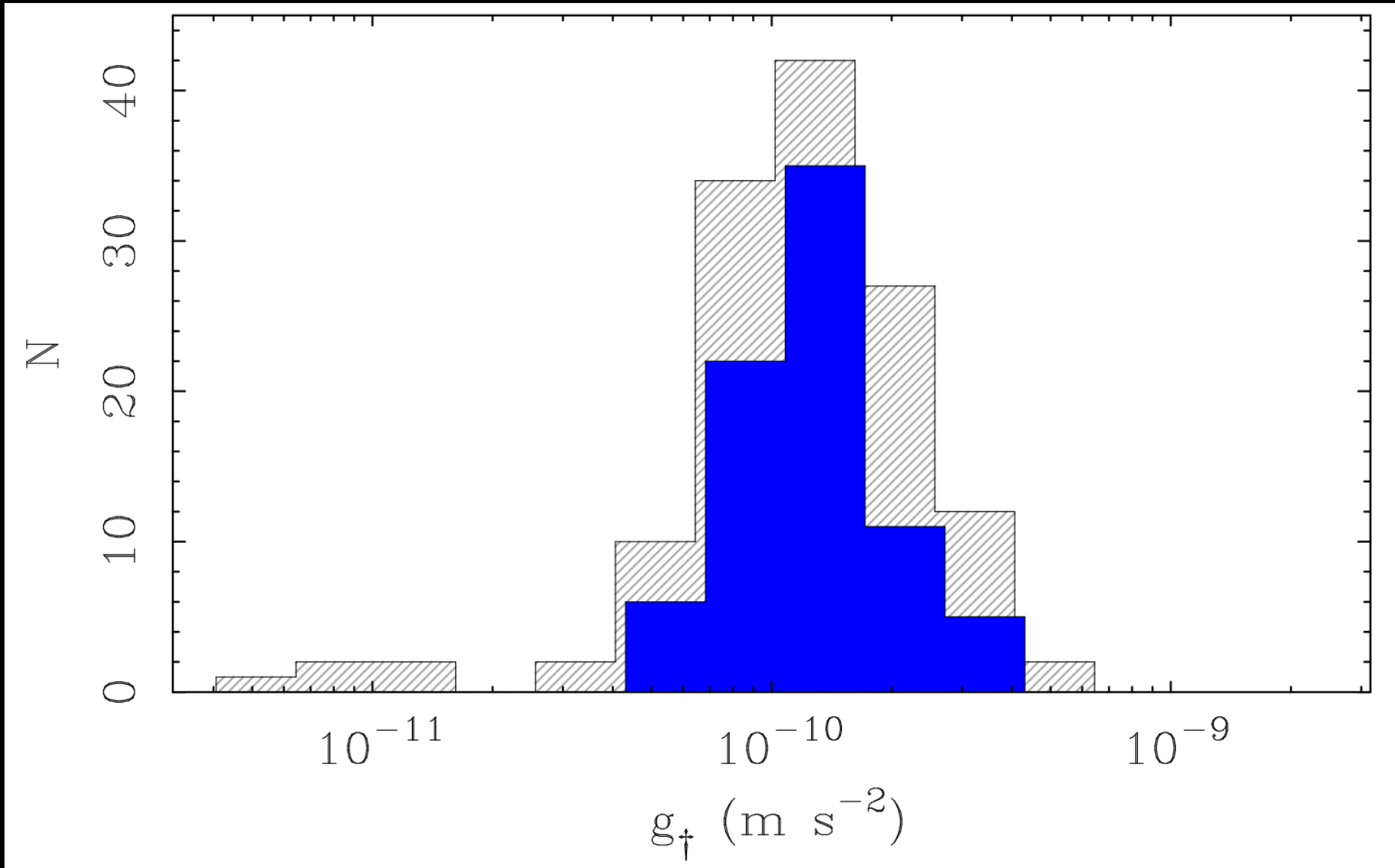
Normalization and slope are almost OK. Success of AM.

Curvature is not observed. Discrepancy with AM.

Scatter is too small. But galaxy formation is stochastic!

More Slides

Slope $\sim 4 \rightarrow$ Acceleration Scale



On dimensional grounds: $g_{\dagger} \sim V_f^4 / (GM_b)$